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ELECTRICAL ENGINEERING

Essential Theory and Typical Applications

by

FRED H. PUMPHREY

Professor of Electrical Engineering
University of Florida

SECOND EDITION

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Foreword

This book is intended for the general engineer and for engineering students who require a knowledge of electrical engineering theory and the applications of that theory in commercial practice.

The first twelve chapters review the basic theory. A fundamental knowledge of physics is assumed. Elementary calculus is used where it helps in the presentation. But in most instances an alternate explanation based on simpler mathematics is also given so that sections including calculus may be omitted if desired. Illustrative examples are given to describe quantitative relationships and the prediction of machine performance, since these factors are a part of essential theory. In the numerical examples, complicating secondary effects are usually not introduced; in a book of this kind, approximate relationships are sufficient generally but, where necessary, important secondary effects are included.

The latter part of the book discusses typical applications of the essential theory. These applications can be used to teach additional theoretical material; they also help the reader to retain his grasp of theory. Motor applications are treated from the point of view of load and power supply. The study of amplifiers as applied to strain gages and of oscillators as applied to high-frequency heating units is an innovation. While these subjects are not usually included in the elementary book, they are very important to civil, mechanical, and electrical engineers.

The author acknowledges his indebtedness to many friends and associates who have contributed to the development of the book. He appreciates the courtesy of the many companies and their representatives in supplying some of the illustrations. Dr. W. L. Everitt of the University of Illinois made helpful suggestions for the manuscript.

FRED H. PUMPHREY

Contents

Introduction

Electricity is one of the basic physical forces utilized by all types of engineers. It provides a source of heat, light, and mechanical power. Its extreme flexibility permits it to be used for measurement and control. By it intelligence is transmitted, so that construction operations may be coordinated on long bridges and huge dams. The chemical engineer finds that many of the instruments used to supervise and control his chemical processes are electrical. The metallurgical engineer uses electric furnaces to purify and heat-treat his metals and electric motors to drive the mills that fashion them. The petroleum engineer uses highly sensitive electrical instruments to locate oil fields. The architectural engineer must equip his building with a system of electric wires that provide illumination, electric power, and various types of communication. The mining engineer uses electric motors to pump water out of his mine and to hoist the ore to the surface. The ceramic engineer uses electricity to control the heat in his kilns and in some cases to supply the heat as well. The civil engineer uses electricity to measure the stresses in bridges when high-speed locomotives pass over them, and the aeronautical engineer uses the same electrical instruments to measure the stresses in the critical structural members of airplanes under severe test conditions.

These various applications are briefly reviewed and discussed in the latter portion of this book. Some of them require an understanding of only the elementary physics of electricity, but others require a considerably broader knowledge of electrical circuits, machinery, and vacuum tubes. It is impossible, therefore, to discuss most of these fields of application without a reasonably complete background of the essential theory underlying the art of electrical engineering.

In order to make a coherent presentation of both the theory and the practice, each has been treated in a separate section of this book. Thus, in the chapters on theory frequent references are made to applications, a more complete treatment of which will be found in the second portion of the book.

Although the order of presentation is not absolutely rigid, it is generally unwise for the beginning reader to attempt to study the later chapters of the theory without first mastering the earlier ones. Also, for the beginning reader it is recommended that a careful reading of the theory precede any serious attempt to read the chapters covering industrial applications. Even the beginner, however, is encouraged to read casually those portions of the latter part of the book that are of particular interest to him so that he may observe his progress in understanding as the theoretical material is covered.

CHAPTER 1

Direct-Current Circuits

Nature of matter and electricity

All matter is considered to be composed of atoms, each of which is made up of a positively charged nucleus of relatively large mass surrounded by negatively charged electrons having orbital motions quite similar to the movement of the planets about the sun. In some atoms there is a vacancy in the pattern of the outer orbital electrons, whereas in others there are one or two electrons beyond the normal stable group. When molecules of these different types become closely associated, there is a tendency for the extra electrons of one to fill in the vacancies of the other, and thus the nuclei share these electrons and form molecules of stable compounds. Not only do atoms form molecules with other atoms, but atoms may combine with atoms of the same kind to share electrons; in doing this they tend to form into a geometric pattern known as crystals. (Molecules also may form into crystals.) Such crystals may and usually do involve tremendous numbers of atoms or molecules; and, since the bond is tight, they are always associated with solid substances. In general, metals have only one or two electrons in their outer orbital group; therefore, when these form into crystals, the bond between the outer electrons and the parent nucleus becomes so loose that it is quite easy for them to migrate from one nucleus to another and thus to progress along the metal. When metals of this type are formed into wire, they may be used as electrical conductors.

Many solids (and liquids as well) have such a tight bond on the electrons that it is difficult for them to migrate; these materials are called *insulators*. Solids have a wide variation in conducting ability; but the engineer, in applying electricity to production problems, is particularly interested in those substances that are *good* conductors and *good* insulators. When such substances are used, it is possible to control the movement of electrons in order that the desired results may be accomplished.

Electrons are caused to migrate along a conductor by reason of an electric field, sometimes called an *electric pressure gradient* or *difference of potential.* The magnitude of the migration, or the flow of electrons, is dependent upon this difference of potential and upon the cross-sectional area of the conductors. The flow is measured in special units, as are also the pressure and opposition to flow. These units must be defined before a real study of direct-current circuits is possible.

The ampere. The unit of electric flow is called the ampere. Many physics texts very properly define the flow of electric current in terms of its magnetic effect, in order that satisfactory relations between electric and magnetic units be obtained. Since the magnetic effects are difficult to measure with high accuracy, the equivalent statement in terms of electrochemical effect is the practical basis of electrical current standards. Thus *the ampere is specified as the constant current that will deposit silver from an electrolyte at the rate of 0.001118 gram/sec.*

Electric currents taken by common electrical appliances are approximately as follows:

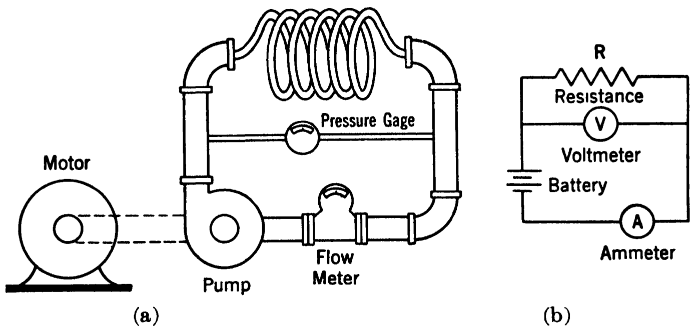
100-w lamp	0 9 amp
600-w toaster	5. amp
½-hp motor	3 7 amp

The coulomb. The unit of electric quantity (or charge) is called the coulomb. *It is the charge that will pass a given point in the circuit each second when a current of 1 amp is flowing.* As indicated above, electric flow consists of the movement of electrons and ions. It would require the movement of approximately 6,300,000,000,000,000,000 (6.3×10^{18}) electrons per second to produce 1 amp. This is a large unit of charge and is seldom used in elementary calculations.

Some idea of the size of the coulomb may be gained from the fact that two charges of 6.8 millionths of a coulomb, when placed 1 ft apart, will have a repelling force of 1 lb acting between them.

The ohm. The unit of resistance to electric flow is called the ohm. It may be defined as *the resistance that will develop 1 joule of heat per second when 1 amp flows through it.* Since this is again difficult to measure or standardize, it is also specified as the resistance that, at a temperature of 0° C, is offered to the flow of current by a column of mercury of uniform cross-section, of a length of 106.3 cm., and of a mass of 14.45 grams. The magnitude of the cross-section so specified is approximately 1 mm^2.

The volt. The unit of electrical pressure, or potential difference, is the volt. *The volt is the potential difference that will cause a current of 1 amp to flow through a resistance of 1 ohm.* A dry cell has a potential difference of about 1.5 v, a three-cell lead storage battery has a potential of 6.6 v, and the usual domestic electrical circuit has a potential difference of 120 v.



Pressure (lb/in.^{2})	Flow (gal/min)	Pressure / Flow	Electric pressure (volts)	Electric flow (amperes)	Volts / Amperes
20	1.6	12.5	8.0	0 92	8 7
35	2 8	12 5	12.0	1.38	8 7
75	6 0	12 5	16.8	1 93	8 7
100	8.0	12 5	22 4	2 57	8 7
150	12 0	12 5	47 0	5 40	8 7

FIG. 1-1. Analogy between Hydraulic and Electric Circuits.

Ohm's law

In order to obtain a better understanding of electrical circuits, reference will be made to the old, but still useful, fluid analogy. In Fig. 1-1 a motor-driven pump is shown. This pump is used to circulate oil* through a cooling coil of small copper tubing. A gage is connected to the ends of the copper tubing to measure the difference in pressure across the coil, and a flow meter is inserted in the pipe to measure the rate at which the oil flows through the tubing.

If the speed of the pump is changed and if readings of the pressure gage and flow meter are taken at each pump speed, a

* Oil is used instead of water because it is a liquid of high viscosity and obeys Ohm's law of hydraulic circuits.

set of data will be obtained, as shown in the table of Fig. 1-1. At each pump speed the pressure divided by the flow gives the same result. In this case that value is 12.5. The flow at any pressure can, therefore, be found by dividing the pressure by 12.5. If the flow is desired at some pressure other than those tested, it might also be obtained by dividing the pressure by 12.5.

Example. What is the flow at 50-psi pressure? According to the relation stated above,

$$\text{flow} = \frac{\text{pressure}}{12.5} = \frac{50}{12.5} = 4 \text{ gallons per minute}$$

pressure being in pounds per square inch. The constant 12.5* is characteristic of this particular size and length of tubing and so can be called the resistance of the coil of tubing.

To the right of this simple hydraulic circuit is shown a similar electric circuit. A battery supplies the electric pressure, or potential difference, that causes an electric current to flow through a coil of copper wire indicated diagrammatically as R. The meter used to measure the electric potential in volts is called a *voltmeter*. The meter used to measure the current in amperes is called an *ammeter*. If taps are arranged on the battery so that different voltages may be applied to the coil of wire, then a set of readings of volts and corresponding amperes can be made. These readings would be comparable to the pressure and flow readings of the hydraulic circuit. In the electric circuit the voltmeter reading divided by the ammeter reading is always 8.7, and this constant is called the *resistance*. It is seen by this analogy that in the electric circuit with a fixed resistance it is also possible to predict the current flow with any given voltage. For instance, if the current corresponding to 65 v were desired, then

$$I = \frac{E}{8.7} = \frac{65}{8.7} = 7.5 \text{ amp}$$

E being in volts. This value of 8.7 is a characteristic of the wire and is called the resistance. It is measured in the unit that has previously been defined as the ohm. The formal statement of the relationship observed above is as follows:

The current in amperes is equal to the potential difference in volts divided by the resistance in ohms.

---

* This constant depends also upon the viscosity of the fluid. In electricity, the variable corresponding to viscosity does not occur.

This statement is known as *Ohm's law* and is the basis for a large portion of electrical circuit theory. It may be expressed mathematically in the three forms below:

$$I = \frac{E}{R}, \qquad R = \frac{E}{I}, \qquad E = RI$$

where I is current in amperes, E is potential difference in volts, and R is resistance in ohms.

A word of caution should be given at this time, for although this is the general rule of behavior of electrical circuits, there are many exceptions. Many of these cases of unusual behavior provide the basis of operation of important commercial equipment.

Exercise 1-1. An electric soldering iron takes 3.8 amp from a 125-v circuit. What is the resistance?

Exercise 1-2. An electric oven has a resistance of 14 ohms. How much current will it draw when connected to a 220-v circuit?

Electric power

The passage of current through a conductor having resistance is always associated with a generation of heat. The relation between the current, voltage, and resistance of the circuit and the conversion of electric energy into heat are important elements in the study of electric circuits.

Referring again to the hydraulic circuit of Fig. 1-1, it is known that, for constant flow, the rate at which energy is put into the circuit by the pump will be doubled if the pressure is doubled. It will also be doubled if the pressure remains constant and the flow is doubled. A similar variation in power exists in the case of the electric circuit. The power, or rate of converting energy into heat in a resistance, may be said to be directly proportional to the product of the current and the voltage. Expressed mathematically, this is

$$P = E \times I$$

P being expressed in watts, E in volts, and I in amperes. This statement may then be used as a definition of a watt. *The watt is the rate at which electric energy is being supplied when a current of 1 amp is flowing at a potential difference of 1 v.*

Several additional equations for power may be derived from the above statement by the use of Ohm's law. These equations are very useful when the information supplied is not given in volts and amperes. They are:

$$P = E \times I = IR \times I = I^2R \qquad (\text{since } E = IR)$$

$$P = E \times I = E \times \frac{E}{R} = \frac{E^2}{R} \qquad \left(\text{since } I = \frac{E}{R}\right).$$

Since power is the rate at which energy is being transferred, the total energy is the product of the power and the time. Thus, a small unit of energy is the *watt-second*, or *joule*. The more common unit, however, is a much larger one known as the *kilowatt-hour*. This unit specifies an energy equivalent to 1000 w, or 1 kw, continued over a period of one hour. It is this unit that is the basis of most of the bills for electric energy issued by the power companies.

Example. An electric toaster requires 4.5 amp at 115 v. (a) What is the power requirement for this toaster in watts? (b) If electric energy costs 3.5 cents per kwh, and the toaster is used 16 hours during the month, what will be the monthly cost?

Solution:

$$\text{Power} = EI = 4.5 \times 115 = 517.5 \text{ w}$$

$$\text{Energy} = \text{kw} \times \text{hours} = \frac{517.5}{1000} \times 16$$

$$= 8.26 \text{ kwh}$$

$$\text{Cost per month} = 8.26 \times 3.5 = 29¢.$$

Exercise 1-3. A salt bath for heat-treating steel is estimated to require 6 kw. What current will be required if the voltage is 220? If electric energy costs 2¢ per kwh, what will it cost to operate per 10-hour day?

Exercise 1-4. A 20-gal domestic water heater has a 1000-watt heater unit. The voltage is 120. (a) What current is drawn? (b) How long will it take to raise the temperature 40° F?

Fuses. One of the important ways in which the heating effect of the electric current is used is the insertion into the circuit of a resistance unit with small current-carrying capacity, so that when the current goes beyond a certain predetermined amount, the resistor is burned up and opens the circuit. This resistance unit is called a *fuse* and is used to protect other and more expensive equipment from harmful effects when the current becomes too large. Fuses are of many different types and range in size from a few milliamperes* up to hundreds of amperes. Since they are placed in a circuit to protect the

* A milliampere is one thousandth of an ampere.

equipment, they should not be replaced by larger fuses or by heavy conductors, because they were specifically designed for the purpose of opening the circuit under overload conditions. Oversize fuses or solid jumpers defeat the purpose of the fuses and permit operation at overload with consequent damage to equipment.

Rating of resistors

Resistors play such an important part in the correct functioning of electrical equipment that it is important to know something of their limitations. Since they tend to heat up with current flow, they will normally operate above the temperature of their surroundings. Each resistor will, in fact, have a maxi-

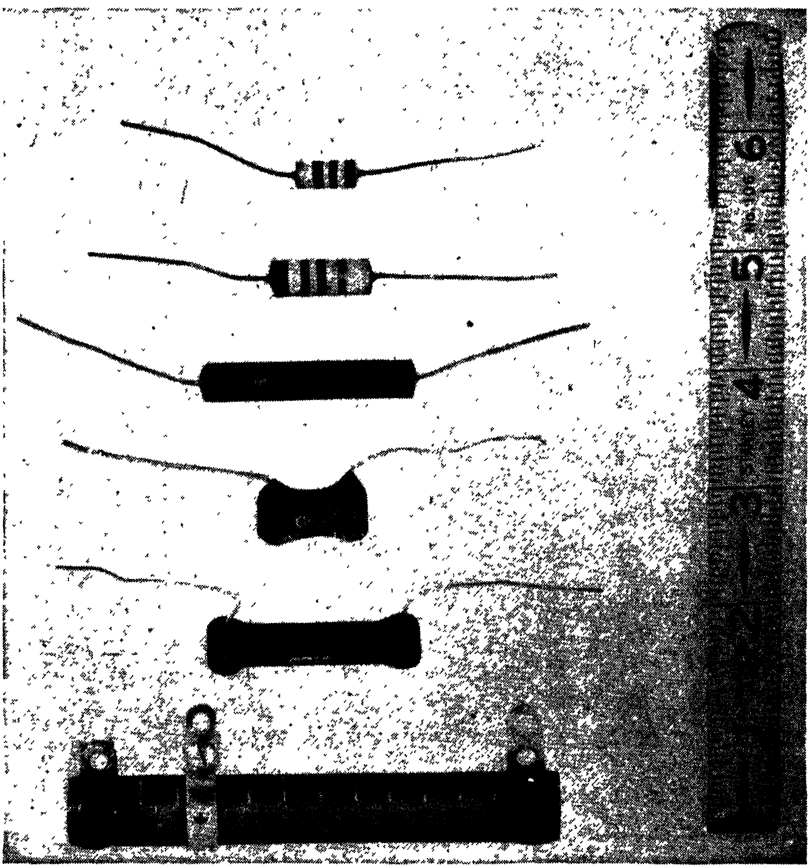


FIG. 1-2a. Resistor Types Used in Electronic Controls.

mum temperature to which it may be raised without damage. It will reach this temperature when a specified current flows or when a specified power is developed within it. As a result, resistors have maximum allowable wattage or current ratings. If these are exceeded, the resistances will probably be damaged and will no longer operate satisfactorily.

Figure 1-2 illustrates the wide variation in size and the corresponding variation in the important characteristics of resistors: (1) resistance and (2) current-carrying or watts-dissipating capacity. The top resistor of Fig. 1-2a might have a resistance of 1,000,000 ohms, but its current-carrying capacity would probably be limited to $\frac{1}{1000}$ amp. That is, it would be rated as a 1-w resistor. In contrast, the resistor in Fig. 1-2b has a resistance of only 2 ohms, but its current-carrying capacity is 2000 amp. Thus it would be rated as an 8,000,000-w, or 8000-kw, resistor.

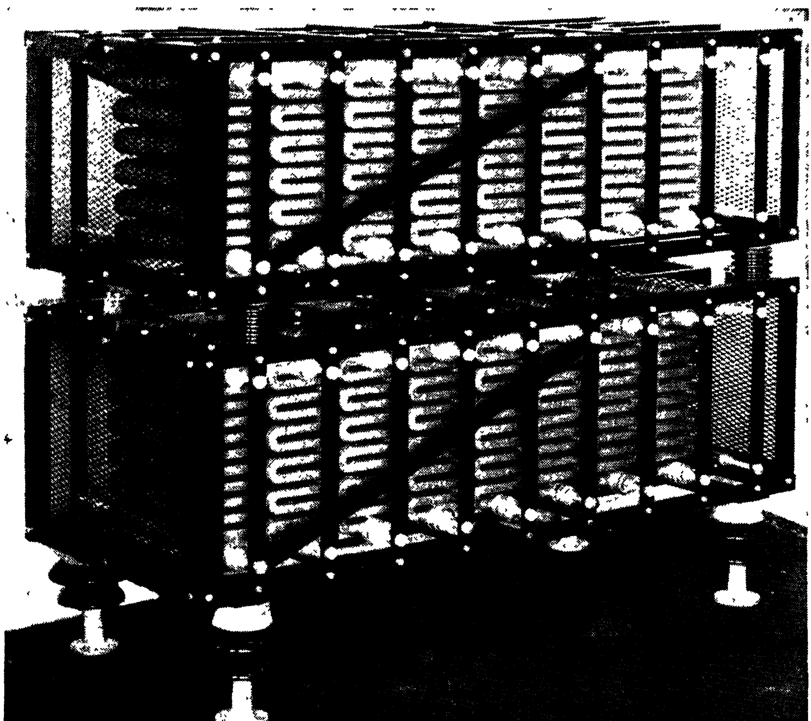


FIG. 1-2b. Indoor Cast-Iron-Grid Neutral Grounding Resistor, Rated 2000 amp, 4000 v, for 1 min (approximately 52 in. high).

Exercise 1-5. Inspect in the laboratory resistances of widely varying value with ratings from 1 w to 5 kw.

Exercise 1-6. A 5-kw resistor has a resistance of 12.5 ohms. What is the maximum current it will carry?

Series circuits

Electrical conductors may be connected following one another so that any current flowing through one must flow through the other. This is shown in Fig. 1-3. When circuits are connected in this manner, the resistances are said to be connected in series. The combined, or equivalent, resistance of R_1 and R_2 connected as in Fig. 1-3 is

$$R_{\text{total}} = R_1 + R_2$$

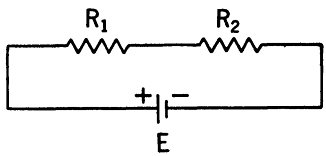


FIG. 1-3. Series Resistances.

The electrical pressure across R_1, when added to the electrical pressure across R_2, will equal the total pressure.

According to Ohm's law, the pressure or voltage across R_1 is R_1I and the voltage across R_2 is R_2I. Since the current is the same in both resistances, the voltage across the individual resistances will be proportional to their resistances. Also, the proportion of the total voltage across R_1 will be $R_1/(R_1 + R_2)$. This relationship is used many times in electrical instruments. Such a combination of resistances to give a reduced voltage is known as a *potentiometer* or *voltage divider*.

Example. A resistor of 20,000 ohms is connected in series with a resistor of 5000 ohms across a 120-v d-c circuit. (a) What current will flow? (b) If the voltage across the 5000-ohm resistor is used to control a gas triode, what would this voltage be?

Solution: The equivalent resistance of the two resistors in series

$$R_t = 20{,}000 + 5000 = 25{,}000 \text{ ohms.}$$

The current will be, by Ohm's law,

$$I = \frac{E}{R} = \frac{120}{25{,}000} = 0.0048 \text{ amp.}$$

The voltage across the 5000-ohm resistor is

$$V_{\text{5000-ohm resistor}} = V_{\text{total}} \times \frac{R_2}{R_1 + R_2}$$

$$= 120\,\frac{5000}{25{,}000} = 24 \text{ v} \qquad (\textit{Ans.})$$

Exercise 1-7. The soldering iron of Exercise 1-1 is too hot. A resistor of 5 ohms is connected in series with the soldering iron. (a) What current is taken from the line? (b) What percentage of the original power is now dissipated in the iron?

Exercise 1-8. It is desired to reduce the oven heat in the oven of Exercise 1-2 by 10 per cent. (a) What additional resistance must be inserted? (b) What current will it carry?

Parallel circuits

Resistors in electric circuits may be connected in parallel as shown in Fig. 1-4. When resistors are connected in this man-

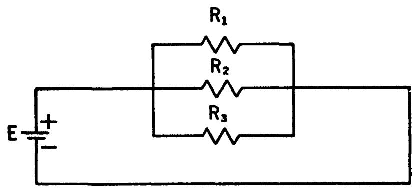


FIG. 1-4. Parallel Resistances.

ner, it is seen that the voltage is impressed across each resistor just as if the other resistor were not there. The current in each resistor is determined by Ohm's law:

$$I_1 = \frac{E}{R_1}, \qquad I_2 = \frac{E}{R_2}, \qquad I_3 = \frac{E}{R_3}.$$

The total current in the circuit is the sum of the currents in the individual resistors, so that

$$\begin{aligned} I_{\text{total}} &= I_1 + I_2 + I_3 \\ &= \frac{E}{R_1} + \frac{E}{R_2} \quad \frac{E}{R_3} \\ &= E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right). \end{aligned}$$

The quantity $1/R$ is a constant called the *conductance* and is indicated by the symbol G. It is that characteristic of a resistor which, when multiplied by the voltage, gives the current. The unit of conductance is called the *mho*. This can be recognized as *ohm* spelled backward, and the name was chosen to be a reminder that the mho is the reciprocal of the ohm.

Example. Determine the equivalent resistance of the following four resistors connected in parallel.

$$R_1 = 20 \text{ ohms} \qquad R_2 = 25 \text{ ohms}$$
$$R_3 = 12 \text{ ohms} \qquad R_4 = 8 \text{ ohms.}$$

Solution:

$$G_1 = \tfrac{1}{20} = 0.050 \text{ mho} \qquad G_2 = \tfrac{1}{25} = 0.040 \text{ mho}$$
$$G_3 = \tfrac{1}{12} = 0.084 \text{ mho} \qquad G_4 = \tfrac{1}{8} = 0.125 \text{ mho.}$$

Total conductance is

$$G_{\text{total}} = G_1 + G_2 + G_3 + G_4 = 0.299 \text{ mho.}$$

Equivalent resistance is

$$R_{eq} = \frac{1}{0.299} = 3.34 \text{ ohms} \qquad (Ans.)$$

Exercise 1-9. Two resistors, one of 10 ohms and one of 12 ohms are connected in parallel across a 120-v line. What is the total current and the equivalent resistance?

Exercise 1-10. Three resistors having resistances of 30, 15, and 10 ohms are connected in parallel across a 230-v circuit. (a) What current is taken from the line? (b) What portion of this current will flow through the resistor having a resistance of 15 ohms?

Exercise 1-11. A 125-v generator supplies a current of 24 amps to two parallel resistors. If one of the resistors takes 10 amps, what is the resistance of each element?

Exercise 1-12. Four resistors are connected in parallel. Their resistances are 6, 8, 16, and 20 ohms. What total current flows when 24 v is applied? What proportion of this current flows in the 16 ohm resistor?

Series-parallel circuits

Many times it is desirable to use combinations of series and parallel arrangements of resistances in electrical equipment.

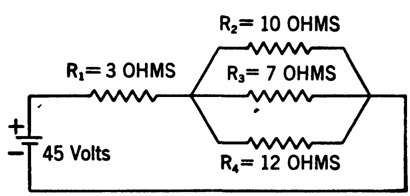


FIG. 1-5. Resistances in Series and Parallel.

The procedure used to solve circuits of this type is to combine the parallel resistance into an equivalent resistance and then to add this equivalent resistance to the other series resistances to

determine the equivalent series resistance. The total current produced by the impressed voltage may then be calculated. This total current will divide in a parallel circuit in proportion to the conductances, and thus the current in any one of the resistances may be found.

Example. In the circuit shown in Fig. 1-5, determine the current in the 7-ohm resistor.

Solution:

$$G_2 = \tfrac{1}{10} = 0.100 \text{ mho},$$
$$G_3 = \tfrac{1}{7} = 0.143 \text{ mho},$$
$$G_4 = \tfrac{1}{12} = 0.083 \text{ mho}.$$

Equivalent conductance is

$$G_{eq} = 0.100 + 0.143 + 0.083 = 0.326 \text{ mho}.$$

Equivalent parallel resistance is

$$R_{eq} = \frac{1}{0.326} = 3.07 \text{ ohms}.$$

Equivalent total resistance is

$$R_t = 3.07 + 3 = 6.07 \text{ ohms}.$$

Total current is

$$I_t = \frac{E}{R_t} = \frac{45}{6.07} = 7.40 \text{ amp}.$$

Current in 7-ohm resistance is

$$I_{7\text{-ohm}} = \frac{G_3}{G_{eq}} I_t = \frac{0.143}{0.326} \times 7.40 = 3.24 \text{ amp}.$$

An alternate method of obtaining the current in any branch of a parallel circuit is to determine the voltage across the parallel portion of the circuit from the equivalent IR drop and then divide this voltage by the resistance of that circuit element to obtain the current flowing in it.

Example. In the circuit shown in Fig. 1-6, determine the current in the 10-ohm resistor.

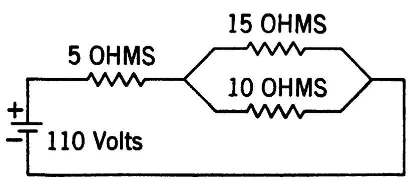


Fig. 1-6. A Series-Parallel Circuit.

Solution: Equivalent resistance of the parallel circuit is

$$R_{eq} = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6 \text{ ohms.}$$

Total resistance of circuit is

$$R_t = 5 + 6 = 11 \text{ ohms.}$$

Total current is

$$I_t = \frac{E}{R_t} = \frac{110}{11} = 10 \text{ amp.}$$

Volts across parallel resistors are

$$E_p = IR_{eq} = 10 \times 6 = 60.$$

Current through 10-ohm resistance is

$$I_{10\text{-ohm}} = \frac{E_p}{R_{10\text{-ohm}}} = \frac{60}{10} = 6 \text{ amp.}$$

Exercise 1-13. Determine the total current and the current in the 5-ohm resistor of the circuit in Fig. 1-7.

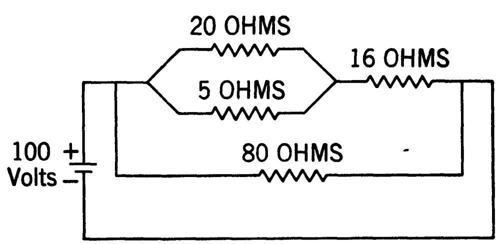


FIG. 1-7. Circuit for Exercise 1-13.

Exercise 1-14. Determine the total current and the current in the 25-ohm resistor of the circuit shown in Fig. 1-8.

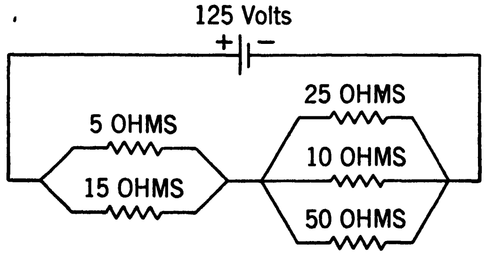


FIG. 1-8. Circuit for Exercise 1-14.

Resistance of electrical conductors

The wires that are used to conduct electricity are usually copper and are available in standardized sizes. The wires used for the distribution of electricity in buildings and factories are insulated with thick coverings, usually in two or more layers, and the current-carrying capacity is specified by the National Electrical Code.* Wire used for the manufacture of electrical machinery and coils is called magnet wire and is provided with thin insulation only. Since usage of these two types of wire is quite different, information regarding them is shown in two separate tabulations.

Table 1-1 shows the wire sizes more commonly used in industrial wiring, with current-carrying capacities for several types of insulation. Included in this tabulation also is the diameter of the copper portion of the wire in mils (thousandths of an inch) and the cross-sectional area in both circular mils and square inches.

Table 1-2 shows the important properties of copper magnet wire.

The circular mil referred to in these tables is a small unit of area so commonly used in electrical work as to justify some

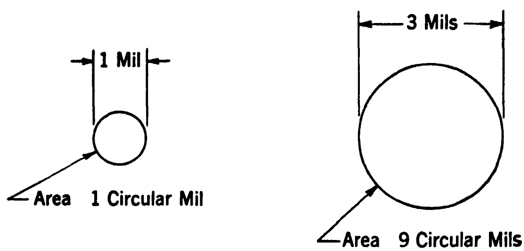


Fig. 1-9. Circular Measure of Areas.

explanation. It is defined as the area of a circle that has a diameter of one mil (one thousandth of an inch). To obtain the cross-sectional area of any cylindrical wire in circular mils, it is therefore necessary only to square the diameter in mils.

A logical development of the rules of series and parallel circuits that have been previously studied is that the resistance of a conductor is directly proportional to its length and inversely proportional to its cross-sectional area. Stated mathematically,

* A set of rules adopted by the National Board of Fire Underwriters.

TABLE 1-1
PROPERTIES OF INSULATED CONDUCTORS FOR POWER WIRING
(From National Electrical Code, 1951)

Size of wire (AWG)	Diameter of solid conductor (in mils)	Pounds per 1000 ft	Area of conductor (in circular mils)	Area of conductor (in in.²)	Ohms per 1000 ft at 20° C	Allowable Current-Carrying Capacity (not more than 3 conductors in raceway)*		
						Code rubber	Heat-resistant rubber	Varnished cambric
14	64.1	12.43	4,107	0 003225	2 525	15	15	25
12	80.8	19.77	6.530	0 005129	1 588	20	20	30
10	101.9	31.43	10,380	0 008155	0 9989	30	30	40
8	128.5	49.98	16,510	0 01297	0 6282	40	45	50
6	162.0	79.46	26,250	0.02062	0 3951	55	65	70
4	204.3	126.4	41,740	0 03278	0 2485	70	85	90
2	257.6	200 9	66,370	0 05213	0.1563	95	115	120
1	289.3	253.3	83,690	0.06573	0.1239	110	130	140
0	325.0	319 5	105,500	0.08289	0.09827	125	150	155
00	364.8	402 8	133,100	0.1045	0.07793	145	175	185
000	409.6	507 9	167,800	0 1318	0.06180	165	200	210
0000	460.0	640 5	211,600	0.1662	0.04901	195	230	235
		758	250,000	0.1970	0.04150	215	255	270
		1,516	500,000	0.3940	0.02075	320	380	405
		2,274	750.000	0.5910	0 01383	400	475	500
		3,032	1,000,000	0.7880	0.01038	455	545	585

* Typical allowable current-carrying capacities per page 351 and 352 of the 1951 National Electrical Code.

TABLE 1-2

PROPERTIES OF COPPER MAGNET WIRE

Size of wire (AWG)	Diameter (mils*)	Ohms per 1000 ft at 20° C	Pounds per 1000 ft	Diameter C† (mils)	Diameter E‡ (mils)	Diameter EC§ (mils)
6	162 0	3951	79.46	170.		
8	128 5	.6282	49.97	134.	131	136
10	101.9	.9989	31.43	107.	104	109
12	80.81	1.588	19.77	85.8	83.0	88 0
14	64 08	2 525	12 43	69 1	66.1	71 1
16	50.82	4.016	7.818	55.8	52.6	57.6
18	40.30	6 385	4.917	45 3	42.0	47 0
20	31 96	10 15	3 092	37.0	33.5	38 0
22	25 35	16 14	1 542	29.4	26 8	31 3
24	20.10	25 67	1 223	24 1	21 3	25 8
26	15.94	40 81	.7692	19 9	17 0	21 5
28	12.64	64 90	4837	16.6	13.6	17 6
30	10 03	103 2	3042	14.0	10 8	14 8
32	7.950	164.1	1913	12 0	8 75	12 8
34	6 305	260.9	1203	10 3	7 01	11 0
36	5 000	414.8	0757	9.00	5 60	9 60
38	3 965	659 6	0476	7.97	4.47	8 47
40	3 145	1049 0	0299	7 15	3 55	7 55

* 1 mil = 0.001 in.
† C means single cotton covered
‡ E means enameled.
§ EC means enameled with single cotton covering.

this is

$$R = \rho \frac{l}{a}$$

where l is the length, a the cross-sectional area, and ρ* the resistance of a conductor having unit length and unit cross-sectional area.

In the United States the length is usually specified in feet, and the cross-sectional area in circular mils. The resistance per unit length and for a unit area is then the resistance of a wire 1 ft long having a cross-sectional area of 1 cir mil. Thus the resistivity of copper is 10.37 ohms for a circular mil-foot at 20° C.

* ρ, the resistance of a conductor of unit length and unit cross-sectional area, is called "resistivity."

Example. Determine the resistance of a copper wire having a diameter of 40.3 mils and a length of 600 ft.

Solution:

$$\rho = 10.37$$
$$l = 600$$
$$a = 40.3^2 = 1624$$

Hence, $$R = \rho \frac{l}{a} = 10.37 \times \frac{600}{1624} = 3.84 \text{ ohms.}$$

Alternate solution:

Look in Table 1-2 to see wire having a diameter of 40.3 mils. This is observed to be #18 wire having a resistance of 6.38 ohms per 1000 feet.

$$R = 6.38 \times \tfrac{600}{1000} = 3.84 \text{ ohms.}$$

Since most copper wire will conform to the standard wire gage, the resistance can usually be obtained directly from tables similar to Tables 1-1 and 1-2. It is often desired, however, to compute the resistance of shapes that are not circular, and so the computation from dimensions is necessary.

Example. Determine the resistance of 400 ft of copper bar which has a $\frac{1}{4}$ by 1-in. cross-section.*

Solution:

$$\rho = 10.37 \text{ ohms per mil ft}$$
$$l = 400 \text{ ft}$$
$$a = \tfrac{1}{4} \times 1 = \tfrac{1}{4} \text{ in.}^2$$
$$= 0.250 \times \frac{4}{\pi} \times 10^6 = 318{,}000 \text{ cir mils.}$$
$$R = \rho \frac{l}{a} = 10.37 \frac{400}{318{,}000} = 0.01305 \text{ ohms} \qquad (Ans.)$$

Exercise 1-15. What is the area in circular mils of a wire having a diameter of 0.05 in.?

Exercise 1-16. What is the resistance of a copper tube 30 ft long, if it has an outside diameter of 1.5 in. and an internal diameter of 1.3 in.?

Exercise 1-17. What is the resistance of a copper strip having a thickness of 0.06 in., a width of 0.75 in., and a length of 164 ft?

It is sometimes necessary to determine the resistance of conductors other than copper. This is easily done if the resistivity

---

* The area of one sq. inch is equivalent to $\frac{4}{\pi} 10^6$ circular mils.

TABLE 1-3
PHYSICAL PROPERTIES OF SOME OF THE MORE COMMON METALS AND ALLOYS

Metals and alloys	Resistivity, ohms (mil. ft)	Resistance relative to copper	Temp. coeff. of resistance per °C (20 C)	Melting point °C
Copper... ..	10.37	1 0	0 0039	1,083
Iron...	60 0	5.80	0 0050	1,535
Zinc......... .	35.5	3 43	0 0035	419
Tungsten....	33.2	3 20	0 0045	3,382
Aluminum... . .	16 1	1 55	0 0040	659
Gold.......	14 5	1 40	0 0034	1,063
Silver...	9 78	0 943	0 0038	960
Radiohm ..	800 0	77 0	0 0007	1,480
Nichrome.	675 0	65 0	0 00017	1,350
Advance	294 0	28 0	0 00002	1,210
High brass	38 7	3 75	0.0016	930
Low brass . .	32 0	3 10	0.0017	1,000
Commercial bronze . .	25.0	2 4	0.0020	1,045

of the other material is known. In Table 1-3 are given the specific resistivities of some of the more common conducting materials.

Example. Determine the resistance of 65 ft of nichrome ribbon which is 0.04 in. thick and has a width of 0.5 in.

Solution:

$$\rho = 675 \text{ ohms per mil ft}$$

$$l = 65 \text{ ft}$$

$$a = 0.04 \times 0.5 = 0.02 \text{ in.}^2$$

$$= 0.02 \times \frac{4}{\pi} \times 10^6 = 25{,}400 \text{ circular mils.}$$

$$R = \rho \frac{l}{a} = 675 \times \frac{65}{25{,}400}$$

$$= 1.73 \text{ ohms.}$$

Exercise 1-18. What is the resistance per 100 ft of aluminum tubing having an outside diameter of 2 in. and an internal diameter of 1.4 in.?

Exercise 1-19. A 100-lb steel railways rail has a cross-section of 9.82 square inches. What would the resistance of this rail be per mile of length?

Exercise 1-20. The wire of a strain gage is made of Advance metal, is 0.001 in. in diameter, and has a length of 6 in. What is its resistance? (See Chap. 18, for a discussion of strain gages.)

Temperature coefficient of resistance

An important consideration in making resistance calculations is the effect of temperature on resistivity. In copper and most pure metals there is an appreciable increase in resistance with an increase in temperature. The change in these pure metals is approximately proportional to the temperature change, as long as very low or very high temperatures are excluded from the discussion. In copper this change amounts to $\frac{1}{254}$, or 0.00393, of the resistance at 20° C for each degree C change in temperature. (This is approximately 0.4 per cent per degree C.) Thus, if the temperature of a coil has risen to 90° C, then the resistance would be

$$
\begin{aligned}
R_{(90)} &= R_{(20)} + R_{(20)}(90 - 20)0.00393 \\
&= R_{(20)}[1 + (70 \times 0.00393)] = 1.275R_{(20)}.
\end{aligned}
$$

In this case the resistance has increased by 27 per cent.

This variation of resistance not only affects the calculations in the design of electrical equipment that operates normally at a temperature much higher than the air, but it is used extensively as a device for the measurement of temperature. The use of resistance coils for temperature measurement is common in recording thermometers and in automatic temperature controllers for comparatively low temperatures.

In making calculations involving the temperature variation of resistance, it is important to remember that the temperature coefficients that are given in the table apply to the resistance at 20° C. If the known resistance is at a temperature different from that of the standard, it is necessary to use the above equation to convert to the standard temperature before making the computation for final resistance.

Example. The resistance of the copper field coil of a machine is measured as 21.3 ohms at 15° F. What will be the resistance when it is operating at 120° F?

Solution:

(1) Convert to Centigrade scale

$$
\begin{aligned}
T_C &= (T_F - 32)\tfrac{5}{9} = (15 - 32)\tfrac{5}{9} \\
&= -17 \times \tfrac{5}{9} = -9.45° \text{ C}.
\end{aligned}
$$

(2) Determine resistance at 20° C

$$R_{(20)} = \frac{R_{(-9.45°)}}{1 - (9.45 + 20)0.00393}$$

$$= \frac{21.3}{1 - 0.116} = \frac{21.3}{0.884} = 24.15.$$

(3) Convert final temperature to C scale.

$$T_C = (T_F - 32)\tfrac{5}{9} = (120 - 32)\tfrac{5}{9}$$
$$= 88 \times \tfrac{5}{9} = 48.9.$$

(4) Determine resistance at final temperature.

$$R_{(120\ F)} = R_{(48.9\ C)} = 24.15[1 + (48.9 - 20)0.00393]$$
$$= 24.15(1 + 0.1135) = 26.9 \text{ ohms}.$$

The above operations 2 and 4 can be combined into one formula for convenience of calculation if a number of computations of this type are made.

An alternate method of computation may be used on the basis that the coefficient of resistance of copper at 70° F is equal to 0.0022 per degree Fahrenheit. This makes it unnecessary to convert to the Centigrade scale.

Exercise 1-21. Develop a single formula for determining the resistance at a specified temperature where the resistance is given at a temperature other than the 20° C reference. (This may be written in the margin of the book for future reference.)

Different metals and alloys have different temperature coefficients. The values of these coefficients are given in Table 1-3. The range for the more common pure metals is from 0.0035 to 0.005. The coefficient of resistance of several of the resistance alloys is not always constant over the higher temperature ranges. The values given in the table will, however, give the data sufficiently close for most purposes. Nichrome wire, for instance, increases only about 14 per cent in resistance in going from room temperature to 800° C.

The coefficient of resistance of Advance (and similar alloys) is so small that it may be neglected for ordinary temperature differences. These alloys are used for standard resistors, and no temperature correction is necessary for ordinary accuracy.

Exercise 1-22. A coil made of tungsten wire is used for temperature measurement. Its resistance at 20° C is 120 ohms. How accurately must this resistance be measured at 600° C if the temperature is to be measured to one degree?

Exercise 1-23. What change in resistance occurs in a nichrome heater when going from 70° F to 1600° F? The resistance at 70° F is 6.05 ohms.

Kirchhoff's laws

Two rules or laws known as Kirchhoff's laws are important tools in the solution of complicated electric circuits. The first of these may be stated as follows. Kirchhoff's current law: *The current flowing into any junction of an electric circuit is equal to the current flowing out of that junction.*

The truth of this statement may be regarded as self-evident since there is no provision for storage of electric charge at the junction points of electric circuits. If the statement were not true, then there would be an indefinite accumulation of charge at the junction that is not possible in simple conductors. In Fig. 1-10, with the current directions assumed as indicated in the diagram,

$$I_1 + I_2 + I_3 = I_4.$$

An alternate statement of this law may be obtained by subtracting I_4 from both sides of the equation to obtain

$$I_1 + I_2 + I_3 - I_4 = 0.$$

This alternate statement is: *The algebraic sum of the currents flowing into any junction of an electric circuit is zero* (when due regard is taken of the assumed positive directions). This statement emphasizes the fact that in complex circuits the direction of current flow is often not known, and, therefore, a positive direction of flow must be assumed. If in the final solution the current value is found to be negative, it indicates that current flows opposite to the assumed positive direction.

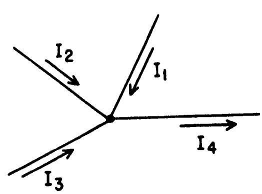


FIG. 1-10. Currents at a Junction Point in Complex Circuits.

The second of the laws has to do with the voltages in any of the loops or meshes of a complex circuit. It may be stated as follows. Kirchhoff's voltage law: *The algebraic sum of the battery or generator voltages around any loop of a circuit is equal to the algebraic sum of the voltage drops in the resistance elements of the same loop.*

The truth of this statement is also self-evident, since in passing around a circuit the potential will vary, but the potential at the starting point will remain the same. The algebraic sum of the voltage variations in passing around the circuit must be zero

in order to arrive at the starting point without change in potential. This leads to the alternate statement for the second law as follows: *The algebraic sum of the voltage changes around any loop of a circuit is equal to zero.*

To illustrate the use of these equations a very simple network, having only two loops, will be studied. In Fig. 1-11 there are two loops with an impressed voltage in each loop. In order to solve this circuit, the current equation at junction A will be written first.

$$I_3 = I_1 - I_2.$$

This equation includes all three unknowns and, therefore, is the only relation that can be obtained from the current law.

The voltage equations around the loop will next be established by carefully observing the following conventions. In

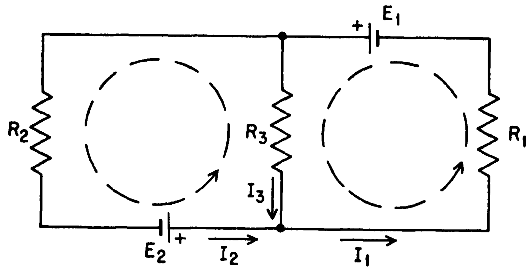


FIG. 1-11. A Two-Loop Network with Current and Voltage Designations for Formulation of Kirchoff Law Equations.

each loop a positive direction around the loop must be chosen. In this case the counterclockwise direction will be assumed as positive for both loops. A voltage tending to produce current in the positive direction is positive. When current flows through a resistance in the positive direction, a reduction of potential is experienced in passing through the resistance; hence this will give a positive voltage drop. If the current flow is in a direction opposite to the assumed positive loop direction, the voltage drop will be negative, or a voltage rise will be experienced in passing through the resistance. Observing these conventions, one obtains the following equations:

$$E_1 = R_1I_1 + R_3I_3, \qquad E_2 = R_2I_2 - R_3I_3.$$

The directions of I_1 and I_3 are both positive in the assumed positive (counterclockwise) direction of traversing the right-hand loop; and therefore, both will produce voltage drops. These voltage drops will be equal to the voltage impressed in the loop. In the left loop, however, the positive direction of

I_3 is opposite to the assumed positive direction of traversing the loop and therefore produces a voltage rise that will be assigned a negative sign in the equation.

Attention is directed to the fact that the positive direction of I_3 was arbitrarily assumed and that there is no assurance that the current is actually flowing in this direction. Whether it is flowing in one direction or the other will depend upon the values of E_1, E_2, and the resistors in the circuit. If the actual flow is not in a positive direction for any of the unknown currents, the solution will give a negative value for that current. Care must be used to maintain the assumed positive directions of current throughout the solution once the assumption has been made.

The current and voltage equations are collected and solved.

$$I_3 = I_1 - I_2, \qquad E_1 = R_1I_1 + R_3I_3, \qquad E_2 = R_2I_2 - R_3I_3.$$

Example. Assume that the voltage and resistance values for the circuit of Fig. 1-11 are as follows:

$$E_1 = 50 \text{ v} \qquad R_1 = 10 \text{ ohms} \qquad R_3 = 20 \text{ ohms.}$$
$$E_2 = 100 \text{ v} \qquad R_2 = 15 \text{ ohms}$$

Determine the current flowing in R_1 and R_2.

Solution:

(1) Set down the current and voltage equations for the circuit.

$$I_3 = I_1 - I_2, \qquad E_1 = R_1I_1 + R_3I_3, \qquad E_2 = R_2I_2 - R_3I_3.$$

(2) Eliminate I_3 by substitution.

$$E_1 = R_1I_1 + R_3(I_1 - I_2) \qquad E_2 = R_2I_2 - R_3(I_1 - I_2)$$

(3) Collect the coefficients of the unknown currents and rewrite the equations.

$$E_1 = (R_1 + R_3)I_1 - R_3I_2, \qquad E_2 = -R_3I_1 + (R_2 + R_3)I_2.$$

(4) Substitute numerical values of voltages and resistances and solve.

$$50 = 30I_1 - 20I_2 \qquad 100 = -20I_1 + 35I_2$$

$$I_1 = \frac{\begin{vmatrix} 50 & -20 \\ 100 & 35 \end{vmatrix}}{\begin{vmatrix} 30 & -20 \\ -20 & 35 \end{vmatrix}} = 5.76 \text{ amp.}$$

$$I_2 = \frac{\begin{vmatrix} 30 & 50 \\ -20 & 100 \end{vmatrix}}{\begin{vmatrix} 30 & -20 \\ -20 & 35 \end{vmatrix}} = 6.15 \text{ amp.}$$

$$I_3 = I_1 - I_2 = 5.76 - 6.15 = -0.39 \text{ amp.}$$

Although the above example is solved by the use of determinants, any of the various forms of solution of simultaneous equations may be used. The results indicate that the assumed direction of I_3 was indeed incorrect and that the current was flowing in the direction opposite to that which was assumed.

Exercise 1-24. Determine the voltage across *ab* of Fig. 1-12.

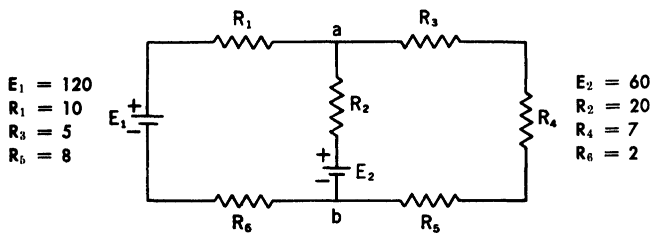
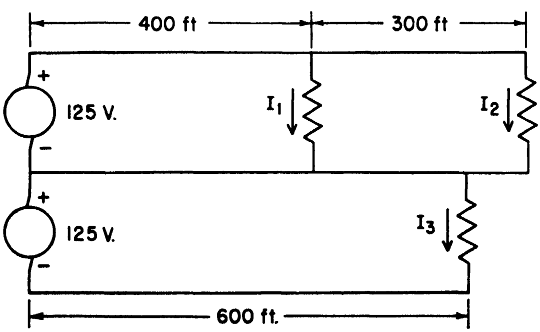


FIG. 1-12. Circuit for Exercise 1-24.

Exercise 1-25. A three-wire distribution system is supplied with two 125-v generators connected in series as shown in Fig. 1-13. Determine the voltage across each load if the distribution wire is all No. 2 AWG copper wire.



$I_1 = 40$ amp.
$I_2 = 35$ amp.
$I_3 = 65$ amp.

FIG. 1-13. Circuit for Exercise 1-25.

Maxwell's mesh equations

An alternate method of solution based on the same fundamental laws was developed by Maxwell and bears his name. This method involves a somewhat different designation of currents, which allows the elimination of the current equations in the Kirchhoff law method and proceeds directly to the voltage equations. This method is preferred by many engineers because it simplifies the solution considerably.

To illustrate this method the same circuit shown in Fig. 1-11

will be used, but the currents flowing in each loop will be designated as I_1 and I_2, as shown in Fig. 1-14. It will be noted that the current in R_3 is then $(I_1 - I_2)$ when the positive direction of current flow is downward as in loop 1. This agrees with the value of I_3 in the Kirchhoff law current equation.

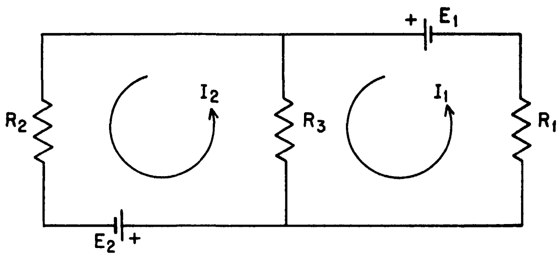


FIG. 1-14. A Two-Loop Network with Current and Voltage Designations for Formulation of Maxwell Mesh Equations.

The voltage equations are formulated as before in loops 1 and 2.

$$E_1 = (I_1 - I_2)R_3 + I_1R_1, \qquad E_2 = (I_2 - I_1)R_3 + I_2R_2.$$

When coefficients of the unknown currents are collected, these equations become:

$$E_1 = (R_1 + R_3)I_1 - R_3I_2, \qquad E_2 = -R_3I_1 + (R_2 + R_3)I_2.$$

It is observed that these equations are identical with the equations obtained by the Kirchhoff law method after I_3 has been eliminated and the coefficients of the unknown currents have been collected. Maxwell's mesh equations are therefore just another form of Kirchhoff's equations.

Principle of superposition

Another method of obtaining the current in a circuit having several voltages is based on the principle that *the current in any wire of a complex circuit is equal to the algebraic sum of the currents produced in that wire by each of the voltages acting independently* (*and with the other voltages shorted out*). This will be explained by solving the illustrative example on page 23 by this method.

Example. (1) In Fig. 1-11 assume that E_2 is shorted out and that E_1 only is effective. This places R_2 and R_3 in parallel and these in series with R_1.

(2) Equivalent resistance of parallel circuit is

$$\frac{R_2R_3}{R_2 + R_3} = \frac{15 \times 20}{15 + 20} = 8.56 \text{ ohms.}$$

(3) Total resistance $= 8.56 + 10 = 18.56$ ohms

$$I'_1 = \frac{50}{18.56} = 2.69 \text{ amps} \qquad I'_2 = \frac{8.6}{15} \times I'_1 = 1.54 \text{ amp}$$

$$I'_3 = \frac{8.6}{20} \times I'_1 = 1.15 \text{ amp.}$$

(4) Assume that E_1 is shorted out and that E_2 is effective. This places R_1 and R_3 in parallel and these in series with R_2.

(5) Equivalent resistance of parallel circuit is

$$\frac{R_1R_3}{R_1 + R_3} = \frac{10 \times 20}{10 + 20} = 6.66 \text{ ohms.}$$

(6) The total resistance $= 6.66 + 15 = 21.66$ ohms

$$I''_2 = \frac{100}{21.66} = 4.61 \text{ amps} \qquad I''_1 = \frac{6.66}{10} \times I''_2 = 3.07 \text{ amps}$$

$$I''_3 = -\frac{6.66}{20} \times I''_2 = -1.54 \text{ amps.}$$

(It is noted that I''_3 will flow in a direction opposite to that assumed in the diagram and will therefore appear as a negative current value.)

(7) The current with both voltages in the circuit will be the algebraic sum of the individual currents.

$$I_1 = I'_1 + I''_1 = 2.69 + 3.06 = 5.75 \text{ amps}$$
$$I_2 = I'_2 + I''_2 = 1.54 + 4.61 = 6.15 \text{ amps}$$
$$I_3 = I'_3 + I''_3 = 1.15 - 1.54 = -0.39 \text{ amp.}$$

It is observed that these values agree with the values obtained previously.

The main advantage in this method is that it does not require a knowledge of the technique of forming and solving simultaneous equations.

Symbols and abbreviations

The circuits that have been used so far have been so simple that there was no possibility of misunderstanding. Later circuit diagrams are more complicated, and so standard symbols are used to indicate circuit elements. The circuit symbols shown in Fig. 1-15 indicate the usual symbols used. However,

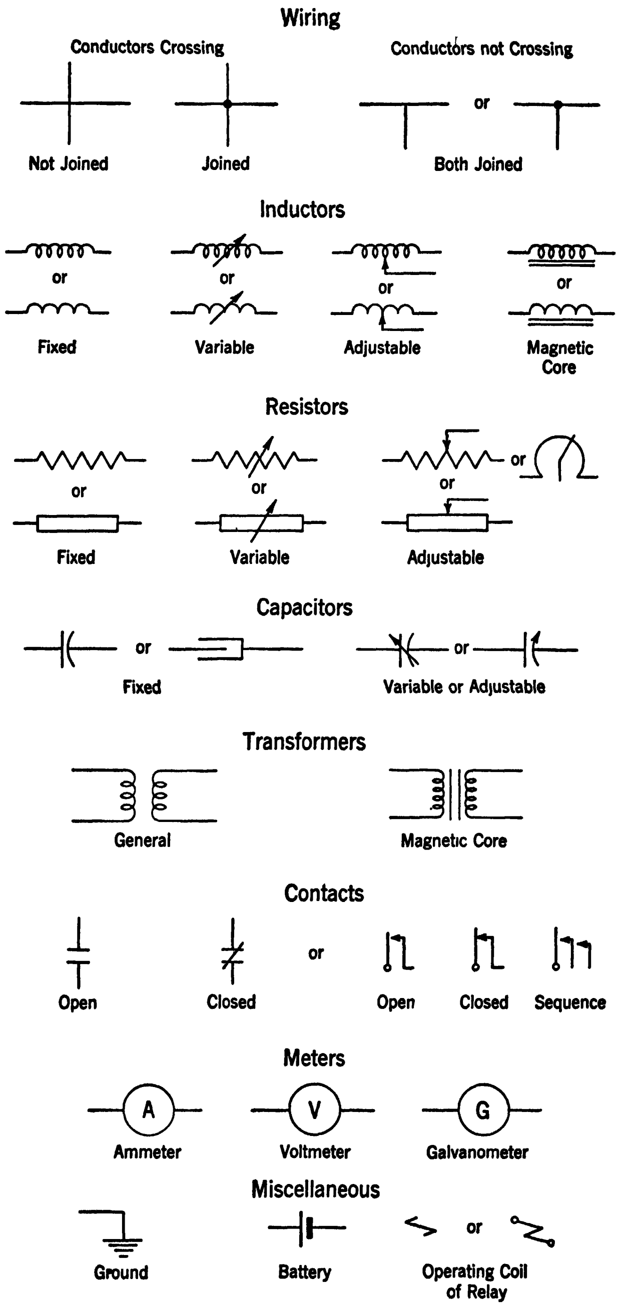


Fig. 1-15. Electrical Graphical Symbols.

some deviation may be made to familiarize the reader with some of the commercial symbols and diagram procedures.

The very wide range of magnitudes of the various electrical quantities has led to the extensive adoption of many units which are decimal parts or multiples of the basic units. The prefix *milli* means one thousandth; for example, one millivolt is one thousandth of a volt. Likewise, *micro* means one millionth. *Kilo* means thousands and *mega* means millions. The use of such units saves many troublesome decimals. *It is necessary, however, to remember that the circuit laws are based on ohms, amperes, and volts and that other units must be converted to these before circuit problems can be solved.*

Nonlinear circuit elements. Varistors

Although most circuit elements conform to the assumption of constant resistance within normal engineering accuracy, there are many special circuit elements in which the resistance varies with the current flow. Such circuit elements are called nonlinear since the rectangular co-ordinate plot of current vs. voltage is not a straight line. Many of these nonlinear circuit elements, or *varistors* (as they are called), are now finding important commercial applications.

CHAPTER 2

Ferromagnetic Circuits

Magnetic concepts

The simpler phenomena of magnetism are known to every scientific student, but a complete understanding of the mechanism of magnetic action is still the subject of advanced research. Magnetism appears to be associated with the movement of electrons. This electron movement may occur in electric wires, in the orbits of molecules of iron and other magnetic materials, or in the spins of these same electrons.

Although the classical use of magnets in navigation as compasses is known to every school boy, the modern uses of magnets are not so well known. Magnetic, as well as conducting and insulating, materials form the basic engineering materials of the electrical industry. Motors, generators, and transformers use most of the several million tons of magnetic materials that are annually consumed. Industry also requires such electromagnetic devices as relays, chucks, couplings, instruments, and other measurement and control devices. Useful magnetic materials either are magnetized and demagnetized easily or are permanent magnets that are magnetized with difficulty but retain their magnetism.

There is no conclusive evidence of any flow of a magnetic substance. However, just as the concept of the flow of an electric fluid aided greatly in the development of electric-circuit theory before the discovery of the electron, we continue to use concepts of magnetic flux or flow to aid in the visualization and prediction of magnetic effects in electrical machinery.

In order to discuss the performance of magnets it is necessary to have a method of representing both the direction and magnitude of the magnetic field. The method adopted by early scientists and still in general use is to indicate the direction of the field by lines that follow the pattern assumed by iron filings sprinkled around a magnet. Such a system of lines is shown in

Fig. 2-1 for a bar magnet. It is observed that at the poles, where the magnetic effect is greatest, the lines are most numerous, or, stated in another way, their density per unit area perpendicular to the lines is a maximum. Thus the density of lines,

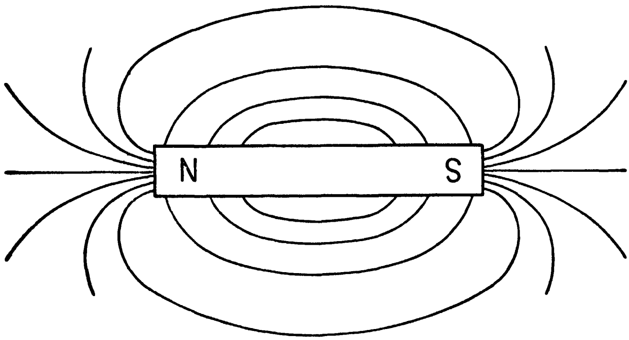


FIG. 2-1. The Magnetic Field of a Bar Magnet.

or *flux density*, is used as a measure of the magnetic flow, which in air is also a measure of the intensity of the magnetic field. In Fig. 2-2 a coil is shown with flux lines almost identical with those of the permanent magnet in Fig. 2-1. If the current in the coil is doubled, the magnetic intensity is also doubled. If the current remains the same and the number of turns is doubled, the magnetic intensity is likewise doubled. If both the current and the turns are doubled, then the magnetic intensity becomes four times the original value. Thus it is inferred that the magnetic intensity or flux density in air is proportional to the *product of the electric current in amperes and the number of turns in the coil.* The unit used by many electrical designers to measure magnetic cause or magnetomotive force is therefore the *ampere-turn.**

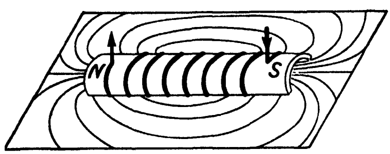


FIG. 2-2. The Magnetic Field about a Coil of Wire Carrying Current.

Iron and many of its alloys are so susceptible to magnetic induction that they are used extensively to guide and concentrate the magnetic effect. In this way, iron and steel act with regard to magnetism in much the same way that a good conduc-

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* The gilbert, $(0.4\pi NI)$, the unit of magnetomotive force in the CGS system is also used extensively.

tor acts toward electricity. Since an ampere-turn, or unit of magnetomotive force, will produce from several hundred to several thousand times as much flux in iron as it will in air, iron is said to be a good conductor of magnetism, and most commercial applications of magnetism involve a circuit of iron that is closed except for a small air gap. Most magnetic problems, therefore, involve a circuit similar in many ways to an electric circuit with nonlinear circuit elements. Since there is no known magnetic insulator, there is always a considerable amount of leakage that may cause significant errors in calculation unless special methods of computation are used. In order to understand the operating characteristics of motors and generators and to properly adjust and maintain the many relays, meters, and control devices used in modern industry, a study of simple magnetic circuits is required.

Magnetic units

Magnetic quantities are measured in units, many of which have distinctive names. In the study of physics these units and their relationships have been defined in terms of the CGS or MKS systems. Most designers in the United States use a slightly different set of units, which is based on the English inch as a unit of measure. The relation of the different sets of units is shown in Table 2-1. From this table conversion factors may be developed as needed.

This complexity of units has tended to obscure the rather simple method of handling magnetic problems. In this text the English, or practical, units will be used for most magnetic circuits involving soft iron (electrical sheets) because that is the form in which engineers generally work these problems. In the case of permanent magnets it is common for the data to be presented and the problems to be worked in the CGS or MKS units. For the formal definitions of these magnetic quantities the student is referred to a good college physics text.

Magnetization curves. Characteristics of nonlinear circuit elements are shown best by curves that indicate the relation between the current and voltage or between magnetic pressure and magnetic flux or flow. Figure 2-3 shows typical curves for a few magnetic materials. The magnetic flux is indicated in lines per square inch and the magnetomotive force is shown in ampere-turns per inch of length of the magnetic circuit required to produce the flux density indicated by the curve.

TABLE 2-1

Quantity	Symbol	MKS Units	CGS Units	English Practical Units
Magnetomotive force	$\mathfrak{F}$	Pragilbert $4\pi NI$	Gilbert $0.4\pi NI$	Ampere-turn NI
Magnetizing force (field intensity)	H	Pragilberts per meter	Oersteds (gilberts per centimeter)	Ampere-turns per inch
Magnetic flux	ϕ	Weber (10^8 maxwells)	Maxwell (or line)	Maxwell (or line)
Magnetic induction (flux density)	B	Webers per square meter	Gauss (maxwells per square centimeter)	Maxwells (or lines) per square inch
Permeability	$\mu = \frac{B}{H}$	$\mu = \mu_0\mu_r$* $\mu_0 = 4\pi 10^{-7}$	$\mu = \mu_0\mu_r$ $\mu_0 = 1$	$\mu = \mu_0\mu_r$ $\mu_0 = 3.19$

* where μ_0 = permeability of free space in the particular set of units and
μ_r = relative permeability or the ratio of flux density in the material under consideration to that in air, with the same field intensity.

For instance, wrought iron will require 30 ampere-turns/in. to produce a flux density of 100,000 lines/in.^{2} If then a bar which has a cross-sectional area of 2 in.^{2} and a length of 10 in. is to be magnetized to this flux density, the total ampere-turns required would be 10 times 30, or 300 ampere-turns, to produce the flux within the bar. The total flux in the bar would be the product of the flux density and the cross-sectional area, or 2 $\times$ 100,000 = 200,000 lines. The magnetomotive-force drop in the remain-

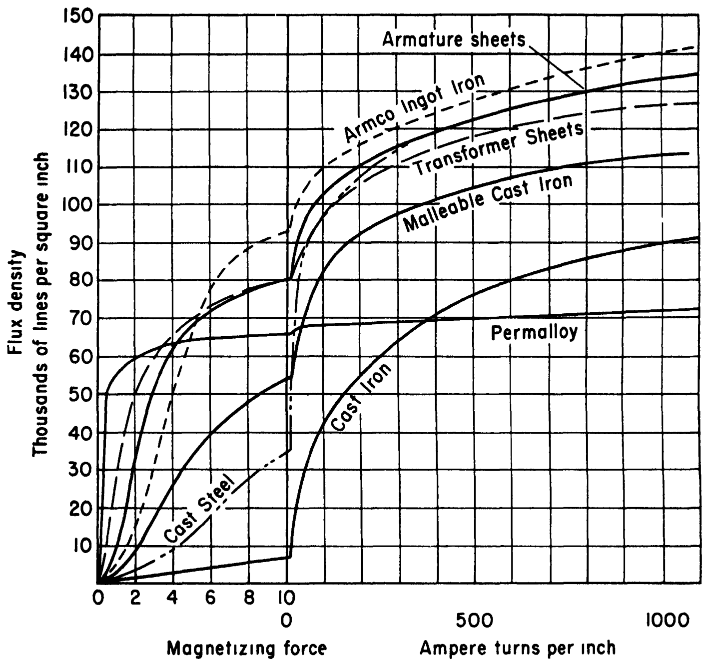


FIG. 2-3. Typical Magnetization Curves.

der of the magnetic circuit must of course be supplied, but the magnitude of that magnetomotive force is a separate calculation.

Magnetic flux in the air gap. Since air and other nonmagnetic materials have no polarized atoms (as will be explained in a later paragraph), the magnetic flux requires a much greater magnetomotive force per unit length of circuit than does iron. Also, the flux is directly proportional to the magnetomotive force, so the magnetization curve is a straight line having an equation of

$$NI_{(\text{per inch})} = 0.313B,$$

where N is the number of turns, I the current in amperes, and B the flux density in lines per square inch.* This curve is plotted in Fig. 2-4.

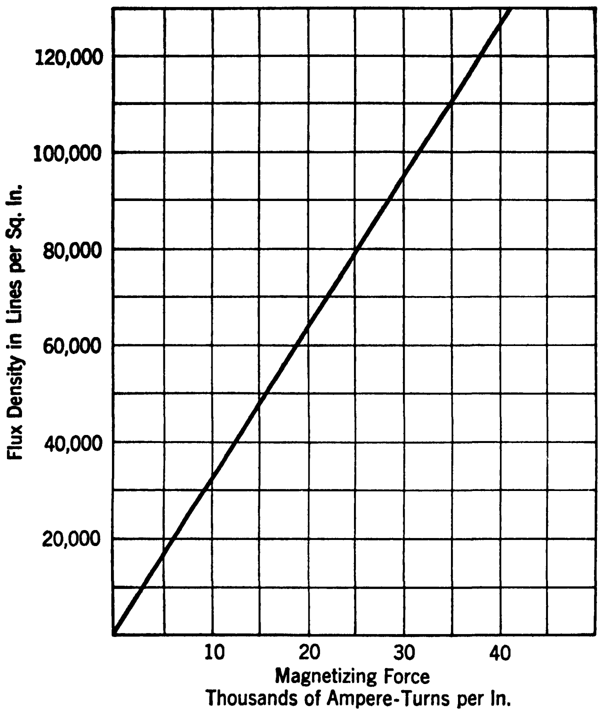


FIG. 2-4. Air Gap Magnetization Curve.

Calculation of simple magnetic circuits

It is now possible to compute the magnetomotive force required to produce a specified flux in a simple magnetic circuit. The procedure is to determine the magnetomotive force required by each series element of the circuit and then to add all of these component parts to obtain the total. (This corresponds to

* This equation is given in most physics textbooks as

$$B = H = \frac{4\pi NI}{10} \text{ (per centimeter)}$$

where $4\pi NI/10$ is the magnetomotive force in gilberts and H is the magnetic field strength in oersteds, which in air is numerically equal to the flux density in gauss. When this equation is converted to English units, it becomes $NI_{\text{(per inch)}} = 0.313B$.

determining the voltage required to force a certain current through a simple series electrical circuit.) The method will be demonstrated by the following example.

Example. How many ampere-turns are needed to produce a total magnetic flux of 20,000 lines in the core of the choke coil shown in Fig. 2-5? The core is made of armature grade electrical sheet steel.

Solution: (1) The cross-sectional area of the magnetic path is $\frac{1}{2}$ in. $\times$ $\frac{1}{2}$ in. = $\frac{1}{4}$ in.^{2}

(2) The flux density is equal to the total flux divided by the cross-sectional area, or $20{,}000 \div \frac{1}{4} = 80{,}000$ lines/in.^{2}

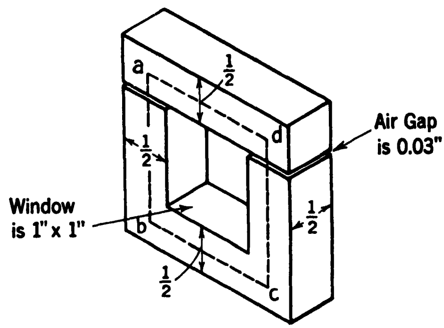


Fig. 2-5. Magnetic Circuit of the Example.

(3) The magnetomotive force required to produce this flux in armature sheets is (from Fig. 2-3) 10 ampere-turns/in.

(4) The mean path may be assumed to be shown by the dotted line *abcd* in Fig. 2-5. The path in the lower part of the iron circuit is $1\frac{1}{4}$ in. from *a* to *b*, $1\frac{1}{2}$ in. from *b* to *c*, and $1\frac{1}{4}$ in. from *c* to *d*. This is a total of 4 in., and the magnetomotive force required is $4 \times 10 = 40$ ampere-turns.

The path in the upper portion of the iron circuit is $\frac{1}{4}$ in. at *d*, $1\frac{1}{2}$ in. from *d* to *a*, and $\frac{1}{4}$ in. at *a* for a total of 2 in. The magnetomotive force is then $2 \times 10 = 20$ ampere-turns for the upper portion of the iron circuit, since it has the same flux density as in the lower portion.

The total ampere-turns necessary to overcome the reluctance of the electric sheet steel portion of the circuit is $40 + 20 = 60$ ampere-turns.

(5) The flux density in the air gap is 80,000 lines/in.^{2}

(6) The magnetomotive force to produce this flux density in air is 25,000 ampere-turns/in. (Fig. 2-4.)

(7) The length of the air gap is 0.03 in. The ampere-turns per air gap is $25{,}000 \times 0.03 = 750$ ampere-turns.

(8) The two air gaps are in series, so the total ampere-turns required for the air is $2 \times 750 = 1500$ ampere-turns.

(9) The total ampere-turns required is the sum of the ampere-turns for air and for the steel.

$$1500 + 60 = 1560 \text{ ampere-turns} \qquad (Ans.)$$

Exercise 2-1. Determine the ampere-turns required to produce 10,000 and then 25,000 lines in the magnetic circuit of the example above. Plot a curve of total flux against the ampere-turns required for the three values computed.

Exercise 2-2. How many ampere-turns are necessary to produce a flux of 6000 lines in the magnetic circuit of the relay shown in Fig. 2-6 if the core is made of Armco iron?

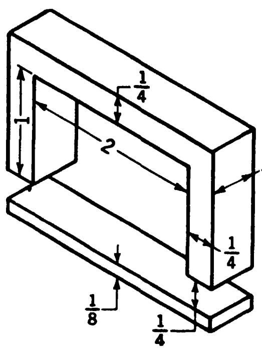


FIG. 2-6. Magnetic Circuit for Exercise 2-2.

The pull of electromagnets

The attraction of an electromagnet for its armature is the basis of operation of most of the electrical control relays as well as some other devices. This attraction is proportional to the square of the flux density and to the area of the air gap. The equation for this pull is

$$P = 1.38B^2A\ 10^{-8}\ \text{lb}$$

where P is the pull in pounds, B is the flux density in lines per square inch, and A is the area of the air gap in square inches. Since the pull is proportional to the square of the flux density, it is important to keep this quantity as high as possible without carrying the iron or steel of the magnetic circuit to saturation.

The pull of electromagnets is used for many purposes in engineering, and it is possible to illustrate but a few of them here. One of the most common types which may be obtained commercially is known as a tractive magnet, because it provides a pull over a specified distance. A picture of such a tractive

magnet is shown in Fig. 2-7, and the corresponding dimensioned sketch of the magnetic circuit is shown in Fig. 2-8. It is observed that the movable element completes the magnetic circuit through the center of the coil and that the air gaps involve both a small gap at the side of the moving element and a larger one at the end. The flux on the two sides of the plunger is approximately equal, and the forces thus balance; but the flux leaving the end of the plunger produces an uncompensated force that *causes the tractive pull of the magnet.* The computation of

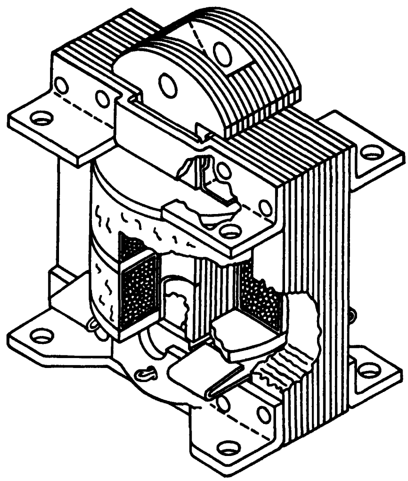


FIG. 2-7. A Commercial Form of Tractive Magnet.

the magnetic forces of such a tractive magnet is illustrated in the following example which also provides a study of magnetic circuits having parallel magnetic paths.

Example. Determine the number of ampere-turns required to produce a 50-lb pull in the tractive magnet shown in Fig. 2-8 when the distance x is 2 in.

Solution: (1) Determine the flux density required in the air gap.

$$\text{Pull} = 1.38B^2A \times 10^{-8}$$

where Pull = 50 lb and $A = 2 \times 1\frac{1}{2} = 3$ in.^{2}

$$50 = 1.38B^2 \times 3 \times 10^{-8}$$

$$B^2 = \frac{50 \times 10^8}{3 \times 1.38} = 12.06 \times 10^8$$

$$B = 34{,}800.$$

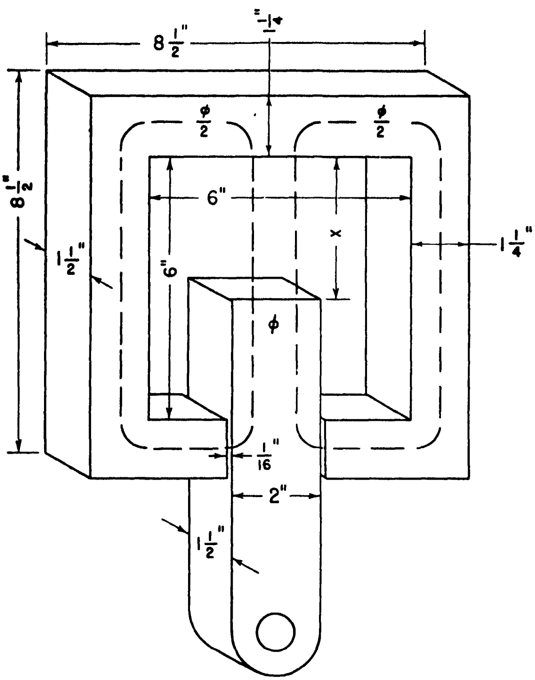


FIG. 2-8. A Simplified Magnetic Circuit of Fig. 2-7

(2) Determine the ampere-turns to force the required flux across the main air gap.

$$\begin{aligned} NI &= 2 \times 0.313B \\ &= 2 \times 0.313 \times 34{,}800 \\ &= 21{,}800 \end{aligned}$$

(3) Determine the ampere-turns required for the secondary air gap. Since only half the flux crosses each secondary gap, the flux across each path is

$$\phi = \frac{B \times A}{2} = \frac{34{,}800 \times 3}{2} = 52{,}100$$

The flux density will then be

$$B' = \frac{\phi}{A'} = \frac{52{,}100}{1.5 \times 1\frac{1}{4}} = 27{,}800$$

$$NI = \frac{1}{16} \times 0.313B = \frac{0.313 \times 27{,}800}{16} = 545.$$

(4) Determine the ampere-turns required for the iron portion of the circuit (armature grade sheet steel).

Since the magnetic path of the sides is really composed of two parallel paths, each taking half the flux, then the flux in each path is 52,100 and the density is the same as in the secondary air gap since the area is the same. From the curve in Fig. 2-3 the NI per inch at a density of 27,800 is about 1. Since the length of each path is about 15 in. in iron, then the total ampere-turns for the iron (other than the plunger) would be

$$NI = 1 \times 15 = 15.$$

The ampere-turns required for the plunger are next calculated. The flux density in the plunger is the same as for the main air gap, which is 34,800. The length of the circuit in the plunger is about 5 in. From the curve of Fig. 2-3 the NI per inch at 34,800 is slightly greater than 1. The ampere-turns for the plunger are therefore

$$NI = 1 \times 5 = 5.$$

The total magnetomotive force for the iron is the sum of

$$15 + 5 = 20 \text{ ampere-turns}.$$

(5) Determine the total ampere-turns to produce the necessary flux.

Since the flux paths are in parallel, the magnetomotive force is that required to force the flux through one of the paths which is

$$\begin{aligned} NI &= 21{,}800 + 545 + 20 \\ &= 22{,}365, \text{ or approximately } 22{,}400 \text{ ampere-turns.} \end{aligned}$$

Exercise 2-3. Determine the current and voltage required in the above example if the magnetomotive force is supplied by a coil composed of 2000 turns of #14 copper wire having a mean length per turn of 14 in.

Ferromagnetic theory

It has been seen that soft iron and steel may be treated as nonlinear magnetic circuit elements in order to obtain engineering solutions to design problems. Although these methods (with such refinements as may be required for the particular problem) are the methods used in engineering design, it must be realized that the magnetization curves are not single-valued curves, but represent only approximate values even for soft iron. In order to understand the limitations of magnetic circuit computations and to be able to intelligently specify permanent magnets, whose commercial importance is rapidly increasing, it is necessary to review more completely the fundamental concepts of ferromagnetic theory.

It has been mentioned that modern magnetic theory associates all magnetic effects with moving electrons. The magnetic characteristics of iron appear to be largely the result of the uncompensated spin of some electrons in one of the orbital groups of the iron atom. The magnetic character of the iron atom causes atoms adjacent to each other (see Fig. 2-9) to align themselves in the form of very minute permanent magnets called domains. These domains are often irregular in shape but tend to align themselves with one of the three axes of the cubical iron crystal. However, there are forces of an interatomic nature that have the general character of frictional forces

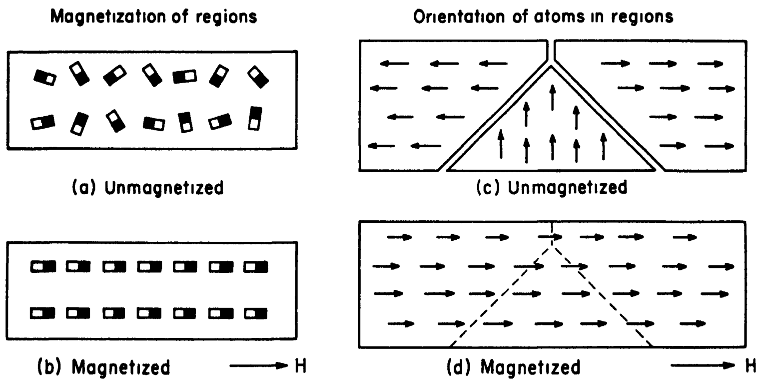


FIG. 2-9. Molecular and Regional Characteristics of Ferromagnetic Materials.

that tend to oppose any change in the magnetic condition of the domains. These forces have characteristics similar to "static friction" in that a certain minimum magnetic field strength is necessary to cause alignment of any single domain; but when this value is reached, that domain suddenly snaps into magnetic alignment, as indicated by Fig. 2-9(d). As the magnetomotive force or field strength is increased, one after another of these domains becomes oriented and the magnetic flux becomes progressively larger. When all domains are oriented, the material is said to be saturated.

The frictional character of these atomic forces tends to hold the domains in alignment and to produce a permanent magnet. Thus the magnetic flux is considerably higher for a given magnetic field strength when it is decreasing than when the field strength is increasing. The magnetization curve is not, therefore, single-valued; but the flux density corresponding to any specified value of field strength is dependent upon the previous

flux value. This frictional phenomenon in ferromagnetic materials is known as hysteresis.

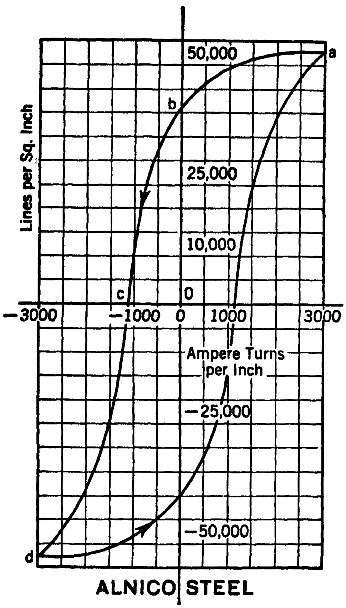
Hysteresis loops

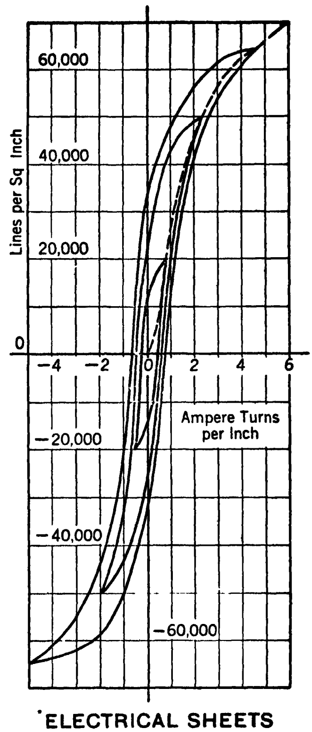
Since much of the magnetic material used in electrical machinery has continuously varying magnetomotive forces applied, the hysteresis is normally specified by a curve of the value of magnetic flux density as it varies from a maximum positive magnetomotive force to zero, to maximum negative, to zero, and back to maximum positive value again. Such a curve is called a *hysteresis loop* and indicates quite completely the magnetic characteristics of the material.

The hysteresis loops of several different materials are shown in Fig. 2-10 and will be used as a preliminary basis for study. The hysteresis loop for Alnico, which is a permanent magnet material, is most broad. It will be used, therefore, to indicate the manner of specifying such a curve and of defining certain magnetic terms. A large value of flux density is reached at *a* with a certain field strength in ampere-turns per inch, which in this case is 3000. The value of this flux density is about 53,000 lines/in.^{2} in this material. If the magnetic field strength is reduced to 1000 ampere-turns/in., the flux density will be reduced only to about 50,000 lines per sq in. In other words, the frictional forces in the magnetic material are so great that even though the field strength is reduced from 3000 to 1000 ampere-turns/in., the majority of the individual domains would retain their magnetic alignment and thus retain a flux density of 50,000 lines/in.^{2}

When the external magnetomotive force is entirely removed, there will still be a flux density, measured by *ob*, of 40,000 lines/in.^{2} in the figure. This value of flux density is called the *residual magnetism* and is the flux that still remains in the iron when the external magnetomotive force or field intensity is reduced to zero.

If the external magnetomotive force or field intensity is reversed and increased to 500 ampere-turns/in., the frictional forces of many of the domains will be overcome; and they will reorient themselves in the opposite direction, thus reducing the flux density to 32,000 lines. By the time the field intensity in the reverse direction has been increased to 1100 ampere-turns/in., enough domains have been reversed to reduce the flux density to zero. The value of field intensity necessary to





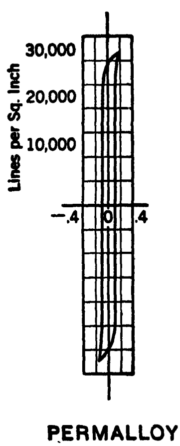


FIG. 2-10. Hysteresis Loops for Alnico Steel, Electrical Sheets, and Permalloy.

accomplish this result is known as the *coercive force;* and in this illustration it is the value *oc*, or 1100 ampere-turns per inch.

As the field intensity increases, the frictional forces of the remaining domains are rapidly overcome until the maximum value of flux density (53,000 lines/in.^{2}) is again obtained at *d*, but in the reverse direction from *a*. If the field intensity is now reduced to zero and then reversed and increased to the original value, the magnetic material will pass through the same cycle it did in going from *a* through *b* and *c* to *d*, but in the reverse direction. The completed curve is known as a hysteresis loop, and in the case of this particular material the high value of the residual magnetism and coercive force indicate that it will be useful as a permanent magnet.

Magnetic characteristics of electrical sheets. By far the greatest volume of magnetic material is in the form of silicon-steel sheets known in the industry as electrical sheets. These steel sheets are used to form the magnetic circuits for nearly all electrical machinery and are, therefore, an important commercial product, millions of tons being produced each year.

Typical hysteresis loops are shown in the upper right portion of Fig. 2-10 for electrical sheets. It will be noted that hysteresis loops are drawn for maximum flux densities of 20,000-, 50,000-, and 65,000-, lines/in.^{2} maximum values.

Since the energy loss for each reversal is proportional to the area of the hysteresis loop, the loss increases rapidly as the maximum flux density increases. It is usually uneconomical therefore to operate a-c machinery at high flux densities. It will be shown later that the loss in the iron includes also I^2R losses resulting from eddy currents that flow in the iron. These two losses are normally combined at any specified frequency and reported in the form of curves of power loss in watts per pound (or per cubic inch) of the material plotted against the maximum flux density.

It will be noted that a dotted line is drawn through the points of the hysteresis loops. This is the magnetization curve shown previously in Fig. 2-3 for electrical sheets. It is now clear that this magnetization curve does not completely specify the flux density in terms of the magnetizing force since there is a wide difference between the flux density on increasing and decreasing values of magnetomotive forces. Where an air gap is involved, however, the effect of this difference on the flux is slight since the ampere-turns to overcome the air gap are usu-

ally much larger than are involved in overcoming the reluctance of the iron.

The ratio of the flux density to magnetomotive force is a measure of the ease with which it is possible to magnetize a material. The term that is used to describe this characteristic of a material is called the *relative permeability*. It may be defined as the ratio of the flux produced in a magnetic material by a specified field intensity to the flux that would be produced by the same field intensity in air. In ordinary electrical sheets the maximum value of this permeability is in the order of 6000 to 12,000.

CHAPTER 3

Direct-Current Measurements

Permanent-magnet moving-coil meters

The practical use of circuit theories that were discussed in Chap. 1 depends to a great extent upon an ability to measure the magnitudes of the currents, the voltages, and the resistances of equipment used in engineering practice. The student's ability to analyze and to interpret correctly the variations in meter readings is the ultimate justification for circuit analysis.

The use of d-c instruments to determine temperature, flow, acidity, and many other quantities important to manufacturing processes makes a thorough understanding of the theory of these devices important for the engineer.

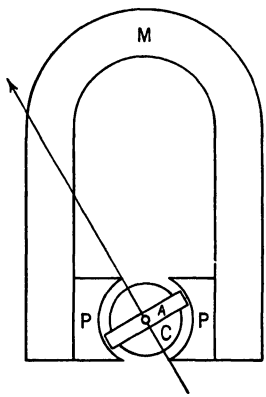


Fig. 3-1. A Permanent-Magnet Moving-Coil Meter.

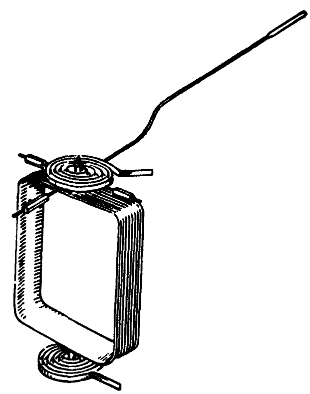


Fig. 3-2. The Moving-Coil Assembly of Fig. 3-1.

Nearly all d-c measurements sooner or later are dependent upon a galvanometer or permanent-magnet moving-coil type of meter, which will be studied first and in some detail. Fig. 3-1 shows the main parts of such a meter. The permanent magnet M supplies a magnetic field that is controlled by the soft-iron pole pieces P and the core C, so that the gaps have a radial field of uniform magnitude. A coil assembly A, shown in detail in Fig. 3-2, is supported in jeweled bearings so that it is free to

rotate back and forth in the air gap. The coil is composed of many turns of fine wire, usually wound on a copper or aluminum coil form. The ends of the coil are connected to hair springs located on the top and bottom of the coil assembly.

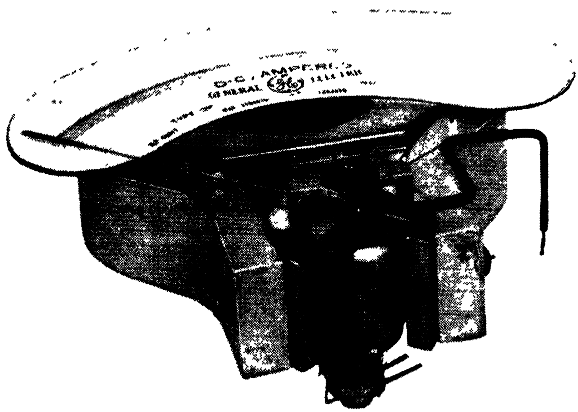


FIG. 3-3. Element of Permanent-Magnet Moving-Coil Instrument Cut Away to Show Construction.

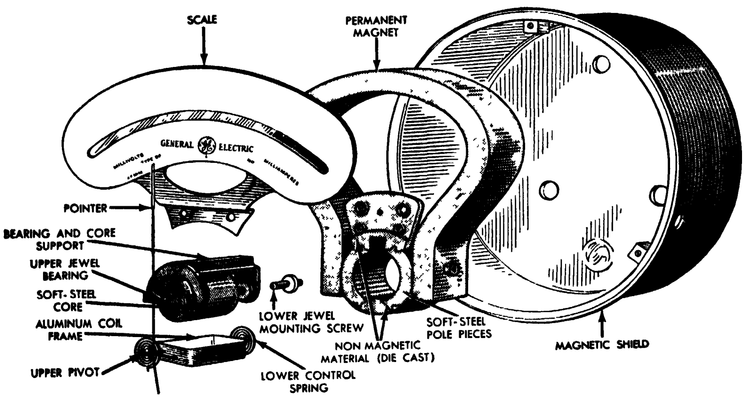


FIG. 3-4. Disassembled Parts of a Typical Permanent-Magnet Moving-Coil Instrument.

These hair springs perform the dual function of providing a torque tending to restore the coil to its zero position and of providing the electrical connections to the coil. The current in the coil reacts with the magnetic field to produce a torque that is proportional to the current. This causes the coil to rotate until the electromagnetic torque and the restoring torque of the springs are equal. The angular deflection is propor-

tional to the current flowing in the coil and is indicated by the pointer as it moves along a calibrated circular scale, as shown in Fig. 3-3. Such a meter is generally known as a permanent-magnet moving-coil instrument. A drawing of the disassembled parts of such a meter is given in Fig. 3-4.

The physics of the permanent-magnet moving-coil meter. When a conductor that is carrying current is located in a magnetic field, a force or sidewise thrust is exerted upon it. The effective thrust is greatest when the wire is perpendicular to both the possible direction of motion and the direction of the field. This is caused by the magnetic interaction of the per-

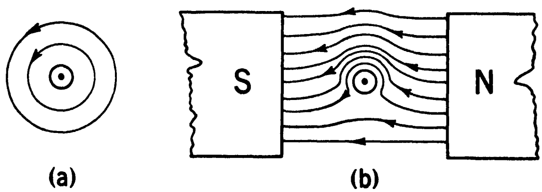


FIG. 3-5. Force Exerted on a Conductor which is Located in a Magnetic Field and Carries Electric Current.

manent magnet field and the field caused by the current, as shown in Fig. 3-5. In this diagram an end view of the wire is shown in (a) with current flowing toward the reader. The magnetic field produced by this current is shown to be in the form of concentric rings of flux around the wire that flows in a counterclockwise direction. When the conductor is placed in a uniform magnetic field as shown in (b), the current flowing in the wire distorts the field so that an excess of lines are concentrated above the wire, whereas below the wire there are less than the usual number of lines. The apparent tendency of the lines to exert a lateral force on each other and on an electric conductor producing such distortion forces the conductor downward. The magnitude of the force on unit length of the conductor is proportional to the flux density in the air gap and the magnitude of the current. Expressed mathematically, this is

$$F = 8.84 \times 10^{-8} BlI \text{ lb}$$

where B is the flux density in lines per square inch, l is the length in inches, and I is the current in amperes.

The manner in which this is applied to the foregoing meters is shown in the following example.

Example. Determine the torque produced on the coil of the meter in Fig. 3-1 when it is carrying 1 ma of current. The coil is square, having a length and width of $\frac{3}{4}$ in., and is composed of 100 turns of fine wire. The air-gap density is 60,000 lines/in.^{2}

Solution: (1) Determine the sidewise thrust on each coil side.

$$\begin{aligned} F &= 100 \times 8.84 \times 10^{-8} \times 60{,}000 \times \tfrac{3}{4} \times 0.001 \\ &= 3.98 \times 10^{-4} \text{ lb} \\ &= 6.36 \times 10^{-3} \text{ oz.} \end{aligned}$$

(2) Determine the torque in inch-ounces.

$$\begin{aligned} T &= F \times D = 2 \times 6.36 \times 10^{-3} \times \tfrac{3}{8} \\ &= 4.77 \times 10^{-3} \text{ in.-oz} \quad (\textit{Ans.}) \end{aligned}$$

As indicated in the example, the forces involved in meters of this type are very small, and it is necessary to use great precision in manufacture and considerable care in the use of these instruments if accurate results are to be obtained. If a meter is to be accurate to $\frac{1}{2}$ per cent of its full scale reading and if the total torque is only about 5×10^{-3} in.-oz, then the friction torque must be limited to less than 2.5×10^{-5} in.-oz. Since friction is one of the important limiting factors in meter accuracy, it can be recognized that a meter that will read 1 amp for full-scale deflection may have an accuracy of only ± 0.01 amp. If then the meter is being used to measure 0.10 amp, the error will be the same, but the maximum percentage accuracy that can be assumed is 10 per cent.

As indicated in the example, the permanent-magnet moving-coil instrument is a current-measuring device. Since the parts are small, the current is usually limited to a few milliamperes and often is less than 1 ma.

Exercise 3-1. A small instrument has a coil $\frac{3}{8}$ in. square. It is wound with 100 turns of wire and the field strength in the air gap is 50,000 lines/in.^{2} What is the torque when it has a current flow of 50 μa?

Exercise 3-2. If the coil in Problem 1 above has an axial dimension (width) of but 0.1 in., how many layers of #40 enameled wire would be required? What air gap would be needed if 0.01 in. of clearance is required on either side of the coil? What is the coil resistance?

Voltage measurement

In order to use this meter to measure voltage it is connected in series with a resistor that will limit the current through the

moving coil to the value that produces full-scale deflection at the maximum voltage for which the meter is calibrated. This resistor is made from manganin or some other resistance wire that has a very low temperature coefficient. This assures that the meter reading will not be much affected by temperature variation.

The smaller the current taken from the voltage source when full-scale voltage is applied, the more *sensitive* is the instrument. Sensitivity is usually expressed in the ratio of the resistance to full-scale volts, or in *ohms per volt.* In a meter having a sensitivity of 1000 ohms/v and a full-scale voltage of 300, the resistance would be 300,000 ohms. Direct-current voltmeters are commonly manufactured in sensitivities of 100, 1000, 10,000, and 20,000 ohms/v.

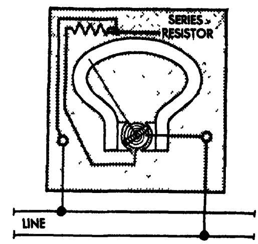


FIG. 3-6. Voltmeter with Internal Resistor.

Exercise 3-3. A voltmeter has a full-scale reading of 150 v and is marked on the scale as having a resistance of 150,000 ohms. What external resistor would be needed to make it possible to use it as a 300-v voltmeter?

Exercise 3-4. If the only resistor available in Exercise 3-3 was one having 100,000 ohms, could the meter be used to measure 240 v? What would the scale reading be?

Exercise 3-5. A meter calibrated as a milliammeter with 0.10 ma full-scale deflection is available and has a resistance of 1000 ohms. It is desired to make a multirange voltmeter with the following full-scale ranges: (a) 1.0 v, (b) 10 v, (c) 100 v, and (d) 500 v.

Show by diagram how you would construct such a meter, indicating the values of resistors you would use.

A voltmeter should make no appreciable change in the circuit to which it is connected. If appreciable current is drawn from the circuit, conditions will change when the voltmeter is connected, and erroneous results will be obtained. In vacuum-tube circuits, therefore, it is almost necessary to use meters with sensitivities of 10,000 and 20,000 ohms/v. In power circuits, where the current values are larger and circuit resistances are lower, the more rugged meters of 100 and 1000 ohms/v are preferred.

By the use of Ohm's law a voltmeter may be used to measure quite high resistances. For instance, a voltmeter having a resistance of 150,000 ohms reads 120 v when connected to the line. When connected in series with the unknown resistance,

it reads 40 v. The current in the meter has been reduced to $\frac{40}{120}$ of its previous value; the total series resistance is, therefore, $\frac{120}{40}$ of the original value, or 450,000 ohms. The added resistance is 300,000 ohms.

Exercise 3-6. It is desired to measure the leakage resistance of the insulation of an electric motor. A 300-volt voltmeter having a sensitivity of 1000 ohms/v is connected in series with the winding so that the only current flow is through the insulation to the ground. When connected to a 250-volt d-c line, the voltmeter reads 56 v. What is the value of the insulation resistance?

The coil of the meter is usually made of copper, which has a high temperature coefficient. If such a meter is used without added resistance to measure very low values of voltage, the results will be subject to a considerable temperature error. As a result, meters that are to measure very low values of voltage are designed for a low coil resistance, so that a series resistor of several times the coil resistance may be added and thus reduce the temperature error to a negligible amount. Such a meter is often calibrated to measure either 200 or 50 mv at full-scale deflection.

Current measurement

Although permanent-magnet moving-coil instruments may be designed to measure directly small values of current, such as a very few milliamperes, the usual use of these instruments for current measurement involves the use of a *shunt*, which is a resistor of low value designed to have a specified millivolt drop at the rated current value. This value is usually 50 mv, although higher values are used for many accurate meters. Manganin is usually used in these shunts to eliminate temperature errors. The arrangement of such a current-measuring device is shown in Fig. 3-7. Here the shunt is connected in the circuit of the current to be measured. A millivoltmeter with calibrated leads connected to the drop terminals of the shunt gives full-scale deflection when rated current is flowing through the shunt. For instance, if the shunt is a 25-amp 50-mv shunt, then the millivoltmeter must give full-scale deflection when 50 mv are across the ends of the calibrated leads.

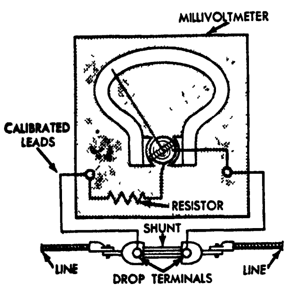


FIG. 3-7. Millivoltmeter with External Shunt.

It is usual to supply a meter scale marked with 25 amp for full scale and so divided as to provide maximum convenience in reading.

Many college laboratories are equipped with shunts of varying size and with millivoltmeters having scales divided into 100 divisions. In such an arrangement the reading of the meter, in the fractional part of the full scale, will be multiplied by the rating of the shunt. Care must be taken to use the special drop terminals on the shunt for the millivoltmeter connection. If the millivoltmeter leads should be connected to the line terminal, an appreciable error would result from contact-resistance drop.

Since the resistances of shunts are quite low, the shunt and millivoltmeter may be used to measure low values of resistance. If a current sufficient to give reasonable deflection on the ammeter is flowing through the resistor of unknown value connected in series with the shunt, then the millivoltmeter may be disconnected from the shunt and connected across the unknown resistor to determine its voltage drop. For instance, if the shunt and millivoltmeter show that the current flow is 80 amp and if the voltage drop across the unknown resistor is 10 mv, the resistance is

$$R = \frac{E}{I} = \frac{0.010}{80} = 0.000125 \text{ ohms}$$

Great care must be exercised so that the millivoltmeter is not damaged in this type of test. Some preliminary method of determining resistance should have determined that the value is sufficiently low that the drop would not cause the millivoltmeter to be thrown off scale.

Exercise 3-7. A phosphor bronze strip has been determined to have a resistance value less than 0.1 ohm. When a current of $\frac{1}{2}$ amp is put through it, the millivolt drop is about 3 mv. What would be your next step in the measurement of this resistance?

Exercise 3-8. The resistance of the series field of a 10-hp 125-volt d-c motor is to be measured. A current of 7.2 amps produces a millivolt drop of 43 across the field circuit. What is the resistance?

Multirange instruments. To obtain greater flexibility in use, instruments are often made for operation with more than one scale. In current-measuring instruments this is accomplished by connecting a tapped shunt as illustrated in Fig. 3-8. As the diagram shows, the entire shunt is in series with the line,

and the coil is connected across the entire shunt. For the low range the line connection is made to the end terminals of the shunt. For the high range the line connection is made to the top of the shunt so that the current flows through only part of the shunt, and the resistance of the remainder of the shunt is in series with the instrument coil. Scale ratios obtained in this manner are usually between 4:1 and 10:1.

Multirange voltmeters are constructed with tapped series resistors as shown in Fig. 3-9. The instruments are normally

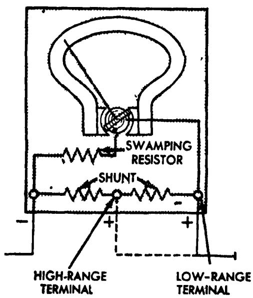


FIG. 3-8. Dual-Range Ammeter with Internal Shunts.

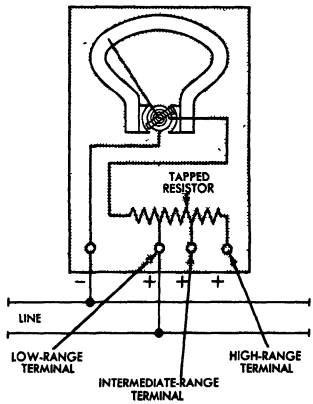


FIG. 3-9. Multirange Voltmeter with Internal Resistors.

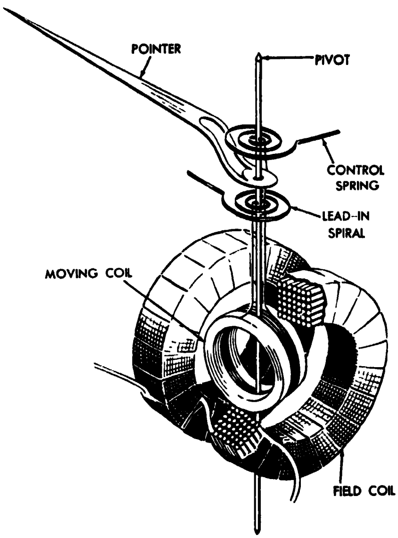
designed for operation at the lowest range, and then added series resistors are provided to extend the range to the desired value.

The dynamometer type of instrument

Electric power in d-c circuits may be obtained from the product of the current and voltage as determined from the permanent-magnet moving-coil meters just described, or it may be measured directly by a wattmeter. The use of the wattmeter is preferable in many cases, particularly where there are rapid changes of load. The wattmeter is an electrodynamometer instrument in which the magnetic field is supplied by a coil carrying electric current rather than by a permanent magnet.

The essential parts of such an instrument are shown in Fig. 3-10. The meter is connected to the circuit as indicated in Fig. 3-11, where it is seen that the current of the circuit flows through

the field coils, and the moving coil is connected across the line through a high resistance to limit the current to a very small value. Thus the current in the moving coil is proportional to the voltage, and the magnetic field is proportional to the current. Since the torque on the meter element is proportional to the product of the current in the moving coil and to the magni-



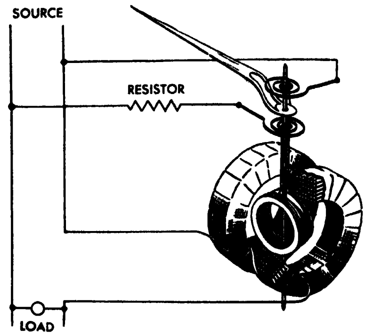


FIG. 3-10 (left). Dynamometer Element.

FIG. 3-11 (above). Dynamo-Element Used as a Wattmeter.

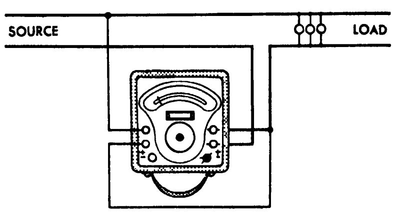


FIG. 3-12. External Connection Diagram for a Wattmeter.

tude of the magnetic field, it follows that the torque and the deflection of the meter are proportional to the product of voltage and current or to the electric power.

The standard connection diagram for such a meter is shown in Fig. 3-12. It will be noted that each coil has one terminal identified as $\pm$. This terminal of the voltage circuit is connected to the moving coil, whereas the other terminal is connected to the resistor. The coil side of the circuit should always

be connected to the side of the line that is carried through the meter, as this connection will eliminate potential stresses on the insulation as well as electrostatic forces that may cause errors.

Electrodynamometer meters may be arranged to measure current or voltage. (In fact, the most accurate a-c meters for current and voltage are of this type.) The connection arrangement for using this type of meter to measure voltage is shown

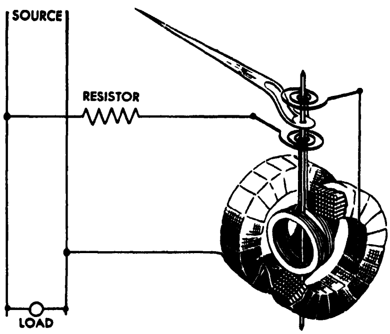


FIG. 3-13. Dynamometer Element Used as a Voltmeter.

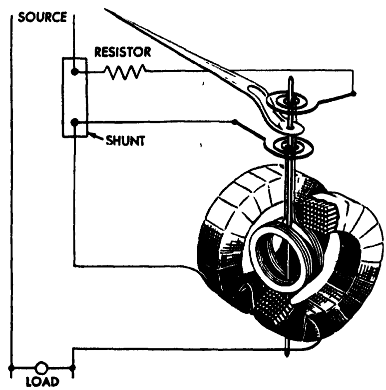


FIG. 3-14. Dynamometer Used as an Ammeter.

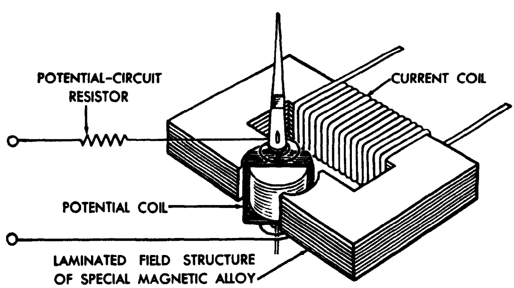


FIG. 3-15. Diagram of Iron-Core Dynamometer Element for Wattmeter.

in Fig. 3-13, whereas the current-measuring connection is shown in Fig. 3-14. In both of these instruments the torque and deflection are proportional to the square of the current flow, so that the scales are not uniform as they are in the permanent-magnet moving-coil type of instruments.

A slight variation in the construction of the dynamometer type of instrument uses an ironcore magnetic circuit as shown in Fig. 3-15. This type of construction is satisfactory as long as the reluctance of the iron portion of the circuit is negligible

in comparison to the reluctance of the air gap. This is easily accomplished with the new low-reluctance alloys. It permits a much smaller movement and uses greater flux density than the conventional construction.

The use of this dynamometer type of instrument for measurement of a-c quantities will be discussed in a later chapter.

The galvanometer. The term *galvanometer* is applied to high-sensitivity permanent-magnet moving-coil instruments

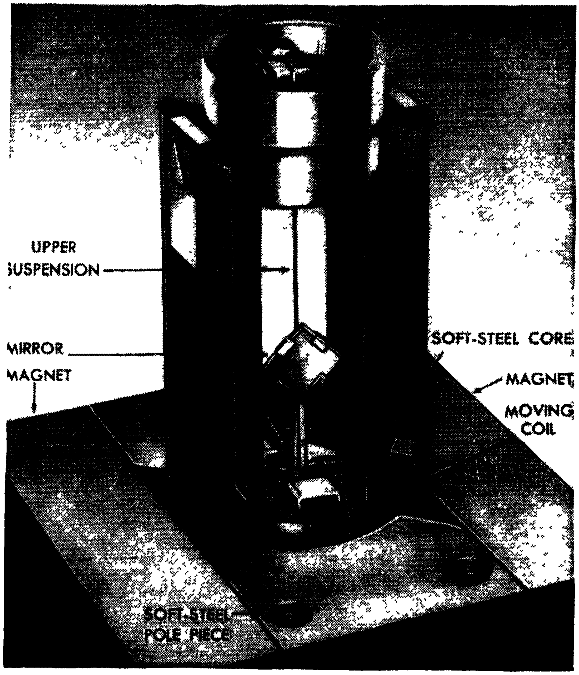


FIG. 3-16. Principal Parts of Light-Beam-Galvanometer Mechanism.

used for detecting or measuring very small values of electric current or voltage. Usually this high sensitivity is obtained by suspending the moving-coil in the magnetic field by means of fine metal ribbon, instead of using a shaft pivoted on jeweled bearings. The coil may be suspended from an upper support by the fine metal ribbon, in which case the upper support and ribbon act as one electrical connection to the coil, and the other connection is brought out at the bottom through a very flexible fine wire. A more rugged construction provides for the coil to be suspended between two strips of gold alloy soldered to spring

mountings maintaining the correct tension on the suspensions. The upper suspension carries one electrical connection to the coil, while the lower one supplies the second connection. The alloy strips not only support the coil but also provide the counter-torque, since a rotation of the coil causes a twist in the suspension. The construction of a typical galvanometer is shown in Fig. 3-16. The moving coil is observed to be suspended in the magnetic field, and in this construction both top and bottom suspensions are used.

The deflection of the galvanometer is indicated by a light beam reflected from the small mirror that is mounted on the

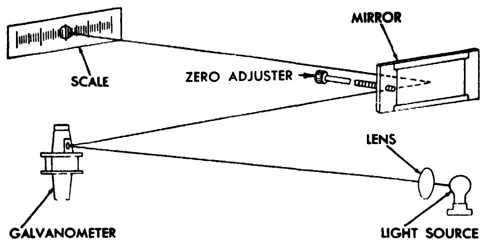


FIG. 3-17. Diagram of Optical System for Typical Light-Beam Galvanometer

coil. This light beam provides the *pointer*, and with it pointer lengths of 1 to 3 ft may be obtained with no added weight to the moving element.

The Wheatstone bridge

The Wheatstone bridge is a special type of parallel circuit by which it is possible to compare the value of an unknown resistor with that of a known or calibrated resistor. It consists of four resistors as shown in Fig. 3-18. Resistors R_1 and R_2 are fixed in magnitude, and R_x is the unknown resistor. The resistor R_s is calibrated and variable. In operation the magnitude of R_s is adjusted until no current flows in the galvanometer G. The bridge is then said to be balanced, and the point d is at the same potential as point c. The following equations result:

$$R_1I_1 = R_2I_2 \quad \text{and} \quad R_sI_1 = R_xI_2.$$

Dividing,
$$\frac{R_1I_1}{R_sI_1} = \frac{R_2I_2}{R_xI_2} \quad \text{or} \quad \frac{R_1}{R_s} = \frac{R_2}{R_x}.$$

Therefore,
$$R_x = R_s\frac{R_2}{R_1}.$$

If R_1 is equal to R_2, as is often the case, then R_x is equal to R_s. This is known as an equal-arm bridge and is most accurate if R_1 and R_2 are of the same order of magnitude as R_x. The accuracy of the results will depend upon accuracy of calibration of the known resistors, the elimination of contact resistance between the bridge elements, and the sensitivity of the galvanometer.

With all of the above elements carefully adjusted, measurements of resistance to an accuracy of 4 or 5 significant figures

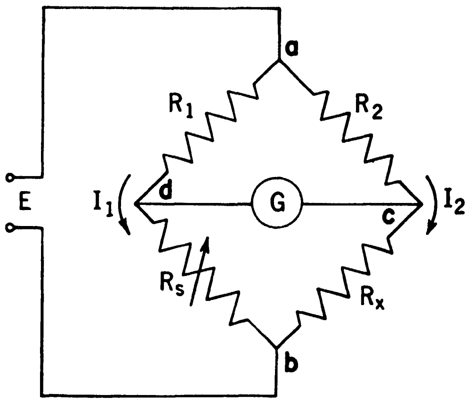


Fig. 3-18. Resistance Bridge.

can be achieved. When the unknown becomes appreciably different from R_1, then it is usually desirable to make R_2 of the same order of magnitude as R_x. In most commercial bridges this is achieved by adjusting R_2 by factors of 10. Thus, if R_1 is 1000 ohms, then R_2 may be 100,000, 10,000, 1000, 100, 10, or 1. The various settings of R_2 would correspond to multiplying factors of 100, 10, 1, 0.1, 0.01, and 0.001. When the multiplying factor differs greatly from unity, as in the case of 100, 0.01, and 0.001, there are effects that tend to reduce the accuracy of measurement. For measurements of resistance values below 1 ohm and above 1,000,000 ohms, special precautions must be observed to obtain resonable accuracy of results.

The Wheatstone bridge is extensively used in industrial instruments for the measurement of temperature, strain, and certain other industrial quantities. The advantage of the Wheatstone bridge is that it is a *null method.* Thus, it is not dependent upon the calibration of the galvanometer but only upon its sensitivity. Therefore, it is possible to use a very

sensitive galvanometer to determine when the bridge is balanced; and if the bridge elements are accurate, excellent results can be obtained. Results are independent of the impressed voltage and resistance of most lead wires.

Exercise 3-9. What is the resistance of the unknown if $R_1 = 1000$, $R_2 = 100$, and $R_s = 5673$ ohms?

Exercise 3-10. If the galvanometer can detect a difference of 100 μv, what maximum accuracy can be expected when measuring an unknown resistance with the following bridge values:

$R_1 = 1000$, $R_2 = 1.0$, $R_s = 473$ ohms? Assume a 2-v battery supply.

The potentiometer

Another instrument involving equipment quite similar to the Wheatstone bridge and using the null method of balancing

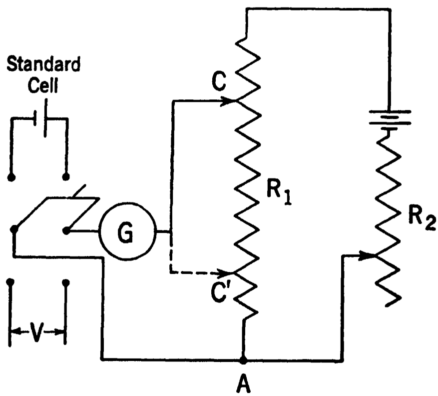


FIG. 3-19. Elementary Potentiometer Circuit.

voltages is known as the potentiometer. It is used to obtain accurate measurements of small voltages, such as the voltages developed by thermocouples.

Fundamentally, the potentiometer consists of a resistor or drop wire carrying a constant current. An elementary form of this circuit is shown in Fig. 3-19. In this figure the resistor R_1 is an accurately calibrated drop wire. Its total resistance might be 1500 ohms. If the variable contact is set so that the resistance from A to C is 1018.3 ohms, then the voltage from C to A would be 1.0183 volts if the current in drop wire were accurately adjusted to one milliampere. Since the voltage of a standard cell is just this voltage, the switch can be thrown in the "up"

position and R_2 then adjusted until a null point is reached. This then gives just one milliampere in the drop wire. This adjustment of the current to the exact amount by comparing the voltage drop to the voltage of a standard cell is often referred to as *standardizing the current.*

When the double-pole switch is thrown to the "down" position and connected to an unknown voltage which is to be measured, the connection at C is adjusted along the resistor R_1 until no galvanometer deflection is obtained. The voltage at C', the point of zero deflection, is now equal to the unknown voltage;

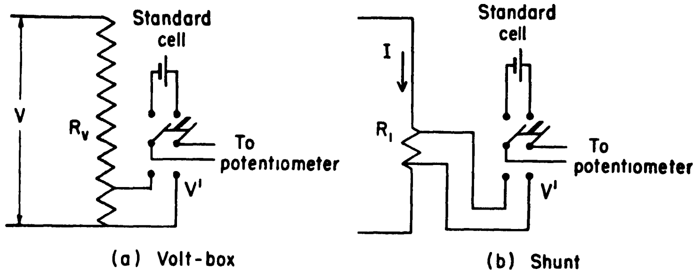


FIG. 3-20. The Use of a Standard Volt-Box and Shunt for Voltage and Current Measurement, Using a Potentiometer.

since the resistance between A and C' is known from the calibration and since the current is 1 ma, the voltage between A and C' is known.

When it is desired to measure voltages that are greater than 1 v, it is customary to use a fixed potentiometer, or volt box, to take off a certain portion of the unknown voltage to measure, as shown in Fig. 3-20 (a). For instance, if it is desired to measure a voltage of 84.37 v, then R_v of Fig. 3-20 (a) would have a total resistance of 100,000 ohms with a tap taken off at 1000 ohms. In this case only 0.01 of the voltage would appear at the voltage terminals V', and the potentiometer would measure a voltage of 0.8437 v.

When it is desired to measure current, a shunt is used, and the potentiometer is used to measure the millivolt drop in the shunt. This circuit is shown in Fig. 3-20 (b). In this case if it is desired to measure a current of 74.67 amp, a shunt having a resistance of 0.01 ohm will be used, giving a drop of 0.7467 v which will then appear at V' to be measured by the potentiometer.

CHAPTER 4

Electromagnetic Induction

Voltage induced in a coil

Just as electric current and magnetic flux are associated, so also a *change* in magnetic flux is normally associated with an electric pressure or voltage. In the case of radio waves the changes of magnetic fields are associated with corresponding changes in electric fields and potentials, which changes are propagated through space and are detected by means of an aerial and radio receiver set. This type of relation between electric and magnetic fields is seldom found in industrial electricity. The study of electromagnetic induction in this chapter will be limited to the restricted, but extremely important, portion of the subject dealing with electrical machinery and other industrial devices.

It should be remembered that electrical engineering is based on the science of physics, and that physics is based on laboratory experiments. All phenomena in electrical engineering are, therefore, based on the fundamental experiments of physics. It is well to review one of these experiments as a basis for the study of electromagnetic induction.

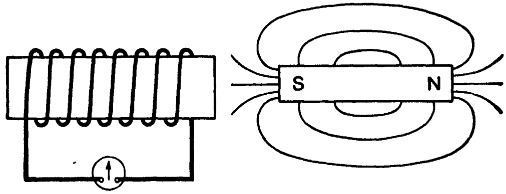


FIG. 4-1. A Coil and a Bar Magnet.

Suppose a wire is wound on a cardboard tube, as shown in Fig. 4-1, and the ends of the wire are connected to a galvanometer or a sensitive meter with a zero center scale. A permanent bar magnet is available whose magnetic field, as shown in Fig. 4-1, is the same as previously discussed in Chap. 2.

When the bar magnet is suddenly thrust into the coil, as shown in Fig. 4-2, the galvanometer pointer is observed to swing to the right. This indicates that a voltage has been produced in the coil, which has forced an electric current through the circuit. This voltage and current are observed, however, to be momentary only, since, when the magnet is stationary in the center of the coil, as shown in Fig. 4-3, the galvanometer

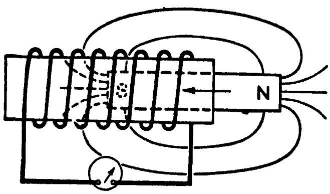


FIG. 4-2. The Bar Magnet Being Inserted in the Coil.

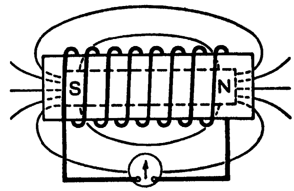


FIG. 4-3. The Bar Magnet at Rest in the Coil.

pointer returns to zero, indicating that no voltage is being produced. It is concluded, therefore, that the voltage produced is associated with the relative movement of the magnet with respect to the coil.

When the bar magnet is suddenly withdrawn from the coil, as shown in Fig. 4-4, the galvanometer pointer swings to the left, which indicates that a voltage is again produced momentarily. This time, however, the direction of the voltage and its associated current has been reversed, since the galvanometer deflection is toward the left instead of toward the right.

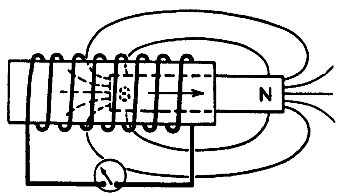


FIG. 4-4. The Bar Magnet Being Withdrawn from the Coil.

These apparently simple phenomena are the keys to a very comprehensive system of interrelationships between the change of magnetic flux, the voltage produced, and the resultant current. Conclusions include the following:

a) Relative motion between a coil and a magnetic field produces a voltage.

b) A reversal of the direction of the relative motion reverses the direction or polarity of the voltage generated.

It can be demonstrated that the current produced in the coil in each case is in such a direction as to oppose the change in flux. That is, a magnetomotive force is produced that tends to oppose the increase of flux when the magnet is being thrust into

the coil, and opposes the decrease in flux when the magnet is being withdrawn. Upon analysis it is found that the current in the coil is also in such a direction that it produces a force that opposes the movement of the magnet, thus conforming to the law of conservation of energy.

The above relationship may be stated as follows:

In any variation of a magnetic field with respect to a coil which will cause a voltage to be generated, the voltage will be in such a direction as to produce a current in the coil, the magnetomotive force of which will oppose the original variation of magnetic flux.

This relationship is known as Lenz's law and may be used to solve many electromagnetic problems.

Magnitude of the induced voltages. If the experiments of Fig. 4-1 to 4-4 were repeated using different rates for insertion and withdrawal of the magnet, it would be observed that the magnitude of the swing of the galvanometer would be proportional to the rate at which the magnetic field moved with respect to the coil.* Stated another way, the voltage induced is proportional to the rate of change of magnetic flux. Mathematically this is expressed as

$$e = K \frac{d\phi}{dt}$$

where e is the voltage induced and K is a constant depending upon the turns in the coil and the units.

Experiment indicates that the voltage e in the above equation is directly proportional to the number of turns in the coil. When the flux is measured in lines or maxwells, then the constant of proportionality needed is 10^{-8}. The following equation, therefore, expresses the quantitative relationship between voltage induced, the turns in the coil, and the rate of change of flux.

$$e = N \frac{d\phi}{dt} 10^{-8}$$

where e is in volts, N is coil turns, ϕ is the lines of flux, and t is the time in seconds.

The product of the number of turns and the lines of flux linking them is called the *flux linkage.*

---

* This assumes that the natural period of the galvanometer is short and that the deflection accurately reflects the current flow.

The voltage of self-induction

If, in the previous figures, the galvanometer is replaced with a battery, a switch, and an oscillograph element to measure the instantaneous current, the resultant circuit is shown in Fig. 4-5. When the switch is closed, a current flows and a magnetic field is set up. This is an identical field with that shown in Fig. 4-2, and it is reasonable to expect that in setting up this field opposing reactions would occur that are similar to those produced when the bar magnet was inserted into the coil. Confirmation is obtained by the results of the oscillographic record when the switch is closed, as shown in Fig. 4-6. This shows that the current does not immediately jump to its full value when the voltage is applied, but rises at a definite rate that gradually decreases as the final current value (determined by the resistance of the circuit) is approached. If a coil of only a few turns is used, the length of time to approach the final value of current would be very short (on the order of

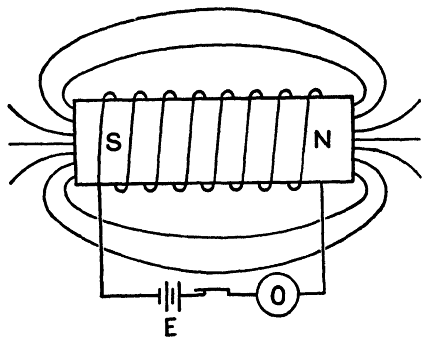


FIG. 4-5. The Magnetic Field of a Coil Carrying Current.

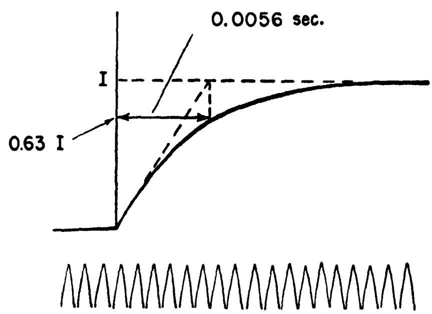


FIG. 4-6. Transient Current Rise in an Inductive Circuit.

0.00001 sec), but if many turns of low resistance wire are used with an iron magnetic circuit, the time to approach the final current value might be as much as several seconds. In the specific case shown in Fig. 4-6 it required 0.0056 sec to reach 0.632 of

the final current value, as is shown by the 1000-cycle timing wave at the bottom of the oscillogram.

If the Ri drop is plotted against time it will follow the same sort of curve as the current. There is, therefore, a difference between the impressed voltage and Ri drop. This difference, indicated by the shaded area of Fig. 4-7, is called the voltage of *self-induction*. It is labeled as e_L, since lower case e is used to represent instantaneous voltage and L is the usual symbol for inductance.

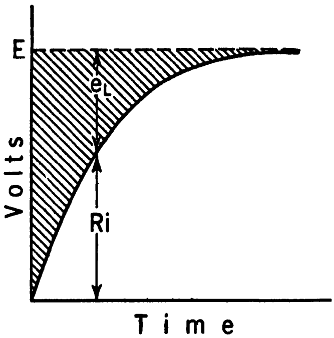


FIG. 4-7. Energy Storage in an Inductive Circuit.

Inductance

This voltage of self-induction, or counter-voltage as it is sometimes called, is proportional to the *rate* at which the current is increasing, which is proportional to the rate of change of flux in the coil. This leads to the definition of unit inductance:

A coil of wire has an inductance of 1 *henry if a current change of* 1 *amp/sec will cause a potential difference of* 1 *v to be produced in the coil.*

The voltage of self-induction may now be stated mathematically as

$$e = -L\frac{di}{dt}$$

where e is in volts, L is in henries, i is in amperes, and t is in seconds. The negative sign indicates that the voltage produced is in a direction that opposes the change of current.

The magnitude of the voltage produced in a coil by a certain rate of change of flux was previously given as

$$e = N\frac{d\phi}{dt}10^{-8}.$$

The direction of this voltage is negative, since it tends to oppose the positive flow of current. It may therefore be more correctly written

$$e = -N\frac{d\phi}{dt}10^{-8}.$$

If these two equations are combined, then

$$-L\frac{di}{dt} = -N\frac{d\phi}{dt}10^{-8}, \qquad \text{and} \qquad L = N\frac{d\phi}{di}10^{-8}$$

This last equation shows that the inductance is proportional to the product of the turns and the rate of change of flux with current. Inductance is, therefore, dependent upon the number of turns and upon the characteristics of the magnetic circuit linking the turns. This permits the calculation of the inductance of many coils having an iron circuit with an air gap if the reluctance of the iron is small in comparison to the reluctance of the air gap.

Example. Determine the inductance of a coil of 5000 turns wound on the magnetic circuit of Fig. 2-5. Assume that the current is limited and that the flux density is such that the reluctance of the iron may be neglected.

Solution: It was determined in the example on p. 35 that a flux of 20,000 lines would require an air-gap magnetomotive force of 1500 ampere-turns. The current required to produce this is

$$\frac{1500}{5000} = 0.300 \text{ amp.}$$

The inductance then is

$$L = N\frac{\Delta\phi}{\Delta I}10^{-8} = 5000\frac{20{,}000}{0.300}10^{-8} = 3.3 \text{ h.*}$$

Exercise 4-1. Determine the inductance of a coil of 2500 turns wound on the magnetic circuit of Fig. 2-5.

Exercise 4-2. Determine the inductance of a coil of 250 turns wound on the core of the relay shown in Fig. 2-6.

Exercise 4-3. Determine the inductance of a coil of 1000 turns wound on the core of the relay shown in Fig. 2-6 if the air gap is reduced from $\frac{1}{4}$ to 0.02 in.

Exercise 4-4. On the magnetic circuit of Fig. 4-8, composed of transformer sheets, how many turns of wire would be required to give an inductance of 1 h? If half of the area in the window of the magnetic circuit can be used for copper (the other half being required for insulation), what is the largest size of wire that may be used?

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* Note. The magnetomotive force necessary to force the flux through the iron portion of the circuit would reduce this value somewhat as the knee of the saturation curve of the material was approached.

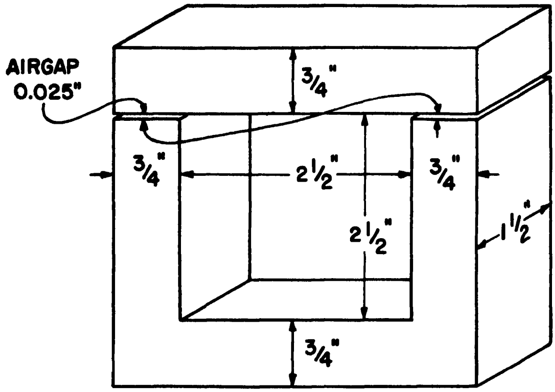

FIG. 4-8. The Magnetic Circuit for Exercises 4-4 and 4-5.

Exercise 4-5. What inductance would 3000 turns of wire give on the magnetic circuit of Fig. 4-8? How much current can the coil carry before saturation of the iron circuit causes 10 per cent reduction in inductance?

Energy stored in a magnetic field

In the circuit of Fig. 4-5 it may be noted that the energy supplied to the circuit is not entirely accounted for by the heat generated in the resistance of the coil. The difference is energy supplied to the inductance during the time of building up the current to its final value. This energy is not dissipated but is stored in the magnetic field and is returned to the circuit when the field disappears.

The magnitude of the energy stored may be obtained by integrating the instantaneous power supplied to the inductance, multiplied by the time differential as follows:

$$W = \int_{t=0}^{t=\infty} P\,dt = \int_{t=0}^{t=\infty} e_L i\,dt = \int_{t=0}^{t=\infty} L\frac{di}{dt}i\,dt$$

The incremental element dt cancels out of the equation, and leaves i as the variable. The limits of integration from the previous paragraph are $i = 0$ and $i = I$, thus

$$W = L\int_{i=0}^{i=I} i\,di = L\left[\frac{i^2}{2}\right]_{i=0}^{i=I} = L\frac{I^2}{2} \text{ joules}$$

If the inductance and current are fairly large as in the case of the field circuits of electrical machines, an appreciable amount of energy is thus stored in the circuit. The mechanism by which this energy is returned to the circuit is through the generation of a voltage in the coil tending to cause the current to continue to flow in the coil. This voltage is proportional to the rate of change of flux linkage as stated on p. 62. If an effort is made to open an inductive circuit suddenly, then the flux must collapse very rapidly and since the rate of change of flux is great, the corresponding voltage is high. This phenomenon is analogous to inertia in mechanics. When a heavy body has attained considerable speed, to stop it suddenly produces disastrous results, as demonstrated by many autos that have hit trees or telephone poles. Similarly, comparatively low-voltage circuits having high inductance can also be dangerous, as some students have learned when opening the field circuit of a medium- or large-sized generator or motor.

Exercise 4-6. How much energy would be stored in the magnetic field of a generator having an inductance of 15 h and carrying a current of 50 amp? What voltage would be produced if the current is reduced to zero in 0.01 sec?

Mutual inductance

Self-inductance has been defined as that characteristic of a coil by which a change of current in the coil produces a voltage within itself because of the corresponding change of flux linkages of the coil. When coils or wires are so located that a change of current in one circuit will cause a change of flux linkage in a second circuit, they are said to have *mutual inductance.* The coil or circuit in which the current is changing (the source of the flux variation) is called the *primary* circuit, whereas the second coil in which the flux linkage is changed and voltage is produced is known as the *secondary* circuit.

Mutual inductance is measured in the same units as self-inductance. Thus, *when a rate of change of* 1 *amp/sec in the primary coil will produce* 1 *v in the secondary coil, the two coils are said to have* 1 *henry of mutual inductance.*

Voltages generated by motion. The flux linkage in a coil may be changed not only by an increase or decrease in the magnitude of the magnetic field but also by a movement of the coil with respect to the field. This is indicated in Fig. 4-9 where a single-turn coil is moving across a uniform magnetic

field perpendicular to the plane of the paper with a velocity v. The length of conductor that is perpendicular to the direction of movement is l. The magnetic field has a flux density B.

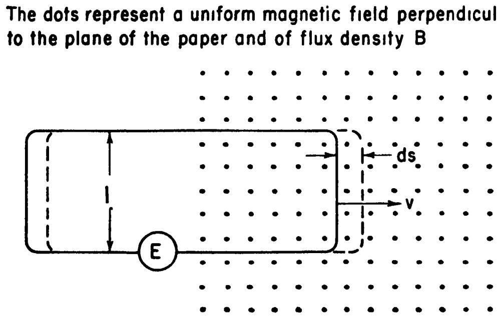

Fig. 4-9. The Generation of Voltage by the Movement of a Conductor across a Magnetic Field.

The voltage generated by the movement of the single-turn coil is

$$e = \frac{d\phi}{dt} 10^{-8} \text{ volts.}$$

The change of flux in time dt is equal to the flux density multiplied by the differential area traversed as

$$e = B \frac{dA}{dt} 10^{-8} \text{ volts.}$$

The area traversed is the length l times the differential of distance moved or

$$e = Bl \frac{ds}{dt} 10^{-8} \text{ volts.}$$

Also the differential of distance with respect to time is velocity, so

$$e = Blv\ 10^{-8} \text{ volts.}$$

Since nearly all commercial electrical power is produced by the movement of electrical conductors (or coil sides) through magnetic fields, this equation for voltage is exceedingly important. In most electric generators the movement of the conductors is perpendicular to the flux, but in the few cases where the movement is not perpendicular it is only the component of velocity perpendicular to the field that produces voltage.

An illustration of this principle is shown in Fig. 4-10a, which shows a single-loop coil rotating in a uniform magnetic field with the voltage taken off from slip rings. The geometry is shown more accurately in Fig. 4-10b, which shows that the velocity v', which is perpendicular to the field, is equal to the peripheral

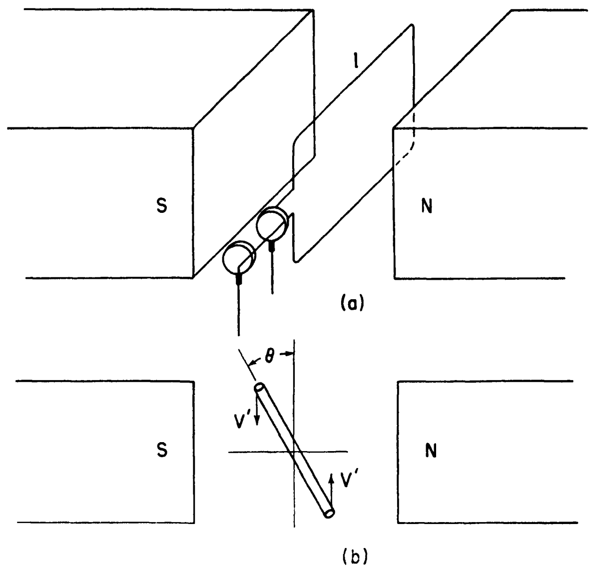

FIG. 4-10. The Generation of a Sinusoidal Voltage by a Coil Rotating in a Uniform Field.

velocity v multiplied by the sine of the angle θ, thus

$$v' = v \sin \theta.$$

If the coil is rotating at a uniform angular velocity, then the voltage generated in the coil by each coil side is

$$e = Blv \sin \theta \times 10^{-8} \text{ volts}.$$

This is an alternating voltage of a form which will be studied later.

CHAPTER 5

Direct-Current Generators

Fundamental physical relations

The theory of d-c generators and motors is based on the relationships between electrons in moving conductors and magnetic fields that have been described in previous chapters. In d-c machines electron movement in a magnetic field is produced in one of two ways. In the first case to be considered, the electrons in a wire are moved physically across a magnetic field and the electric force exerted on the electrons drives them to one end of the conductor, producing a difference of potential, or voltage, between the ends of the conductor, as indicated in Fig. 5-1.

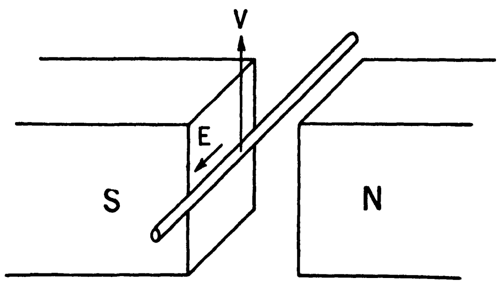

FIG. 5-1. Voltage Produced in a Conductor Which Is Moving across a Magnetic Field.

The magnitude of the voltage is proportional to the velocity, the strength of the field, and the length of the conductor in the field thus,

$$e = Blv\ 10^{-8} \text{ volts}$$

where B is in lines per square inch, l is in inches, and v is in inches per second. In the second and related case, a voltage is applied to a conductor and there results an electron drift or electron movement in the conductor in the form of an electric current. The force exerted on the electrons results in a sidewise thrust

on the wire, as indicated in Fig. 5-2. The magnitude of this force is equal to

$$F = 8.84BlI\ 10^{-8} \text{ lbs}$$

where B is in lines per square inch, l is in inches, and I is in amperes.

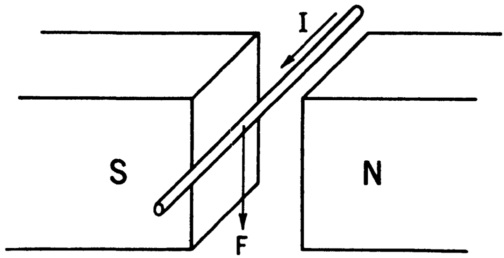

Fig. 5-2. Force Exerted on a Conductor Which Is Located in a Magnetic Field and Carries Electric Current.

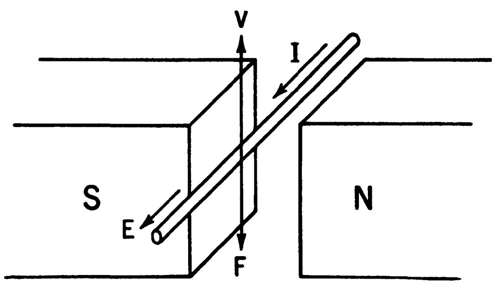

Fig. 5-3. Relation of Velocity, Electromotive Force, and Magnetic Field in a Generator

In generators and motors both conditions occur simultaneously since in both, conductors are moving across magnetic fields and these same conductors have currents flowing in them.

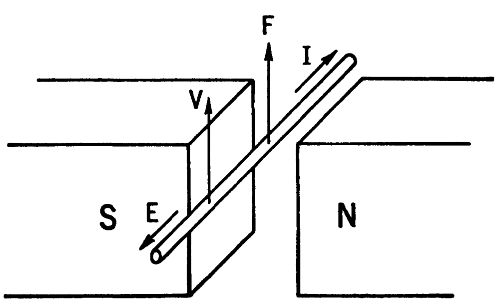

Fig. 5-4. Relation of Velocity, Electromotive Force, Current, Force, and Magnetic Field in a Motor.

in the generator the current flows in the same direction as the voltage generated and produces a force opposing the direction of motion, as shown in Fig. 5-3. This means that the generator must be driven against the electromagnetic force by an external mechanical force. In the motor, as shown in Fig. 5-4, the current flow is opposed to the direction of voltage generation and the force of the conductor is in the same direction as the velocity and will maintain its own motion.

The Gramme-ring type d-c generator. In order that the construction of d-c machines may be studied more effectively, it is necessary to have a preliminary knowledge of how the various parts function in the production of voltage. In Fig. 5-5 a very simple form of d-c generator, known as the Gramme-

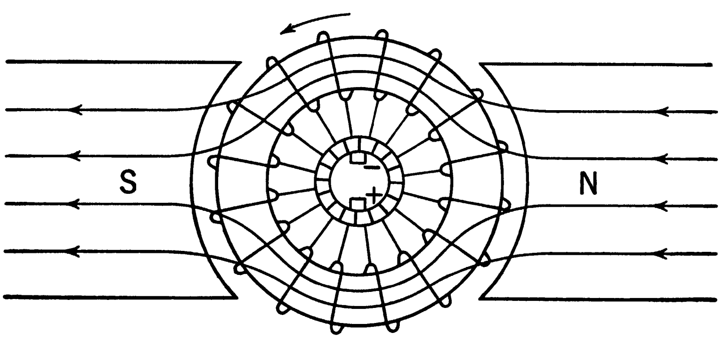

Fig. 5-5. An Elementary D-C Generator.

ring type, is shown diagrammatically. In this generator a hollow cylindrical ring composed of laminated sheets of iron is mounted on bearings so that it can rotate between two magnetic poles. The flux enters the ring at the north pole and leaves the ring at the south pole. An insulated copper wire is wound around the iron ring as shown in the diagram. Each turn* is connected to an insulated copper bar mounted in an assembly (shown in the center of the diagram), called the commutator. The commutator is composed of a number of tapered copper bars separated from each other by mica insulation, clamped together, mounted on the shaft, and machined to form a smooth cylindrical surface. Such a commutator assembly for a commercial machine is shown in Fig. 5-11. Carbon blocks

* For higher voltage machines every other turn, or possibly every fourth or tenth turn, may be connected to the commutator bar. Any integral number may be used, but the winding should be entirely symmetrical.

called brushes are held against the outside surface* of the commutator on opposite sides as shown in Fig. 5-5.

When the armature is rotated, the conductors on the outside surface cut the flux under the poles and a voltage is produced. If the rotation is counterclockwise, the conductors under the north pole produce a voltage out of the page. Each of these voltages is added to the other so that the difference of potential between the brushes is the sum of the voltages produced by all of the conductors on that side of the armature. The voltages generated by the conductors under the south pole are into the page, and these will also be additive, so that the difference in potential is the same as it was for the side under the north pole. The direction of current flow will thus be from the negative brush, into the machine, and then out of the positive brush for both windings.

A study of the diagram shows that the rotation will change the position of the individual conductors on the surface of the armature but will not alter the voltage generated with respect to the brushes. *This is a d-c generator.*

When the brushes are connected to an external circuit, a current will flow through the external circuit from the positive to the negative brush, and the same current will flow from negative to positive within the machine, dividing equally between the two sides of the machine. The currents in the windings flow in the same direction as the voltage generated, so that the force produced by the conductors is opposed to the motion of the conductors and tends to cause the generator to slow down. This set of forces must be overcome by the torque of a driving motor or engine, usually called the *prime mover*. The prime mover supplies the mechanical energy that is converted to electrical energy in the generator. These relationships may be shown by an example.

Example. The laminated iron ring of Fig. 5-5 has an outside diameter of 6 in., an axial length of 4 in., and is rotating at 1800 rpm. It is wound with 100 turns of wire uniformly spaced. Each pole produces a flux density of 40,000 lines/in.^{2} and has a pole arc or peripheral length of 5 in. (a) What is the voltage generated? (b) If the generator delivers 10 amp to an external circuit, what must be the torque of the prime mover?

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* Note: In order to simplify the drawing, the brushes in the diagram are shown pressing against the inside of the commutator. This is electrically equivalent but not mechanically feasible.

Solution: (1) Determine the voltage produced by a single conductor which is under the pole face.

$$v = \pi \times \text{diameter} \times \text{rps} = 6\pi \frac{1800}{60}$$
$$= 566 \text{ in./sec}$$
$$l = 4 \text{ in.}$$
$$B = 40{,}000 \text{ lines/in.}^2$$
$$e = Blv \times 10^{-8} = 40{,}000 \times 4 \times 566 \times 10^{-8}$$
$$= 0.91 \text{ volts.}$$

(2) Determine the number of conductors in series that are under the pole and are producing voltage.

$$\text{pole pitch} = \frac{6\pi}{2} = 9.45 \text{ in.}$$

The portion of the total series conductors that are active is the ratio of pole arc to pole pitch, or 5/9.45.

There are a total of 100 conductors, half of which are in series under each pole.

The active series conductors are therefore

$$50 \times \frac{5}{9.45} = 26.5.$$

(Since this indicates a ratio only, the decimal may be retained.)

(3) The voltage generated is the product of the active conductors and the voltage per conductor.

$$E = 0.91 \times 26.5 = 24 \text{ v} \qquad (\textit{Ans.})$$

(4) Determine the force on a single conductor. The current in each conductor, being one half of the current delivered, is 5 amp.

$$F = 8.84BlI \times 10^{-8}$$
$$= 8.84 \times 40{,}000 \times 4 \times 5 \times 10^{-8}$$
$$= 0.071 \text{ lb.}$$

(5) The number of active conductors is again 26.5 per side, or 53 for the entire periphery. Therefore, the tangential force is

$$F = 53 \times 0.071 = 3.76 \text{ lb.}$$

(6) The torque is the product of the force and the lever arm, which is 3 in.

$$\text{Torque} = 3 \times 3.76 = 11.3 \text{ in.-lb.} \qquad (\textit{Ans.})$$

Exercise 5-1. Make the same determination as above when the outside diameter of the ring is 8 in. and the length is 5 in. Assume

that the pole arc is $\frac{2}{3}$ of the pole pitch, that flux density remains at 40,000, and that the current delivered is 20 amp.

Construction of commerical machines

There have been a few direct-current machines built in form similar to the one shown in Fig. 5-5. The design is not efficient, however, and other types have now superseded this design. In Fig. 5-6 a drawing is shown of a modern four-pole machine.

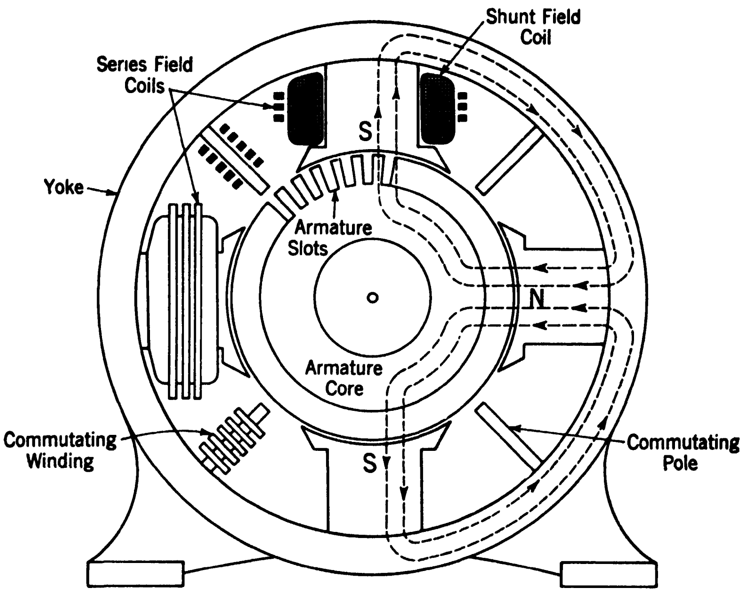


Fig. 5-6. D-C Machine Showing the Magnetic Circuit and Field Windings.

A cylindrical rotor made of laminated steel punchings, with the armature conductors placed in slots, rotates within a frame composed of electromagnets. The chief structural element of the machine is the *yoke,* which usually is formed of heavy rolled steel stock. It supports the pole pieces and acts as conductor of magnetic flux from one pole piece to another. The pole pieces are made of steel laminations which are stacked and riveted together to form a solid block, as shown in Fig. 5-7. These pole pieces are bolted to the yoke and support the field coils, which usually have many turns of small wire. The field coils are connected in series. They are then connected to the main terminals of the generator through a control resistance or rheostat. Such an arrangement of field coils is called a *shunt*

field because the coils are shunted across the generator terminals. This is shown in Fig. 5-8, which shows in diagrammatic form how the electrical connections are made to various ele-

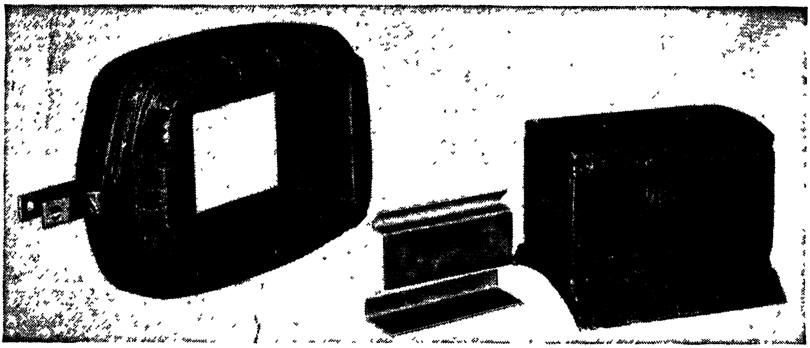


FIG. 5-7. Main Field Coil and Pole, and Spring Pads, for Shunt-Wound Stabilized D-C Motor, 50 hp, 850 rpm, 230 v.

ments of a d-c generator. The main current of the generator is conducted through a few turns of wire wound around the main field poles and also around some narrow poles between the main poles called *interpoles* or *commutating poles*. The function of these commutating poles will be explained later. The rotor is formed of thin steel sheets or laminations which are punched to the proper shape and stacked to form a steel cylinder with slots running axially along the surface. These slots are usually straight sided in the larger machines, in order that form-wound coils similar to those in Fig. 5-9 may be laid into the slots. The tips of the teeth are notched so that a hard wood or fiber wedge may be used to hold the armature coils in position. In construction, the *armature core*, as the laminated steel assembly is called, is pressed on the shaft. The commutator assembly is also pressed on the shaft. The preformed coils are then laid in the slots, wedged in position, and the ends of the coils are connected to the commutator, as shown in Fig. 5-10. The rotor is supported in bearings mounted, in most machines, in

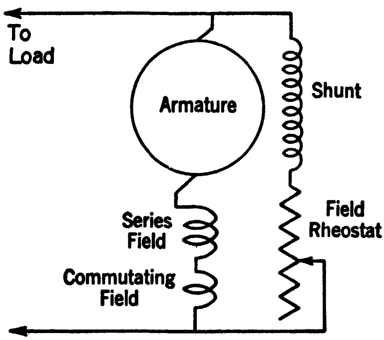


FIG. 5-8. Conventional Representation of Dynamo Connections.

a framework or end bell bolted to the yoke. The end bell also supports the assembly of brush holders in which the carbon blocks or brushes are held.

A series of views of the parts of the stator of a commercial machine are shown in Fig. 5-12 and will assist the reader in identifying the above parts of the machine as they are found in an actual manufacturing design. Fig. 5-13 shows a view of the assembled motor.

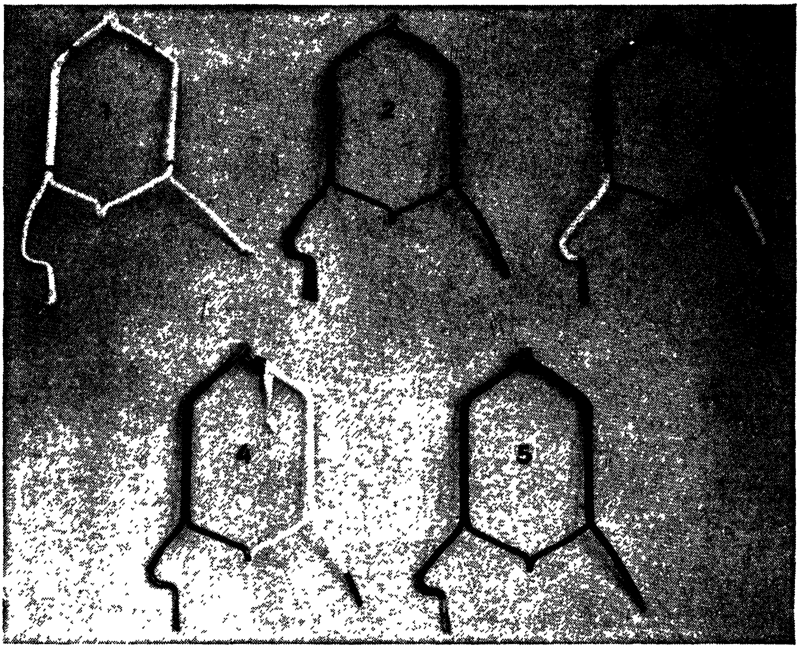


FIG. 5-9. Five Armature Coils for D-C Motor or Generator. View Approximately One-sixth Size, Showing Successive Steps in Molding and Insulating.

Armature windings

The manner in which the coils are placed in the slots on the surface of the armature is shown in Fig. 5-10. The method of connection to the commutator is also shown. In a photograph, however, it is very difficult to trace the circuit, and so a developed armature-winding diagram such as that in Fig. 5-14 is used to illustrate one of the common types of direct-current armature windings. It also illustrates a common form in which the winding specifications are supplied to the shop and repair man.

In this diagram the winding is unrolled or flattened out so that the connections and circuits are easily followed. The brushes are so located that they short-circuit the coil, which has coil sides between the poles. If the student follows the coils from the right-hand commutator bar under the central negative brush of the diagram in the direction of the arrows, he will eventually come to the positive brush after traversing ten active conductors all producing voltage in the direction in which the circuit is being traced. Thus the voltage between the negative and positive brushes is equivalent to the sum of the ten

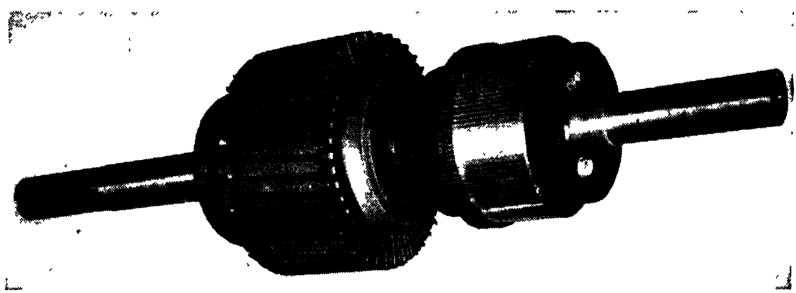


FIG. 5-10a. Unwound Revolving Armature for D-C Motor or Generator.

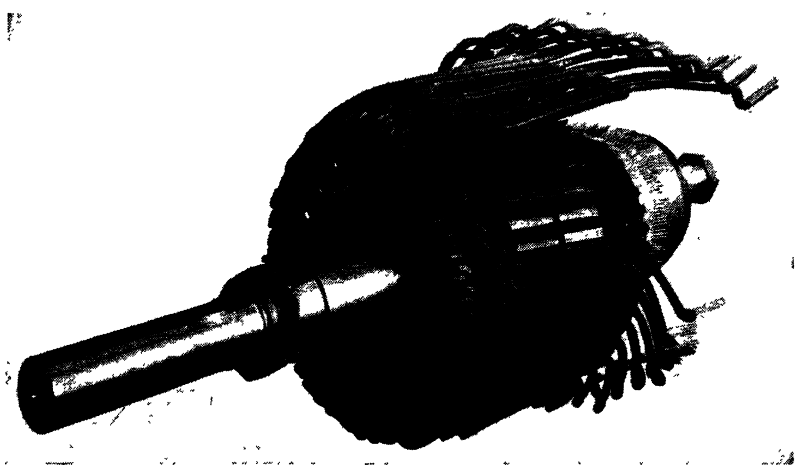


FIG. 5-10b. Revolving Armature for D-C Motor or Generator in Process of Being Wound, Showing Coils with Half in Slot Bottom, and Other Half Ready to Be Placed in Slot Top.

conductor voltages. If now the student will follow the circuit in the opposite direction from the same negative brush he will find that he again traverses ten conductors all producing voltage in the direction of the tracing before he reaches the positive brush at the left of the diagram. Since the machine is symmetrical, this voltage is the same as the other voltage, and so the two positive brushes may be connected. Two additional circuits, both having the same voltage, will be found leading away from the negative brush at the right of the diagram. Thus there are four parallel circuits within the machine leading from the negative terminal to the positive terminal. With this

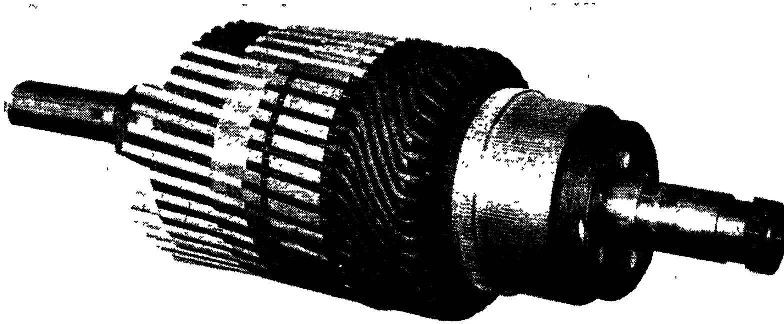


FIG. 5-10c. Revolving Armature for D-C Motor or Generator Nearing Completion, Showing Coils in Place, Some of Slot Wedges in Place, and Some Ready to Be Driven into Place.

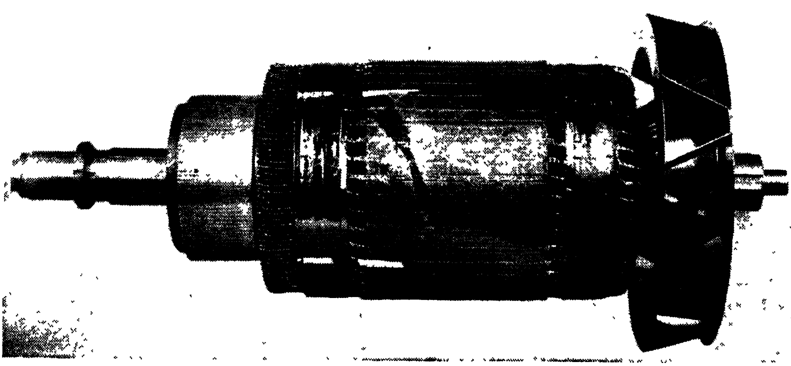


FIG. 5-10d. Completed Revolving Armature with Fabricated Steel Fan for D-C Motor or Generator.

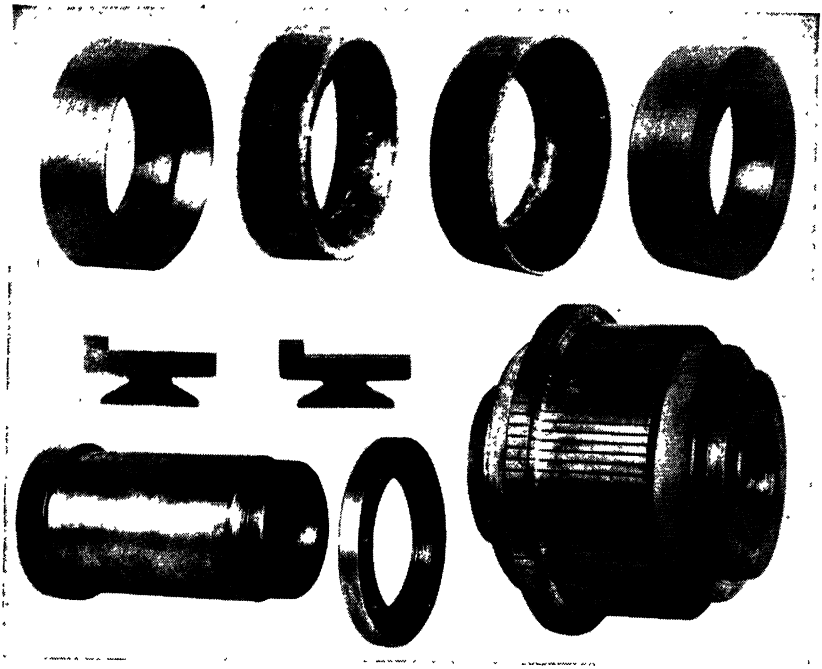


Fig. 5-11. Construction of a Typical Commutator for a D-C Motor.

Fig. 5-12a. End Shield.

Fig. 5-12b. Ball-Bearing Housing with Brush Yoke and Brush Rigging.

type of winding, which is known as a *lap winding*, there are as many parallel paths as there are poles. An eight-pole machine, for instance, would have eight parallel paths, two leading from each set of negative brushes. There are also as many positive

Fig. 5-12c. Wound Stationary Field for Shunt-Wound D-C Motor.

brushes as there are pairs of poles and a similar number of negative brushes. For simplicity, the diagram shows a single conductor for each coil side. Coils of several turns are often used instead of the single-turn coil, in order to obtain the desired terminal voltage. The manner of connection is, however, the same as if the coils had but one turn.

Although the above type of winding is a common one, other types are in extensive use. All of these windings are arranged so that the voltages generated by the conductors under the pole faces add to produce the final voltage between the brushes.

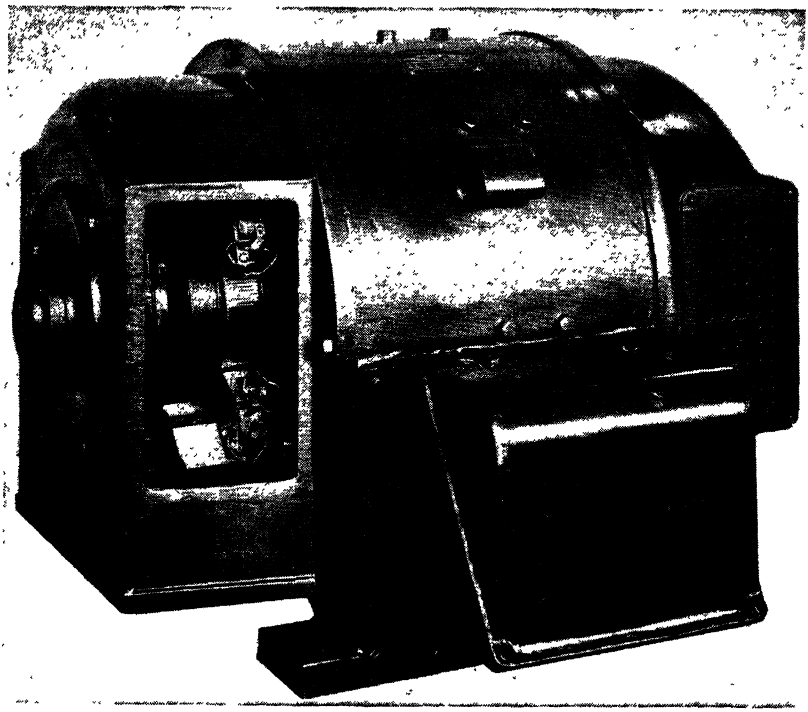


Fig. 5-13. Ball-Bearing Adjustable-Speed D-C Motor, 25 hp, 500/1500 rpm, 230 v.

Generated voltage

The computation of generated voltage and torque for a machine of this type follows the same procedure as with the simple ring winding previously calculated.

Example. In a four-pole machine such as is illustrated in Fig. 5-12 the armature is 10 in. in diameter, 8 in. long, and has 40 slots and commutator bars. The pole is 8 in. long and the pole arc is 6 in. The flux density is 45,000 lines per square inch, and the armature is driven at 2000 rpm. (a) How many turns are required in each armature coil to obtain 230 v at the terminals? (b) What is the torque of the prime mover when the machine is delivering 300 amp? (c) What is the horsepower input?

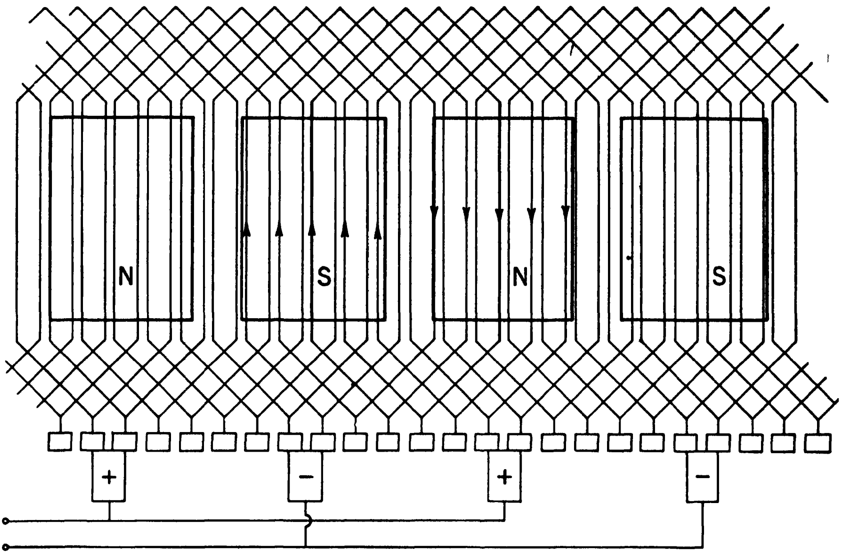


FIG. 5-14. Lap Winding for a Four-Pole Machine.

Solution: (1) Determine the voltage generated by a single conductor.

$$\begin{aligned} e &= Blv \times 10^{-8} \\ &= 45{,}000 \times 8 \times 10\pi \times \tfrac{2000}{60} \times 10^{-8} \\ &= 3.78 \text{ v.} \end{aligned}$$

(2) Determine the number of series conductors required.

Active series conductors needed = 230/3.78 = 61.

Total number of series conductors needed is equal to active series conductors multiplied by the ratio of pole pitch to pole arc

$$= 61 \times \frac{(10\pi/4)}{6} = 80.$$

(3) Determine the number of turns per coil.

There are as many coils as slots; therefore there are 40 coils. Since there are four parallel paths, the number of coils in series is 10. There are 2 conductors per coil per turn, so that each turn provides 20 series conductors. The turns required are therefore

$$\tfrac{80}{20} = 4 \text{ turns per coil.} \qquad (\textit{Ans.})$$

(4) Determine the force on a single slot.

There are 8 conductors per slot (two coil sides in each slot). Each conductor carries $\frac{1}{4}$ of the total current.

$$\begin{aligned} F &= 8.84BlI \times 10^{-8} \text{ lb} \\ &= 8.84 \times 45{,}000 \times 8 \times \tfrac{300}{4} \times 8 \times 10^{-8} \\ &= 19.1 \text{ lb.} \end{aligned}$$

(5) Force due to all active slots:

$$\text{Active slots} = \text{total slots} \times \frac{\text{pole arc}}{\text{pole pitch}}$$

$$= 40 \times 0.763 = 30.4$$

$$\text{Tangential force} = 30.4 \times 19.1 = 580 \text{ lb.}$$

(6) The torque is equal to the tangential force times the radius.

$$\text{Torque} = 580 \times \tfrac{5}{12} = 242 \text{ lb-ft.} \qquad (\textit{Ans.})$$

(7) The horsepower input is

$$\text{hp} = \frac{2\pi \text{ torque} \times \text{rpm}}{33{,}000} = 2\pi \times 242 \times \frac{2000}{33{,}000} = 92. \qquad (\textit{Ans.})$$

Exercise 5-2. What would the voltage be if the flux density in the above example were cut to 35,000 lines per sq in.?

Exercise 5-3. What would be the effect on the torque of reducing the flux density to 35,000 lines per sq in., if the current remained the same?

Exercise 5-4. What would be the effect on the voltage if the rpm were reduced 20 per cent?

Exercise 5-5. What would be the effect on the torque if the rpm were reduced 20 per cent (assuming that the current and the flux remain constant)?

Exercise 5-6. (a) Compute the terminal voltage for a machine similar to that in the example above but with the following changes.

Armature diameter	7 in.
Armature length	4 in.
Armature slots	36 slots
Turns per coil	4 turns
Flux density	40,000 lines per sq in.
Pole arc	3.5 in.
Speed	1200 rpm

(b) Determine the torque and horsepower input required to drive this generator when it is delivering 200 amp.

The above example and exercises emphasize that for any given machine the voltage is affected primarily by the flux density and the speed. Thus, if it is desired to control the voltage of a constant-speed generator, it is usually done by

varying the flux. This is normally accomplished by varying an external resistance in the circuit of the shunt field. A change in speed will also change the voltage generated. This characteristic is particularly important in the study of the operation and characteristics of the d-c motor.

The torque depends upon flux density and current flowing in the armature. When current is drawn from the generator, the energy represented by the product of the current and voltage is taken from the prime mover. *Except for internal losses, the mechanical input and electrical output are equal.*

Commutation

Although a brief explanation of the function of the commutator has been given, a further study is necessary, since the proper adjustment and maintenance of brushes and commutator are very important to the satisfactory operation of d-c machinery.

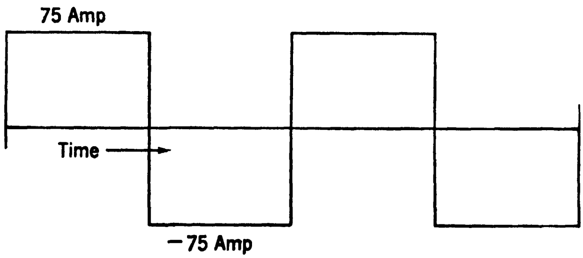


FIG. 5-15. Instantaneous Current in an Individual Coil of a D-C Generator or Motor.

In the machine used for the example on page 59, each conductor carries 75 amp. The diagram of Fig. 5-15 shows that the current in a single coil reverses each time the coil passes a brush. This diagram shows the current changing instantaneously from plus 75 to minus 75 amp. This, of course, is impossible, as the inductance of the coil will not permit instantaneous changes of current. Actually, the current must reverse during the short time that the coil is short-circuited by the brush. If the brush width is equal to the width of one commutator bar, then the time during which this change of current occurs, in the machine of the example, is $\frac{1}{40}$ of a revolution. This is 0.00075 sec. The rate of change of current is

$$\frac{\Delta I}{\Delta t} = \frac{150}{0.00075} = 200{,}000 \text{ amp per sec}$$

The inductance of such a coil would normally be about 0.03 millihenry. Even with this small inductance, however, a reactance voltage amounting to 6 v will be generated if an average rate of change is maintained.

It is customary to use carbon blocks or brushes as the connection to the commutator. The comparatively high resistance of these brushes (the voltage drop between the commutator and brush is usually about one volt and is quite constant with varying current) tends to maintain uniform current density over that portion of the commutator in contact with the brush. The change in the brush-contact area during commutation

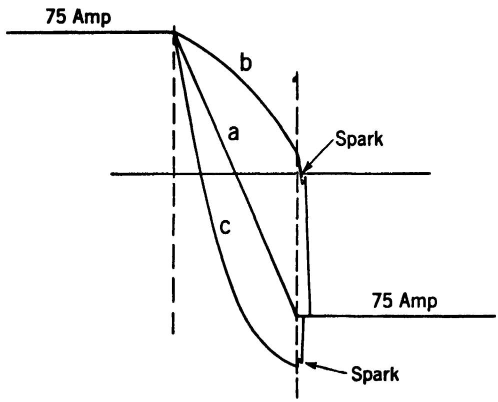


FIG. 5-16. Actual Coil Currents During Commutation: *a*, correctly adjusted, *b*, undercommutated, *c*, overcommutated.

causes satisfactory current reversal in the coil as long as the reactance voltage does not greatly exceed the one-volt brush drop. For higher reactance voltages, however, such as in the example above, other methods must be used.

In nearly all modern machines a voltage is introduced in the coil undergoing commutation which is approximately equal to the reactance voltage. This is done by placing narrow poles called *interpoles* or *commutating poles* between the main poles. These are excited by a series winding so that the flux and therefore the voltage is proportional to the armature current. These interpoles are shown in Fig. 5-6. With these interpoles the current changes almost uniformly with time, as shown in curve *a* of Fig. 5-16. Without the interpoles, the change would follow curve *b*, and severe sparking would occur when the coil left the brush. If an excessive commutating voltage is intro-

duced, it is possible to cause the current not only to reverse but also to overshoot, and then sparking may also occur. It is important, therefore, that the commutating voltage be of the correct magnitude.

When the armature current is small, only a small reactance voltage develops, and so only a small commutation voltage is needed. The series winding on the interpole provides this variable voltage automatically. Minor deviations are absorbed in the brush-contact drop.

Excitation

Most systems supplying d-c power operate on the basis that a constant voltage will be maintained between the power lines. It is then possible by connecting equipment across these lines to draw from them such power as is needed. Since constant-voltage systems do predominate, the generator that supplies this voltage will be given primary consideration.

If the generator is excited by a shunt field such as has been described in the previous paragraph on the construction of d-c machines, it will give approximately constant voltage. This is true because the field coils are connected across the constant-voltage terminals and this holds the magnetizing force and field flux constant. With the armature rotating at constant speed, the conductors will cut the flux at constant velocity and the voltage will be approximately constant regardless of the magnitude of the armature current that supplies the load.

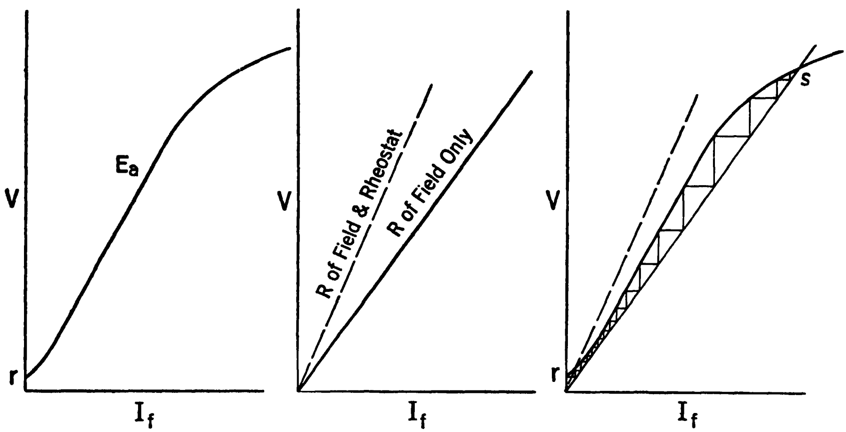
Magnetization curve

Since the magnetic circuit is made up largely of iron, the relation between generated voltage and field current assumes a curve similar to the magnetization curve for iron. Such a curve is shown in Fig. 5-17(a), where field current is plotted as abscissa and terminal voltage as the ordinate. This is called the *magnetization curve* of the generator. It is noted that this curve does not start from zero, as there is a small amount of residual magnetism in the iron from its previous use.

Self-excitation of a shunt generator

Since a shunt generator depends upon its own voltage to produce the field current, and since there can be no appreciable voltage without field current, the manner of simultaneous development of field current and armature voltage is of interest.

Fig. 5-17(b) shows the amount of field current resulting from any given armature voltage and is often called the *field resistance line.* The solid line which is of primary interest represents this relation when no external resistance is inserted in the field circuit. When this curve is combined with the magnetization curve, which has the same coordinates, the basis of Fig. 5-17(c) is obtained. When the prime mover drives the generator at rated speed, a small voltage due to the residual magnetism is



Magnetization Curve
(a)

Field Resistance Lines
(b)

"Building Up" of Shunt Generator
(c)

FIG. 5-17.

obtained as shown at *r*. This voltage causes a small current to flow in the field, which then causes an increase in voltage which causes more current, and so on, as indicated by the steps in the diagram. This process, known as *building up,* will continue until the two curves cross at *s*. If there is considerable external resistance in the field circuit, the field current for a given voltage will be much reduced, as shown by the dashed line in Fig. 5-17(b). When this curve is placed on the magnetization curve, it is evident that the voltage will not build up. It is important, therefore, when starting a d-c generator, to be sure that the field resistance has been removed from the circuit so that it will build up quickly.

D-c generators that have been installed and operating will usually cause no trouble in building up if the field resistance is

reduced to zero and if the normal rotational speed is maintained. In new installations, several additional possibilities of difficulty exist. If the direction of rotation is reversed, then the residual voltage will cause field current which will oppose instead of aid the residual magnetism. This same effect is caused by reversed field connections in assembly after shipping or after a disassembly for repair. Either of these may be corrected by reversing the direction of rotation or by reversing the shunt-field connections. Occasionally, after considerable vibration during shipment, the residual magnetism has been so reduced as to be ineffective. This can be remedied by placing a storage battery in the field circuit to aid in producing the field current. The battery will act in the same manner as the residual flux, and so is subject to the same possibilities of difficulty, if the polarity of its connection is wrong. After the voltage has built up once, the battery may be removed and the machine should build up by itself.

Armature reaction

Although the flux in a shunt generator will continue to be approximately the same magnitude regardless of the load supplied, the load currents flowing in the armature do produce magnetomotive forces which in a small degree affect the voltage generated. The magnetic effects of this armature reaction are shown in Fig. 5-18. In the lower right portion of the machine, the magnetic effect of the armature conductors alone has been shown. Here it is seen that the effect is to cause flux to flow into the pole on one pole tip and out on the other pole tip. When this flux is combined with the flux of the main poles produced by the shunt field, the effect is to concentrate the flux in the trailing pole tip instead of having it uniformly

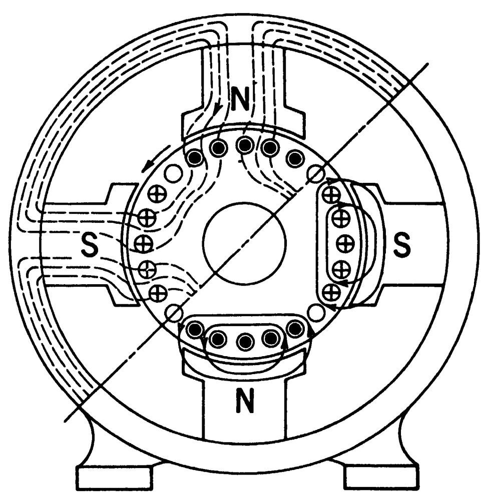


Fig. 5-18. Field Distortion Caused by Armature Reaction.

distributed. This is shown in the upper left portion of the machine. This effect has a tendency to reduce the over-all flux somewhat, since the extra flux obtained in the trailing pole tip is not as great as the reduced flux in the leading pole tip. This reduction is caused by the saturation of the armature teeth where the flux is concentrated in the trailing pole tip.

In large machines, subject to sudden changes in load, this concentration of voltage across particular coils and commutator bars may cause them to arc across the mica insulation of the commutator. When one of these small arcs occurs, it is likely to be progressive and to cause a short circuit from brush to brush, which usually damages the machine. In some large machines it is common practice, therefore, to place fixed conductors in the pole face. These conductors are in series with the load and their magnetomotive force will neutralize the magnetomotive force of the armature. Such windings are called *compensating windings* or *compensating fields*. Since the addition of these windings is expensive, they are omitted in machines that do not require the improved operating characteristics obtained with them.

Voltage characteristics

The voltage of a shunt generator is reduced slightly as the load is increased. This reduction is caused by the following

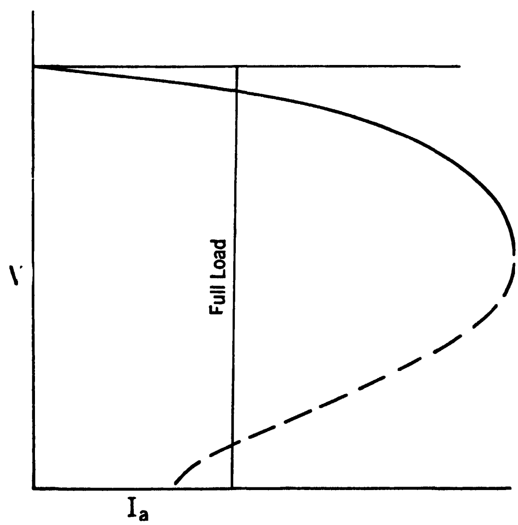


Fig. 5-19. The Load-Voltage Characteristic of a Shunt Generator.

cumulative effects. First, there is a resistance drop in the armature and brushes, which increases with load and is subtracted from the generated voltage. Second, the armature reaction distorts the field, which reduces the total flux being cut. The voltage reduction produced by these two causes reduces the voltage across the shunt field so that the excitation is reduced and this still further reduces the terminal voltage. These effects are not excessive as long as the load does not go beyond the rated value, but at higher values of load they cause a rapid decrease in voltage. This is shown in Fig. 5-19, where a normal load-voltage characteristic is plotted for a shunt generator. Such a dropping characteristic is not entirely satisfactory. It gives a reduced voltage at the generator terminals when the line drop is a maximum, and so the ideal of a constant voltage at the load is not obtained. In order to overcome this drop in voltage, a few turns of a series winding are placed upon the main poles to give additional magnetomotive force as the load is increased. A generator with such a winding is called a *compound generator.* These windings are shown in the drawing of Fig. 5-6 and are used on many machines. When only enough series-field turns are used to raise the full-load voltage to no-load voltage, as shown in Fig. 5-20, the machine is said to be *flat-compounded.* When sufficient series field is used to raise the voltage at full load above that at no load, the generator is said to be *overcompounded.*

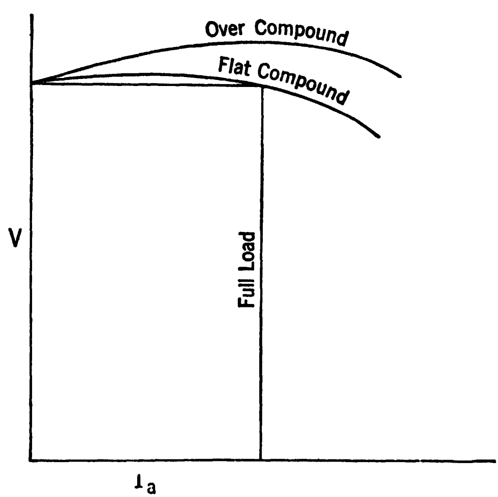


FIG. 5-20. The Load-Voltage Characteristics of Compound Generators.

Voltage regulators

Compound generators do not maintain the voltage absolutely constant, although the variations are usually not large. In order to obtain a more constant voltage, regulators are used to adjust the current in the shunt field. Most modern voltage regulators consist of field rheostats that are varied by some form of solenoid or torque motor which operates against a spring. They are very sensitive, so that a small difference between the pull of the spring and the torque of the motor will make a large change in the field resistance. The torque on the motor (or pull in the solenoid) is proportional to the terminal voltage, and so a balance can be obtained between this pull and the spring tension. In this way a constant voltage can be obtained without the use of the series winding.

Load limitations and rating

Several limitations of the d-c generator have been mentioned. The effects of armature reaction and of commutation are two of these. With very careful attention to commutation and armature reaction in design, the machine can usually be made to handle heavy loads for short periods of time. If heavy loads are continued, however, they cause overheating in one or more parts of the machine, which will cause damage. The most critical limitation is, therefore, the energy losses that cause heating. These include the losses involved in forcing a current through the armature and field against the circuit resistance, which are usually called I^2R losses, since they are equal to the product of the current squared and the resistance. The energy losses also include the brush losses at the commutator, iron losses in the armature iron, and friction losses in the commutator and bearings. The machine is given a rating, which indicates the load that can be carried continuously without becoming dangerously hot. Usually this is assumed to represent a temperature rise above the surrounding air of 50° C. If, however, special insulations are used, a higher temperature rise is permitted.

This rating is based on the assumption of a temperature of the surrounding air of about 20° C or 70° F. If the equipment is used in much hotter surroundings, the load that the machine will carry satisfactorily is somewhat reduced. Likewise, if the equipment is being used in a cold climate, the load may be increased somewhat without damage to the machine.

Special generators

The shunt and compound generators discussed above comprise most of the generators that are used for the general supply of d-c power. In many cases, the terminal voltage of these machines is controlled by an automatic voltage regulator.

Many power requirements need individual machine characteristics, and for these applications, special designs are made to supply the excitation in the proper form. One power load requiring special characteristics is arc welding. This load needs a current that is quite constant, even though the voltage varies considerably. Many different types of d-c generators have been designed to meet this need.

A generator that is of considerable importance in the measurement of speed is one with a permanent magnet for the field. In such a machine, the no-load voltage is a direct measurement of the rpm of the rotor, and therefore it may be used as a tachometer.

CHAPTER 6

Direct-Current Motors

From generator to motor

Direct-current motors are identical in construction with d-c generators, and ordinarily the same machine may be used either as motor or generator. Because of this, practically all of the material of Chap. 5 is applicable to a discussion of d-c motors. The essential difference is in the direction of current flowing in the armature conductors. In the case of the generator, the armature is driven by a prime mover, and the current flow is in the direction of voltage generation. In the case of the motor the current flows in a direction opposite to that of voltage generation and thus the force on the conductors, instead of retarding motion, is in a direction that will continue the motion.

In order to demonstrate more effectively the difference between the operation of d-c generators and motors, an analysis of the shift from generator to motor action of a shunt machine will be made. Let it be assumed that a d-c shunt generator is being driven at 2000 rpm and that it is delivering its rated full-load current of 100 amp to a 250-volt d-c system being supplied also by a number of other generators. The armature resistance drop is assumed to be 10 v; therefore, the generated armature voltage must be 260 v. The power output of the machine is 25 kw, and, therefore, the retarding torque on the armature is such as would produce this power at 2000 rpm. (An additional torque would be required of the prime mover to overcome friction, windage, and other generator losses.) Let it be further assumed that other d-c generators are connected to the system so that, regardless of what is done to the generator under study, the d-c system voltage remains at 250 v.

If the prime mover is a gasoline engine, let the throttle setting be lowered. This means a reduced fuel supply and, therefore, a reduced engine torque. The engine torque would no longer be equal to the retarding torque of the generator, and it would slow down slightly. This might reduce the generated

voltage to 255 instead of the previous 260 v. The armature current would be reduced to 50 amp because the terminal voltage must still remain 250 and only 5 v are now available for the armature-resistance drop. This reduces the generator-retarding torque to one-half its previous value. If the prime mover can supply this torque, the generator will stabilize at that point. If the prime-mover torque is greater than this 50 per cent value, the generator speed and voltage will increase, and the current will also increase until a point of equilibrium is reached.

Now assume that the fuel supply to the prime mover is entirely stopped. The prime-mover torque will be reduced; the generator will slow down until the generated voltage is 250; thus no current will flow and no retarding torque will exist in the generator. But with the friction and windage and other losses in both generator and prime mover the speed will decrease still further, and as the speed decreases, the voltage of the generator also decreases. Suppose it decreases to a value of 248 v. It is known that a value of 10 v of armature drop will produce 100 amp of armature current. A 2-volt difference in voltage between line and armature-generated volts will, therefore, produce 20 amp but in a direction of flow opposite to that when acting as a generator. When the current reverses, the direction of the torque also reverses, and now a torque equal to one-fifth of full-load generator torque is provided by the d-c machine in a direction tending to continue the rotation. The machine is now operating as a motor and is driving not only itself but the prime mover as well. The speed of the motor has in the meantime been reduced from 2000 rpm to

$$\tfrac{248}{260} \times 2000 \text{ rpm} = 1910 \text{ rpm}.$$

If now the d-c machine is connected to a centrifugal pump instead of to the prime mover, it will still continue to run. If the load on the pump is increased, then a slight reduction in speed will occur until the generated voltage in the motor has been reduced so that an added armature current can flow and thus produce the necessary driving torque. The motor therefore has a natural system for controlling the power taken from the line in order to maintain an almost constant motor speed.

Motor-generated voltage

The voltage generated in a motor is computed in the same way that it is in a generator. Since it opposes the flow of motor

current, however, it is sometimes referred to as the *counter-electromotive force* of the motor. It is proportional to the flux of the main poles and to the speed. Since the number of effective conductors in series and the length of the conductors are fixed in any particular machine,

$$E_m = K\phi S$$

where E_m is the motor cemf, K is a constant of the machine, ϕ is the main field flux, and S is the speed in rpm.

The motor-generated voltage added to the armature resistance drop must equal the impressed or circuit voltage. Thus

$$E_m + I_aR_a = V$$

where E_m is the motor-generated voltage, I_a is the armature current, R_a is the armature circuit resistance, and V is the impressed or circuit voltage. This relationship is very helpful in analyzing many motor problems. Except under starting conditions the armature circuit resistance is small, so the motor-generated voltage must be approximately equal to the impressed voltage.

Exercise 6-1. A 10-hp 230-v 1200-rpm d-c shunt motor has a full-load current of 38 amp. The shunt field takes $1\frac{1}{2}$ amp. What is the full-load motor-generated voltage if the armature resistance is 0.4 ohm?

Exercise 6-2. A 50-hp 230-v 1200-rpm d-c shunt motor has an armature resistance of 0.09 ohm. The full-load current is 180 amp. If the effects of armature reaction are neglected, what speed regulation may be expected from no load to full load? Why?

Motor torque. The torque produced in a motor can be computed from the flux density, number and length of active conductors, and the current flow as was done for a generator in the example on p. 84. For any machine the number and length of conductors are fixed so that the motor torque is proportional to the product of field flux and armature current. Thus

$$T = K'\phi I_a$$

where T is the motor torque, ϕ is the main field flux, I_a is the armature current, and K' is a constant determined by the number and length of active conductors.

It is observed that in a shunt motor where the field flux is constant the torque is directly proportional to the armature current.

Motor commutation

The commutating problem in a motor is the same as in a generator. The current in the individual coils must be completely reversed during the time the coil is short-circuited by the brush. This is normally accomplished by the use of commutating poles just as in the generator. Since the current in the armature is reversed, the direction of the commutating voltage must also be reversed, and this is automatically accomplished since the field winding on the commutating pole is in series with the armature. No change in connections of the commutating pole winding is therefore necessary to change from generator to motor action.

The change in load on motors is often more sudden than in generators, and the problem of high coil voltages produced by armature reaction, as discussed in Chap. 5, is more often critical. The use of compensating or pole-face windings is quite common on motors used for blooming mills and other continually reversing motors.

Speed-torque characteristics for shunt motors. In a motor one of the important operating characteristics is the variation of speed with load. In a shunt motor the field circuit is connected across the line, and as long as the line voltage remains constant, the field excitation also remains constant.

If it is assumed that the field flux also remains constant, then the torque will be directly proportional to armature current. As the load changes, the armature current will change, and since

$$E_m = V - I_a R_a$$

the motor-generated voltage must change slightly in order to compensate for the change in armature resistance drop. This armature resistance drop varies from about 5 per cent in large d-c motors to about 10 per cent in small motors. A variation of from 5 to 10 per cent will therefore be required in the motor-generated voltage.

Exercise 6-3. In a 10-hp 230-volt 1200-rpm d-c shunt motor with armature resistance of 0.4 ohm the full-load current is 38 amp. If the effect of armature reaction is neglected, determine the no-load speed at which the motor operates when the rated full-load speed is 1200 rpm.

Exercise 6-4. When drawing a full-load armature current of 96 amp, a 230-v shunt motor runs at 1750 rpm. The armature

resistance is 0.16 ohm and the brush drop is constant at 2 v. It is desired to reduce the speed by adding resistance to the armature circuit. (a) How much resistance must be added to give a speed of 800 rpm at full-load torque? (b) How much resistance to give 1200 rpm at half of full-load torque? (c) What is the efficiency for each of these conditions?

The generated voltage was shown previously to be

$$E_m = K\phi S.$$

Since the field flux was assumed to be constant, the motor voltage is directly proportional to speed, and therefore only a 5 to 10 per cent drop in speed is normally expected of a shunt motor when full load is placed on it. Computations of these speed variations may be made on the basis of the above equations when the necessary line voltage, armature resistance, and armature current are given.

The actual performance of shunt motors is not quite so simple as outlined above because of the effects of armature reaction on the main field flux. As discussed in the chapter on generators, the effect of armature reaction is to concentrate the flux in one pole tip. In the case of the motor this concentration comes in the leading, instead of the trailing, pole tip. In either case, however, the effects of saturation tend to reduce the net field flux per pole.

When the armature reaction reduces the field flux, two effects are obtained. In the first place, the armature current must be increased slightly to overcome the reduction in torque produced by the lowered magnitude of field flux. As far as speed is concerned the lowered value of field flux requires a corresponding increase in speed to obtain the necessary generated voltage to limit the armature current. This effect tends to compensate for the effect of armature resistance drop in reducing the speed.

The magnitude of the effect of armature reaction will depend upon the degree of saturation of the field circuit. In modern machines where maximum use is made of the magnetic materials, the effect of armature reaction may not only completely neutralize the effect of the armature resistance drop on the speed of the motor, but may actually cause an increase in speed as load is applied to the motor shaft. Under conditions of overload this increase in speed may become critical, and so it is customary to add a few turns of series winding to the main field

to hold the total field flux constant or even increase it. Such a winding is known as a *stabilizing* winding.

To summarize, therefore, the speed of a shunt motor is essentially constant, regardless of the load applied to the motor shaft. This characteristic is highly desirable for many types of loads, and so the shunt motor is extensively used where d-c power is available.

Speed-torque characteristics of compound motors. For many loads it is desirable that the speed be reduced somewhat as the torque is increased. To obtain such a characteristic it is only necessary to operate the machine on a lower portion of the magnetization curve at no load and add some turns of series windings to the main field. The effect is similar to that of the stabilizing winding, but since saturation effects are negligible, a definite increase of field flux results with increase in load. Since the flux increases with load, a speed reduction is necessary in order that the motor counter-electromotive force may be reduced to a value that will allow the necessary armature current to flow.

The speed reduction at full load may be controlled by the magnitude of the series field. Thus, it is possible to obtain a wide variety of speed-current curves, as is indicated in Fig. 6-1. If it is desired to reduce the full-load speed to $\frac{2}{3}$ of the no-load value, a series field would be used of such a magnitude that the series ampere-turns at full load would be approximately 50 per cent of the shunt-field ampere-turns. (This is approximately

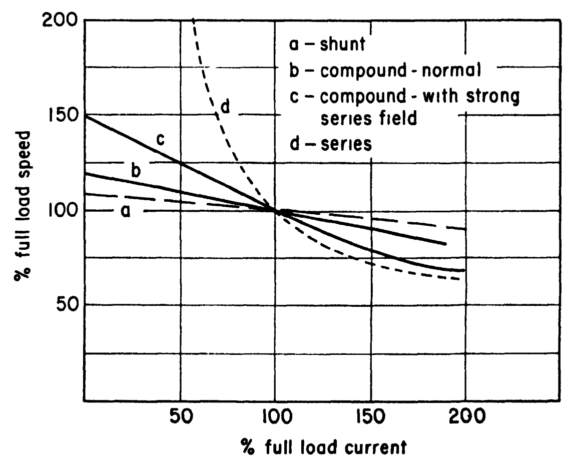


FIG. 6-1. Typical Speed-Current Curves of D-C Motors.

true since the saturation effects would tend to neutralize the effects of armature resistance drop on speed.)

Exact computations of the design of compound machines are complicated by the interdependence of the various effects. It is often possible, however, to obtain approximate results by neglecting the effects of minor magnitude.

Speed-torque characteristics of series motors. In certain types of loads, such as in electric railways and crane hoists, constant speed is not desired, but very large torque at low speeds is important. In such motor applications it is customary to omit the shunt-field winding entirely and use only the series-field winding. Such motors are known as *series* motors.

Since the field flux is proportional to armature current (at field values below saturation), the torque is proportional to the square of the armature current.

If the effects of the armature resistance drop and of saturation are neglected, the speed may be said to be inversely proportional to the armature current.

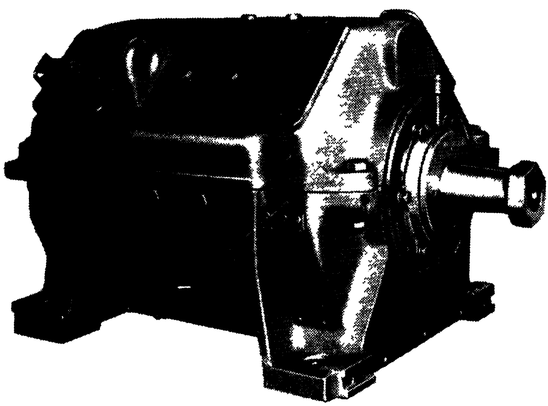


Fig. 6-2. A Mill-Type D-C Motor for Use on Hoists, Cranes, and Other Heavy-duty Industrial Applications.

The tendency of the series motor to increase in speed without limit as the retarding torque is removed makes it a very dangerous machine unless the motor is permanently connected to a load that will itself limit the speed. Its use is normally restricted to traction applications (that is, streetcars, electric locomotives, and suburban trains) and crane hoists, where the motor is permanently geared to the load and there is enough

residual load to adequately limit the speed. A typical motor for this type of service is shown in Fig. 6-2.

Many small motors are wound as series motors. They may be used on either d-c or a-c, since the torque remains in the same direction when both armature and field are reversed. They are called universal motors and are often used on electrical appliances, such as vacuum cleaners and sewing machines.

Starting of d-c motors

The previous discussion of d-c motors has assumed that they were operating at normal speed and has considered the changes in speed with load in order that sufficient armature current would flow to produce the proper torque. In general, the armature resistance drop is a small part (5 to 10 per cent) of the impressed voltage.

When the motor is stationary, as at starting, there is no counter-electromotive force so the entire voltage must be absorbed in the I_aR drop of the armature circuit. Excessive armature currents would be damaging to both the motor and the power circuit, and so it is normal to limit the starting current to approximately twice full-load current. This limitation is largely determined by the ability of the commutator and brushes to handle only this amount of armature current without sparking.

The motor current at starting is limited by a resistor inserted in the armature circuit. Sections of this resistor are then shorted out in sequence as the motor armature develops speed. The use of a starting resistor is common to all types of d-c motors, as indicated in Fig. 6-3. In each of the diagrams the starting resistor is gradually shorted out by moving the lever from left to right.

The action of the starter is such that since the current is limited only by the armature circuit resistance, a torque is produced in excess of that necessary to overcome the shaft load. Therefore, the motor speeds up and develops a generated voltage that reduces the net voltage and also the current. When this reduction has been achieved, part of the resistance may be cut out of the circuit, and the armature current will again increase. This process is repeated until all resistance has been removed from the rotor circuit.

When the starter handle is moved all the way to the right, it is held by a small electromagnet which in *a* and *c* of the dia-

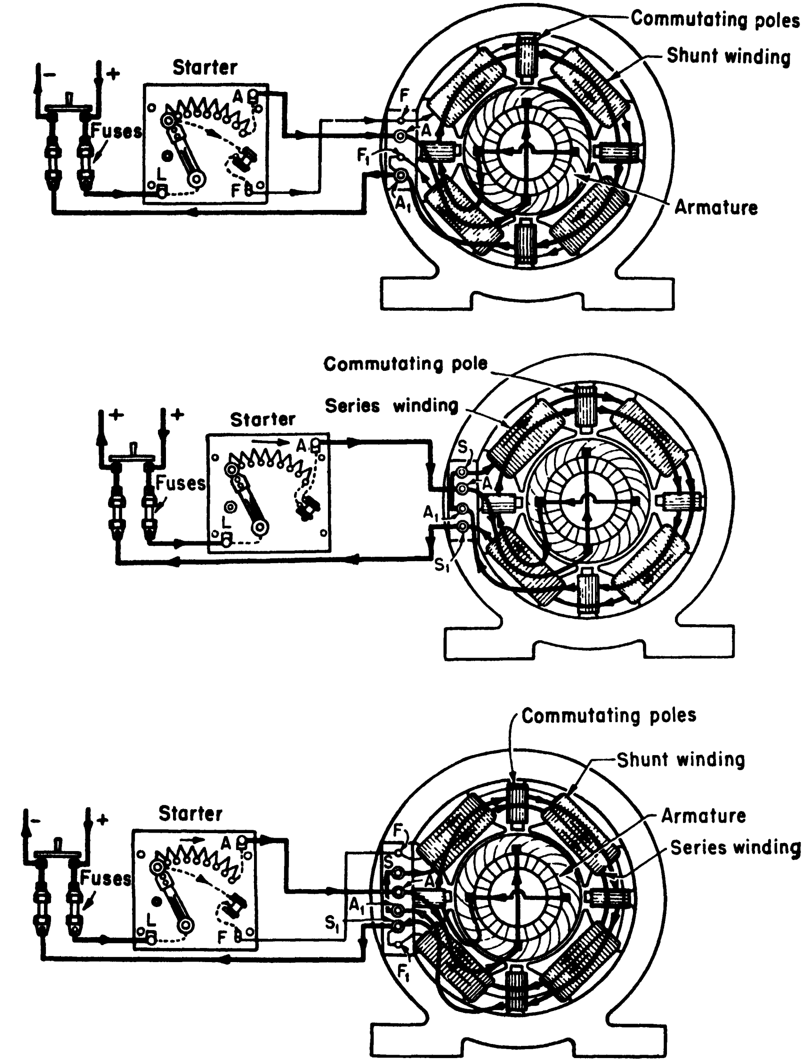


FIG. 6-3. Starter and Motor Connections for Shunt, Series, and Compound D-C Motors.

gram is in series with the shunt-field circuit. In *b* of the diagram it is in series with the armature circuit. In case the power fails while the motor is operating, this coil releases the arm, which is thrown back to the starting position by a spring. Such a starting box is called a three-point, or three-terminal, starter. In many starters the electromagnet no-voltage release

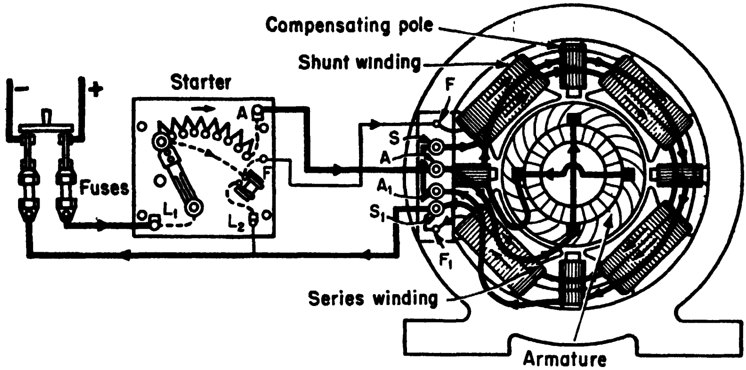


FIG. 6-4. Compound Motor with Four-Terminal Starter.

is connected across the line, as shown in Fig. 6-4. Such a starter is called a four-terminal starter.

Speed control of d-c motors

The speed of the direct-current motor may be changed without appreciably reducing the motor efficiency; therefore, it is often used for variable speed service. The key to variable speed operation lies in the equation for counter-electromotive force given previously.

$$E_m = K\phi S, \qquad \text{or} \qquad S = K' \frac{E_m}{\phi},$$

where E_m is the motor counter-electromotive force, K is a constant of the machine, ϕ is the field flux, and S is the motor speed.

From a study of this equation it can be seen that the speed will vary inversely with the magnitude of the field flux, and directly with the magnitude of the voltage impressed. Since it is easy to reduce the field flux by increasing the resistance of the field rheostat and thus cause an increase in the motor speed, this is the most common of the speed controls of d-c motors.

If the field flux is maintained constant and the armature voltage is varied, the speed will vary in direct proportion. The armature voltage is often varied by inserting resistance in the power circuit as in starting. This method is applied most generally to series motors on crane hoists. The armature voltage may also be varied by having a separate generator for the motor and controlling the generator voltage. This type of control,

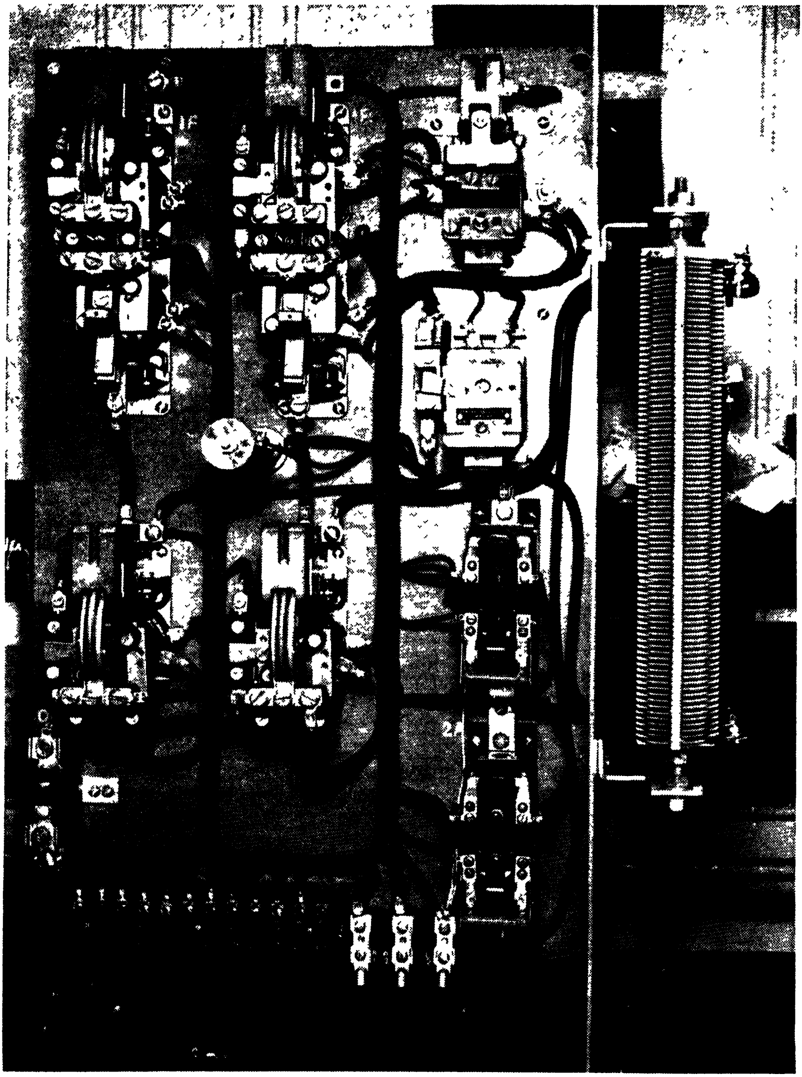


FIG. 6-5. General-Purpose, Adjustable-Speed, Reversing Magnetic Controller with Dynamic Braking, and Jogging. (Courtesy General Electric Co.)

which is used on elevators and steelmill motors, is known as the Ward-Leonard control and will be described later.

Adjustable-speed motors. An adjustable-speed motor is one in which the speed of the motor may be adjusted, but, once adjusted, this speed is held essentially constant with load. D-c shunt motors with field rheostats are adjustable-speed motors.

The limit of speed adjustment depends upon the design of the motor, although even general-purpose constant-speed motors may have their speed increased to twice rated value by introducing a field rheostat. To increase the speed beyond this value may lead to mechanical failure caused by excessive centrifugal forces, or the decrease in field strength may lead to instability caused by armature reaction.

Adjustable-speed motors are designed to withstand the increased centrifugal forces at the higher speeds. The effect of armature reaction becomes much more troublesome at the reduced shunt field magnetomotive forces required at high speeds. Special consideration is therefore given, in the design and construction of these motors, to minimize or neutralize the effects of armature reaction. This results in less change of speed with load, and prevents excessive concentration of voltage on the commutator. As a result it is quite possible to obtain a motor with a four-to-one speed range (in special designs, six-to-one) that will operate at high efficiency at all speeds.

The power limitation in these motors is usually the armature heating. Since this is primarily dependent upon the armature current, this is a fixed limitation on the machine. If the impressed voltage is the same, the power delivered is essentially independent of the speed. The reduction in torque caused by the reduction in flux (for constant armature current) will normally just compensate for the increased speed.*

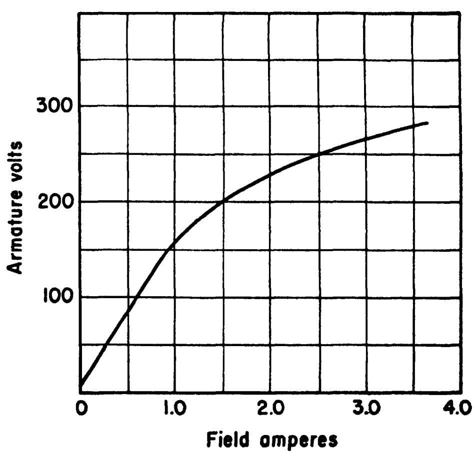


FIG. 6-6. Magnetization Curve for a 15-hp, 230-volt, Adjustable-Speed Motor when Operating at 700 rpm.

Exercise 6-5. A 15-hp, 230-volt d-c adjustable-speed motor has a magnetization curve as shown in Fig. 6-6. The resistance of the shunt field measures

* Where very wide range of speed is required, it is often desirable to use a standard adjustable-speed motor with field control in the higher speed range and with armature resistance control below the standard base speed.

70 ohms and a field rheostat of 400 ohms is supplied. (a) If the armature resistance drop is neglected, determine the speed range. (b) What is the speed range if an armature resistance drop of 15 v is assumed?

Variable-speed d-c motors. Variable-speed motors are those that vary widely in speed with load. This applies to series motors and compound motors with strong series fields. In these motors a change in load torque produces a change in armature current, which, in turn, changes the field flux, and this requires a change in speed to obtain the proper motor-generated voltage. Motors of this type have a very strong field for heavy loads, which gives maximum torque for a given armature current on the basis that

$$T = K'\phi I_a.$$

Such motors are particularly useful, therefore, where variable loads occur and where constant speed is not necessary or desirable. Typical variable-speed motor applications are for cranes, hoists, bridges, and streetcars.

Where a fixed maximum speed is desired, the compound motor is used. Such loads as punch presses, shears, crushers, conveyers, etc., often call for compound motors.

Exercise 6-6. A motor having a magnetization curve similar to that of Exercise 6-5 has a series-field magnetomotive force equivalent at full load to 2 amp of shunt-field current. It has a fixed shunt-field current of 1 amp, and the power supply is 240 volts. If the armature resistance drop is neglected, what is the no-load speed, the full-load speed, and the speed at 150 per cent load?

Adjustable voltage control for d-c motors. For machines requiring a speed range greater than can be obtained by field control or for applications requiring frequent and rapid reversing, the adjustable voltage system of control is most satisfactory. In its simplest form this control system or drive consists of the following:

(1) Shunt motor with separate field excitation.

(2) Shunt generator with separate excitation and a rheostat or potentiometer for its control.

(3) Exciter or source of constant-voltage direct current.

(4) Driving motor for generator and exciter. These are shown diagrammatically in Fig. 6-7.

By operating the rheostat in the generator field, it is possible to vary the armature voltage over a wide range, often from the residual value to normal rated value, and with such

voltages across the motor armature a wide range in speed is obtained. Since the motor flux is constant, full-load torque will be obtained with full-load amperes at any speed. Such drives are consequently best adapted for constant-torque loads. The stabilized speed of the motor will vary as the motor-generated voltage, as previously shown.

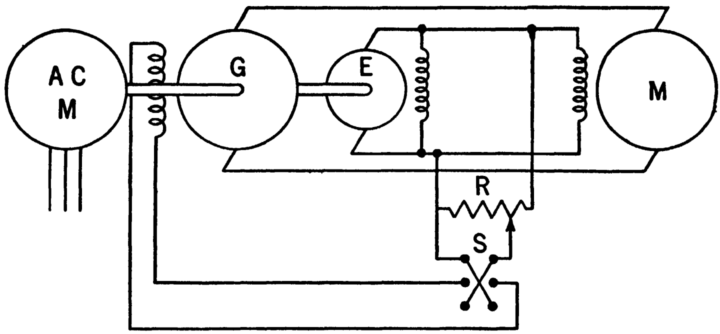


FIG. 6-7. Ward-Leonard Control for D-C Motors.

The wide speed range possible, the simplicity of the control, smooth acceleration, elimination of armature resistors, and large contactors make this system ideal for many industrial machines such as large power shovels.

Efficiency of d-c machines

As in nearly all types of electrical machinery the efficiency is higher in the larger sizes. The average efficiency for 1-hp motors is about 80 per cent, while for 50-hp motors this increases to about 90 per cent. The variation however depends not only on size but on rated speed as well. This range of efficiencies is shown in Fig. 6-8, where the shaded area indicates normal

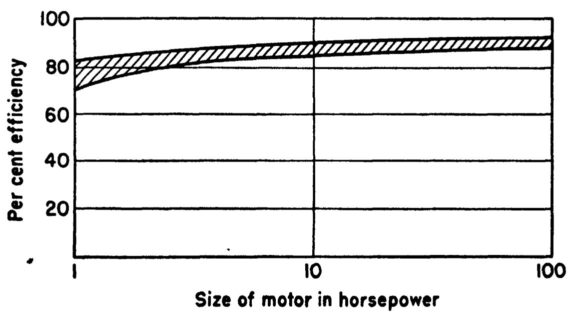


FIG. 6-8. Variation of Efficiency with Motor Size.

variation caused by speed and design procedures of different manufacturers.

It is characteristic of electric machinery that the efficiency remains essentially constant throughout the life of the motor since the losses are of a type that do not change with use. In fact, any appreciable change in losses will most certainly cause overheating and a resultant failure of the equipment.

Rating and performance

Motors are specified in terms of the horsepower they will deliver, the speed at which they operate, and the voltage and type of power supply required. Similarly, generators are specified in terms of the current and voltage they will supply and thus the possible power output and the speed of operation.

The machine rating is determined by the design. As discussed in Chap. 5, the armature must have enough series conductors to produce the rated voltage when rotating at rated speed in normal field flux. The armature conductors must be able to carry rated current in the case of both generator and motor. In the motor this rated current will produce sufficient torque that the rated horsepower will be delivered at the shaft. In other words, electrical machinery is rated on the basis of output.

Most generators and motors are constant-voltage machines, and output will depend upon the current in the armature. The current-carrying ability of the armature, therefore, becomes the limiting factor for any motor or generator. The limits to the current-carrying ability of the machine are set by commutation and by overheating, which may damage the insulation.

Commutation limits have been mentioned in connection with starting of motors. Most motors will commutate satisfactorily momentary overloads of current equal to double the rated current. Overloads of 150 per cent rated current for as long as one minute will also be within commutation limits.

The real limitation for continuous operation is, therefore, the heating limit. In a 10-hp motor, having an efficiency of 85 per cent, approximately 1300 w are converted from electrical energy into heat. This will cause the temperature of the motor to rise until radiation and convection dissipate the heat as rapidly as it is converted.

The limiting temperature for long life of ordinary insulation has been standardized at 40° C above the surrounding air.

This limits the average temperature to 70° C even in summer heat, and impregnated cotton and other common insulation is not harmed below this temperature. Where Fiberglas and mica insulation is used, the temperature may be permitted to rise another 10 to 35° C without damage to the machine. Heating is not always uniform. In fact, there are usually some spots where heat dissipation is not as good as normal. When sudden overloads occur, these spots become the critical ones, as they heat up much more rapidly than the rest of the machine.

Thus, a machine is rated normally on the current or horsepower that it will deliver continuously without exceeding the 40° C temperature rise. Since it takes considerable time to reach the limiting temperature, it is possible to give special ratings for intermittent duty somewhat above the continuous rating. Furthermore, the motor can be expected to carry momentary or short-period overloads without overheating.

N.E.M.A. standards. In order to obtain the benefits of standardization, the National Electric Manufacturers Association has been established to set standards of dimensions, voltages, speeds, horsepower and kilowatt ratings, and performance.

The standards of dimensions specify the distance from the base to the center of the shaft, the size of the shaft, and the position of the mounting bolts for each size of motor. Thus, it is possible for the industry generally to design to these standards, and any motor of proper size will fit the mounting.

Voltages for d-c machines are specified as 125 or 250 v for generators and 115 or 230 v for motors. All sizes of motors are built for 230 v, but 115-v motors are limited to 30 or 40 hp, since in the larger sizes the current becomes excessive.

Motors are manufactured in the following standard sizes: 1, $1\frac{1}{2}$, 2, 3, 5, $7\frac{1}{2}$, 10, 15, 20, and 25 hp. Both larger and smaller sizes are standardized, but these standards have not been included in this list. A wide range of standard speeds may be obtained in nearly all of these sizes. The standard speeds conform to standard speeds of induction motors, as will be studied in a later chapter. The most common base speeds are 3500, 1750, 1150, and 850 rpm. Lower base speeds down to 100 rpm may be obtained.

Adjustable-speed motors are those that maintain essentially constant speed with load, but whose speed may be adjusted. In d-c motors these are usually shunt motors that are so designed that a field reduction may be used to obtain speeds up to three

or four times base speed. With such motors the increased speed of rotation gives better ventilation, and this permits increased horsepower ratings. The N.E.M.A. standards specify the increase in accord with the curve in Fig. 6-9.

To illustrate let it be assumed that a 10-hp adjustable-speed motor with a base speed of 500 rpm is being considered. At base speed, with full-shunt field, it may be loaded up to rated load, and a 50° C temperature rise is allowed. From 500 to 750 rpm the rating remains constant, but the allowable temperature rise at 750 rpm at rated load is reduced to 40° C. At three times base speed (1500 rpm) the motor may be rated at

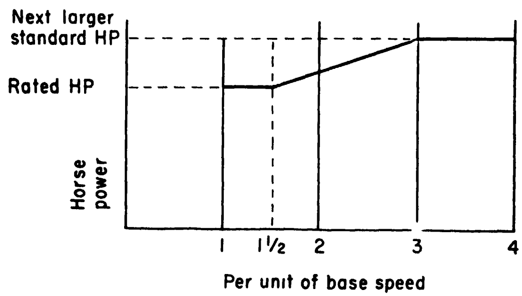


Fig. 6-9. Tapered Rating of Adjustable-Speed D-C Motors.

the next larger standard size, which is 15 hp. Between 750 and 1500 rpm the hp rating increases on a straight-line basis. From 1500 to 2000 rpm, the limit of standard speed variation, the hp rating is constant at 15 hp. Large d-c motors do not operate above 3600, even if the base speed is 1800, because of the excessive centrifugal forces on commutator and windings.

It is noted that these ratings are somewhat arbitrary, but they assure standardization among manufacturers. In operating electrical machinery, some judgment is permitted to the engineer in charge if that judgment is based on a sound knowledge of conditions. For instance, in winter, with low temperatures, an increased loading might be permitted without exceeding safe operating temperatures.

Where Class B insulation is used, a considerably higher temperature is permitted, and these motors may be given a continuous rating with temperature rises up to 75° C. It is particularly desirable to use such motors where they are located near furnaces or other places where high initial temperatures are experienced.

In selecting motors, both economy and quick delivery are

obtained by making the selection within the standards. Sufficient variety in both size and performance is available to meet almost any need. When special motors are specified, it usually means that the purchaser will pay a much higher price and will have to wait for a standard motor with a special name plate.

Exercise 6-7. A 10-hp 230-v 1800-rpm open motor drives a belt conveyor on a construction job in Minnesota. The winding fails and the only other motor available is a 5-hp 230-v 900-rpm d-c motor. The electrician suggests that it be used with a field resistance to increase the speed to 1200 rpm, which would permit operation at reduced capacity. Do you approve his recommendation (give full justification for your answer)?

Protection of d-c machines

All generators including d-c generators should be protected by a circuit breaker or fuse that will break the circuit in case of a short circuit somewhere on the system. Such a breaker should not trip out on momentary overloads, such as are experienced in the starting of motors but must disconnect the generator quickly if the load exceeds the commutating limit of the machine. It is usual, therefore, to set breakers for d-c generators for instantaneous trip somewhat above twice normal load.

Motors may be protected by fuses or by switches. Both fuses and switches are preferably arranged with heat-storage devices, so that they will not open on starting and on momentary overloads but will open on continued overloads that will overheat the motor. In some small motors, thermostats are included in the motor so that if the motor temperature rises beyond a safe value, the motor circuit is opened.

CHAPTER 7

Alternating-Current Circuits

Alternating current and voltage

An alternating current or voltage is defined as a current or voltage in which the direction changes periodically. In other words, the current flow, or electron drift, is first in one direction in the circuit and then in the other, this reversal occurring at regular intervals.

The frequency with which a complete change occurs may be 60 times a second (as in the case of electric power supplied to most residences), from 20 to 10,000 times per second (for voice and music waves in telephone communication), or up to millions and billions of times per second (as in the case of the radio signals that are used in communications and other signal purposes). In many of the applications of alternating current the variation with time is smooth and regular, following the variation of the sine of a constantly varying angle. Such an alternating current or voltage is said to vary sinusoidally with time or to be a sine wave.

Sine waves in nature. A tuning fork produces sound or variation in the atmospheric pressure that varies sinusoidally with time. The pendulum of a clock is shifting energy back and forth from kinetic to potential in its sinusoidal movement. If a hacksaw blade is clamped in a vise, a weight placed on the end of it may be made to oscillate with a sinusoidal movement that also shifts energy back and forth from kinetic to spring, or strain, energy.

The vibration of a gasoline engine, caused by unbalances in the rotating elements and uneven forces exerted on the pistons, appears in the form of sinusoidal movements of the engine itself. The oscillations in the tuned circuit of a high-frequency electric heating unit are also sinusoidal.

Sine waves in electrical equipment. In electric power equipment every effort is made to assure a sinusoidal voltage at the power outlet. This is so nearly achieved in modern power cir-

cuits that a sinusoidal voltage may be assumed without appreciable error.

In high-frequency heating units, such as are used for preheating thermoplastics for molding operations and for surface heating and hardening of small gears, the oscillations assume the sinusoidal form by the very nature of the energy interchange between the magnetic and electric fields.

Since such sinusoidal variations of current and voltage are so extensively used in all a-c equipment, their characteristics will be studied in considerable detail.

Time-phase plotting of sine waves

A sinusoidal variation of current with time* may be conveniently represented by plotting the instantaneous values of

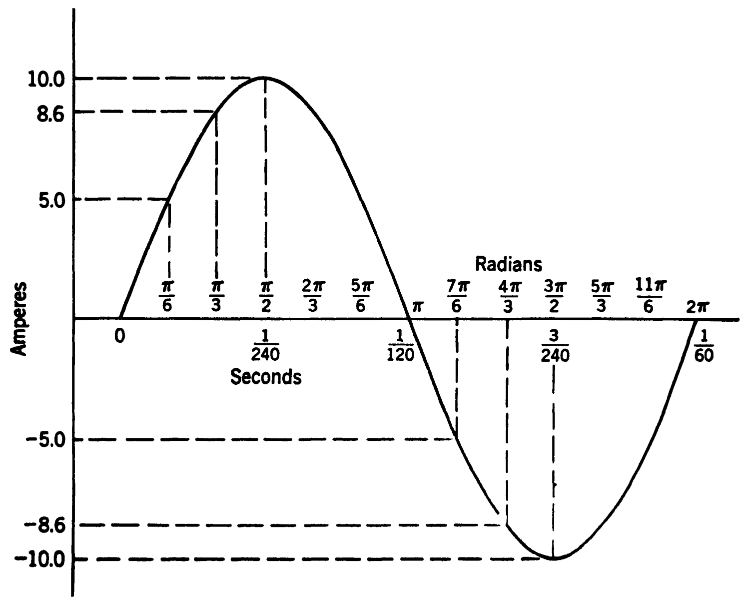


FIG. 7-1. Sinusoidal Variation with Time.

the current as ordinates against the corresponding values of time as abscissas. Such a curve is shown in Fig. 7-1 for a current that has a maximum value of 10 amp. The time scale is shown just below the axis with the major points specified in fractions of a second. It is seen that a complete cycle requires

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* *Note.* The generation of a sinusoidal a-c voltage by a rectangular coil rotating in a uniform air gap is discussed on p. 69 of Chap. 4.

one-sixtieth of a second or that there would be 60 complete cycles in one second. Such a sine wave is said to have a frequency of 60 cps.

A scale of angular measure in radians is shown above the time scale. This angular scale demonstrates the sinusoidal character of the variation, as the ordinate in each case is ten times the sine of the angle. There is, moreover, a very definite relationship between the time and the angle. This can be stated mathematically as

$$\theta = 2\pi f t$$

where f is the frequency and t is the time in seconds. The value of the current at any time may be written as

$$i = I_{max} \sin (2\pi f t),$$

which, in the case of the current shown in Fig. 7-1, is

$$i = 10 \sin (2\pi \times 60t) = 10 \sin 377t.$$

Exercise 7-1. Plot a sine wave of a-c current having a maximum value of 25 amp and a frequency of 400 cps.

Exercise 7-2. Calculate the current in a 60-cycle circuit having a maximum value of 100 amp 0.005 sec after the current passes through zero.

Alternating current in a resistance

According to Ohm's law the voltage across a resistor is equal to the product of the resistance and current. Thus the instan-

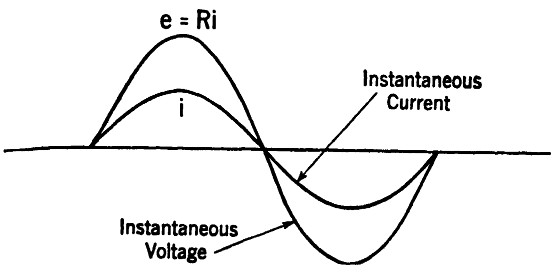

FIG. 7-2. Alternating Current and Voltage Relations in a Resistor.

taneous voltage may be written as

$$\begin{aligned} e &= Ri = RI_{max} \sin (2\pi f t) \\ &= E_{max} \sin (2\pi f t) \end{aligned}$$

This relationship is shown in Fig. 7-2, where e is a sine wave exactly similar to the current but with a magnitude equal to Ri.

The power at any instant is the product of the resistance and the square of the current. This may be written as

$$p = i^2R = RI^2_{max} \sin^2 (2\pi ft).$$

Since $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, the above equation may be written

$$p = \frac{RI^2_{max}}{2} - \frac{RI^2_{max}}{2} \cos (4\pi ft).$$

Since the average value of the second term on the right is zero for a complete cycle, the average power is

$$P_{avg} = \frac{RI^2_{max}}{2}.$$

These values are shown in Fig. 7-3, where the curves of instantaneous and average power are added to the curves current of and voltage of Fig. 7-2.

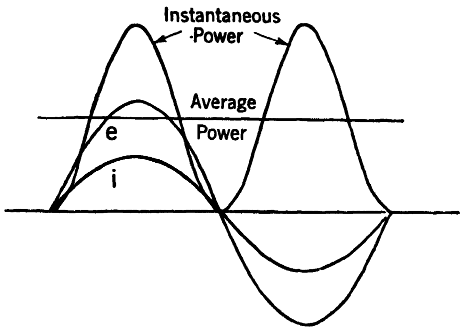

FIG. 7-3. Instantaneous A-C Power in a Resistor.

Exercise 7-3. Plot curves of current, voltage, and power when a current of 2 amp flows in a resistance of 5 ohms. The frequency is 1000 cps.

Exercise 7-4. Plot the current and voltage in a resistor* when the a-c current flowing has a maximum value of 25 ma and the resistance is 120 ohms. Assume a frequency of 2000 cps.

Maximum and effective values of a-c waves

The magnitude of a-c waves has so far been specified in terms of the maximum value. Although this method is satisfactory

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* *Note.* Such a resistor is typical of the resistance strain gage explained in Chap. 18.

for some purposes, when a-c currents were first used it was decided that the effective value of an a-c current should be defined *as that current which would give the same average heating effect when flowing in a resistance as a d-c current of the same number of amperes.* By definition, therefore,

$$P_{avg} = I_{eff}^2 R.$$

In the preceding paragraph, however, it was shown that

$$P_{avg} = \frac{I_{max}^2 R}{2}.$$

Therefore,

$$I_{eff}^2 R = \frac{I_{max}^2 R}{2}, \qquad \text{and} \qquad I_{eff} = \frac{I_{max}}{\sqrt{2}}.$$

TABLE 7-1

DETERMINATION OF AVERAGE AND EFFECTIVE VALUES OF A SINE CURRENT HAVING 10 AMP MAXIMUM VALUE

Time (sec)	Degrees	i (amp)	i^2
.00046	10	1.74	3.03
.00093	20	3.42	11.79
.00139	30	5.00	25.00
.00185	40	6 43	41 35
.00231	50	7 66	58 67
.00278	60	8.66	75 00
.00324	70	9.40	88 36
.00370	80	9 86	97 22
.00416	90	10 00	100 00
.00463	100	9 86	97 22
.00509	110	9.40	88 36
.00555	120	8 66	75 00
.00602	130	7.66	58.67
.00648	140	6 43	41 35
.00695	150	5 00	25 00
.00741	160	3.42	11.79
.00787	170	1 74	3.03
.00833	180	0 00	0 00
	Sum	114.34	900 8
	Average	6.36	50.0

Equivalent d-c current = $\sqrt{50.0}$ = 7.07 amp.

In Table 7-1 the values for current and current squared are given for each 10 deg, or 0.00046 sec for a 60-cycle current. This table demonstrates by the use of simple arithmetic the conclusion that the average of current-squared values is equal to

the maximum current squared, divided by 2. It also indicates that the reason for referring to the effective value as root-mean-square, or rms, value is that the effective value is the square root of the mean, or average, of the squared values of the current.*

Alternating voltages as well as currents are expressed in terms of effective values. The ratio of the maximum value to the effective value remains equal to $\sqrt{2}$. The average power is then equal (for a resistance load) to the product of the effective current and effective voltage, or

$$P = EI.$$

This may be developed from the average power in the following manner:

$$P_{avg} = \frac{I^2_{max}R}{2} = \frac{I_{max}}{\sqrt{2}}\frac{I_{max}R}{\sqrt{2}} = \frac{I_{max}}{\sqrt{2}}\frac{E_{max}}{\sqrt{2}} = IE.$$

Some types of a-c meters rectify the a-c current and thus give a reading that is proportional to the average of the half-wave value of the current or voltage.

The different forms used to specify sinusoidal a-c values are:

a) Instantaneous value

$$i = I_m \sin (2\pi ft) = I_m \sin \omega t.$$

b) Average value (of the half wave)

$$I_{avg} = \frac{1}{\pi}\int_0^\pi I_m \sin (\omega t) d(\omega t) = \frac{2}{\pi} I_m = 0.636 I_m.$$

c) Effective value (rms value)

$$I_{eff} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m.$$

Exercise 7-5. Plot the power flow to a lamp taking 0.25 amp at 120 v. What is the frequency of the power pulses if the current frequency is 25 cps?

---

* The above relationships have been developed on the assumption that the current, and therefore the applied voltage, were sinusoidal. The definition of the *effective* value of the current for irregular waves is still "that current which will give the same heating effect or average power as a d-c current." The use of effective current, even with irregular waves, makes it possible to determine whether or not a resistor will overheat or whether it will carry the specified current satisfactorily. The ratio of the maximum current to the effective current will not ordinarily be $\sqrt{2}$ in nonsinusoidal currents.

Radius-vector or phasor method of representation of sine waves. Although the sinusoidal curve provides a very satisfactory representation of the time variation of sinusoidal quantities, it has the disadvantage of being difficult to draw, and the curves become confusing when more than three or four are placed on the same diagram. It has become common practice,

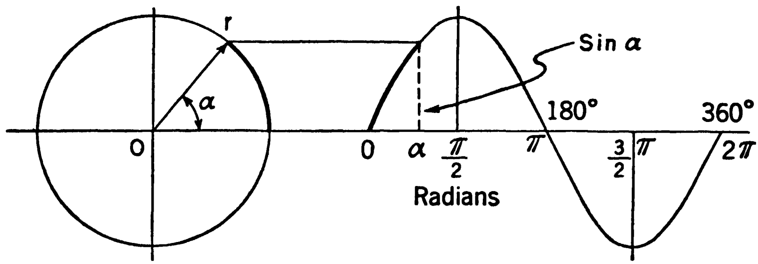

FIG. 7-4. A Rotating Radius Vector or Phasor May Be Used to Produce a Sine Wave.

therefore, to use a rotating *radius vector* or *phasor* to represent a sinusoidally varying quantity. The line *or* of Fig. 7-4 is a line of unit length that is being rotated at a uniform angular velocity. In the portion of the diagram at the right, the angular measure in radians is plotted as abscissa, whereas the vertical projection of the rotating line *or* is plotted as ordinate for each corresponding position. The rotating line, called a *radius vector* or *phasor*, may then be said to generate a sine wave and may be

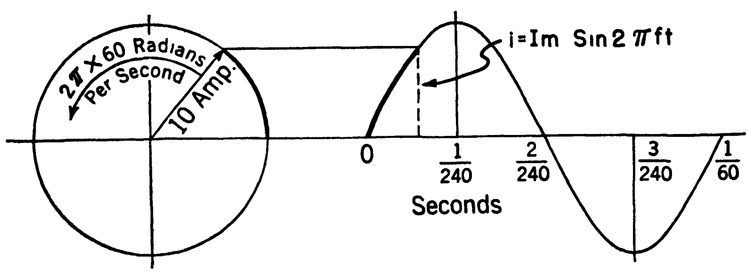

FIG. 7-5. A Rotating Radius Vector and Sine Wave Show the Variation of Current in a Conductor Carrying 60-Cycle Alternating Current.

used to provide a conventional representation of a sinusoidally varying quantity.

The phasor in Fig. 7-5 is given a scale length equal to 10 amp and is rotated at an angular velocity equal to $2\pi \times 60$ radians/sec. This angular velocity (usually called ω) is equal to $2\pi f$, and in this case produces a sinusoidal wave having a

frequency of 60 cps. The phasor representation of sinusoidally varying a-c currents and voltages has the advantage of simplicity and so is used almost exclusively for this purpose.

Phase difference of alternating currents and voltages. When a-c quantities pass through zero and reach the maximum positive value at the same time, as do the current and voltage in Fig. 7-2, these quantities are said to be *in phase*. When alternating currents and voltages are *not in phase*, the quantity that reaches the maximum positive value first is said to *lead* the other quantity. Similarly, the one which reaches its maximum later is said to *lag* the other a-c current or voltage. If, for instance, two wires are carrying different a-c currents, one of which is 10 amp maximum and the other one 5 amp maximum and lagging 60 deg behind the first, they could be represented by the diagram of Fig. 7-6. In this figure the current in each wire is represented by its individual sine wave and phasor. Since the periods of the sine waves are similar in the sine-wave portion of the diagram, they will maintain the same relation to each other. Likewise, since the angular rotation of the phasors is the same, the phasors will also maintain a constant relationship with one another.

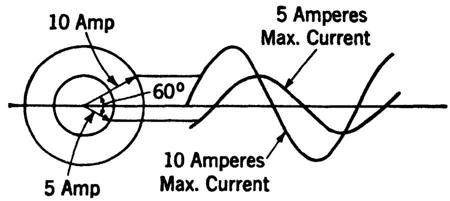

FIG. 7-6. A-C Currents Which Are Out of Phase.

Adding alternating currents and voltages

Let the two wires carrying 10 and 5 amp, respectively, be joined as in Fig. 7-7a, so that although the current in each remains as before, the currents are added in the common wire. The current in this common wire is then the sum at every instant of the currents in the other two wires.

This is shown in the sine-wave diagram of Fig. 7-7b as the resultant current and is itself a sine wave with a maximum value of 13.2 amp. In the phasor portion of the figure, if a parallelogram is completed and a diagonal drawn, this diagonal is also found to be equal to 13.2 amp, or the maximum value of the resultant current. Furthermore, the phasor will reach the vertical position at the same time the resultant current is maximum, as shown by the instantaneous addition of the sine-wave values.

Thus, sine-wave quantities may be added algebraically at any instant, or, if they are represented by phasors, the addition must be a vector addition. When a subtraction is to be made,

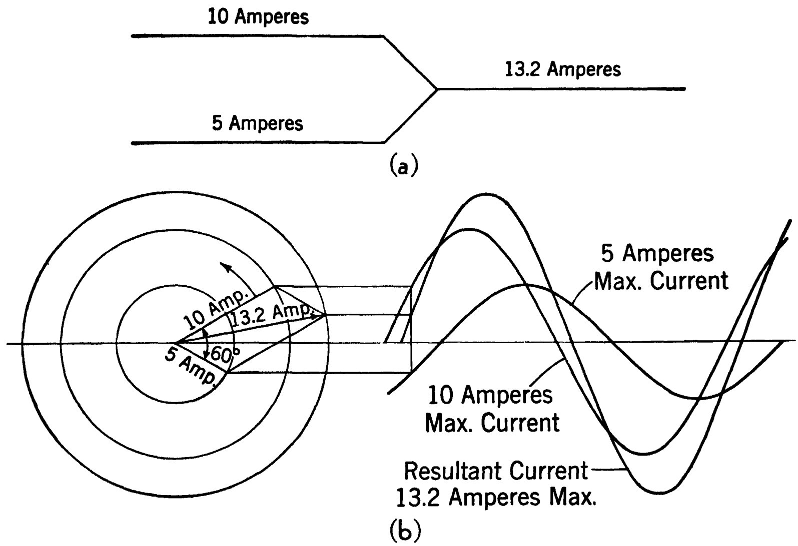

FIG. 7-7. Addition of Alternating Currents.

it may be done on the basis of instantaneous algebraic subtraction of sine waves, or one phasor may be subtracted from the other. To subtract a phasor, it is reversed and then added vectorially. This procedure is shown in Fig. 7-8, where the same 10-amp and 5-amp phasors are shown, but it is desired to subtract the 5-amp phasor from the 10-amp phasor. The resultant vector in this case is 8.66 amp and leads the 10-amp current by a phase angle of 30 deg, or $\pi/6$ radians.

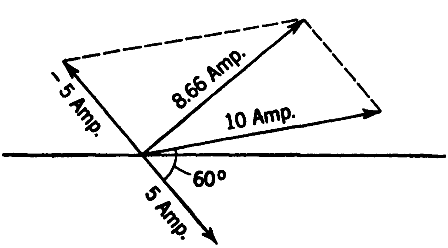

FIG. 7-8. Vector Subtraction of Alternating Currents.

Exercise 7-6. Plot the sine wave of current in a wire when it is composed of the current from two branches, one of which carries 25 amp and the other carries 10 amp leading the first by 30 deg. The frequency is 60 cps.

Interpretation of phasor methods. Either the sine wave or the phasor may be used to represent an a-c quantity. Sometimes one and sometimes the other is more convenient or more

effective in showing the characteristics that are to be studied. Hereafter the method will be used that seems to be most effective, and only occasionally will both representations be used on the same problem. Either type of diagram is useful in showing the magnitude and phase relationships of currents and voltages. It is common practice in such diagrams to use one scale for voltages and a different scale for currents.

The phasors have normally represented the maximum value of current or voltage. Since the ratio of effective to maximum values is constant for all sine waves, it is customary to use phasors to represent effective values. As long as a single phasor diagram represents only effective values or only maximum values, no difficulty will be experienced.

Rate of change of current in a sine wave

Important characteristics of a-c circuits depend not only upon the amount of the current but also upon the rate at which the current is increasing or decreasing. Expressed mathematically, the rate of current change is the derivative of the instantaneous current, and if the instantaneous current is

$$i = I_{\max} \sin 2\pi f t,$$

then

$$\text{rate of change of current} = \frac{di}{dt} = I_{\max}(2\pi f) \cos (2\pi f t).$$

If the maximum current is 10 amp and the frequency is 60 cps, this becomes

$$\begin{aligned} \text{rate of change of current} &= \frac{di}{dt} = 10(2\pi 60) \cos (377t) \\ &= 3770 \cos (377t) \quad \text{amp/sec} \end{aligned}$$

This may be demonstrated without the use of calculus in the following manner. Since the vertical change in the sine wave represents the change in current and the horizontal change represents the change in time, the current change divided by the corresponding time change is, by definition, the rate of change of the current. This ratio is also the slope, or steepness, of the sine curve. In Fig. 7-9 the same current of 10 amp maximum is shown that has been studied before. At the point a a tangent is drawn, and the slope can be determined by dividing the distance i' in amperes by the distance t' in seconds. If this same procedure is followed for a number of points on the current curve and the results plotted against the corresponding time, a

curve will be obtained for the slope. This curve will be a sine wave displaced by 90 deg. It will be observed that the maximum value occurs at the time the current is passing through zero, which means that it is a sine wave that leads, or precedes, the current wave by a time-phase angle of 90 deg, or $\pi/2$ radians. The maximum value of the curve of the rate of change of current is $2\pi f$ times the maximum of the current curve, where f is

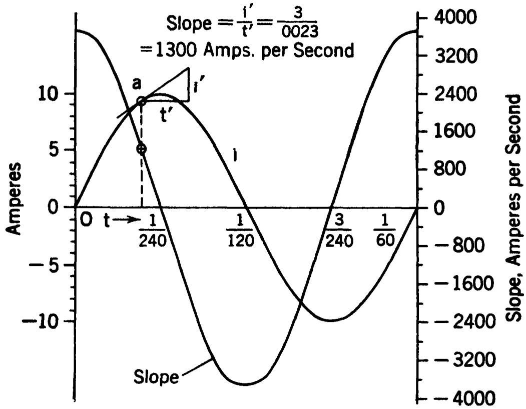

FIG. 7-9. The Rate of Change of Current with Alternating Current.

the frequency. This maximum may be explained by the fact that the rate of change, when the current is going through zero, is represented by the terminus of the radius vector moving at full velocity. Since its angular velocity is $2\pi f$ radians, or f complete revolutions per second, its velocity, or rate of change, will be $2\pi f$ times its length, and the length is equal to the maximum value of the current.

Either method of demonstration shows that the rate of change of current is a sinusoidal wave that has a maximum magnitude equal to $2\pi f$ times the maximum value of the current. This relationship is important, as it is the basis for the determination of the reactance of a-c circuits.

Inductive reactance

When an alternating current flows in an inductance coil, the current in the coil and the voltage across the coil are no longer in phase. Let it be assumed that an a-c current is flowing in an inductance coil and that this current is represented by the sine-wave and phasor diagram of Fig. 7-10. If the coil has an inductance of L h, then the voltage of self-induction

will be*

$$e = L\frac{di}{dt} = 2\pi fLI_{max}\cos(2\pi ft).$$

The direction of this voltage will be such (by Lenz's law) as to oppose the change of current. Thus, when the current is increasing at a maximum rate, the voltage of self-inductance has a maximum negative value, as shown by the dotted curve of Fig. 7-10.

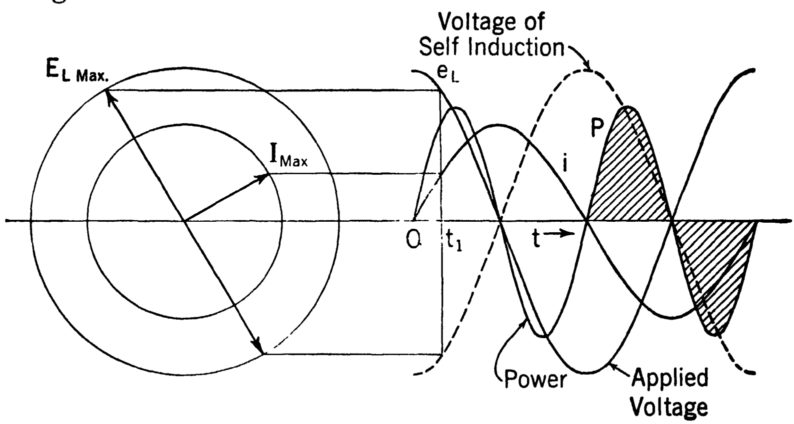

FIG. 7-10. Current, Voltage, and Power in an Inductive Circuit.

In order to maintain an a-c current through the inductance, a voltage must be impressed upon the coil, which will both neutralize this reactance voltage and, in addition, overcome the resistance drop. If the resistance drop is small, as it is in many cases, then the impressed, or applied, voltage will be essentially equal and opposite to the reactance voltage. The curve of impressed voltage is shown by the solid line on the sine-wave diagram of Fig. 7-10 and is equal and opposite to the reactance voltage. Since this voltage wave comes to a maximum 90 deg, or $\pi/2$ radians, before the current wave, it is said to lead the current wave, or *the current wave may be said to lag* 90 *deg behind the wave of impressed voltage.* The absolute magnitude of this maximum voltage is

$$E_{L_{max}} = (2\pi fL)I_{max}.$$

The quantity $2\pi fL$ is a characteristic of the inductance coil and frequency and is called the *inductive reactance of the coil.* It is

* See p. 64 of Chap. 4.

usually given the symbol X_L and, since it is the ratio of a voltage to a current, it is expressed in ohms. The effective values of current and voltage have the same ratios as the maximum values, so the relation may be expressed as follows:

$$E_{L\max} = X_L I_{\max},$$

and

$$E = X_L I,$$

or

$$I = \frac{E}{X_L}.$$

Example. A coil having negligible resistance and an inductance of 15 mh is connected to a 120-v 60-cycle source. Determine the current flowing and draw the diagram of vectors.

Solution: (1) Determine the reactance. The reactance of the coil is

$$\begin{aligned} X_L &= 2\pi f L \\ &= 2\pi \times 60 \times 0.015 \\ &= 5.65 \text{ ohms.} \end{aligned}$$

(2) Determine the current. No mention is made to the contrary, so it is assumed that 120 v is the effective value.

$$I = \frac{E}{X_L} = \frac{120}{5.65} = 21.3 \text{ amp.}$$

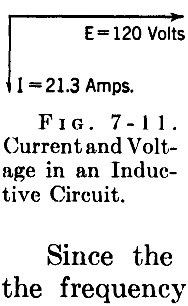

FIG. 7-11. Current and Voltage in an Inductive Circuit.

(3) Draw the phasor diagram. (Fig. 7-11.) Let E = 120 v be the reference voltage that is impressed upon the coil. Then I = 21.3 amp is the current flowing. It lags 90° behind the voltage and is not drawn to the same scale as the voltage phasor.

Since the inductive reactance is directly proportional to the frequency, its value becomes quite large with high frequencies. Inductance coils are often used, therefore, to limit the magnitude of a-c currents when the frequencies become greater than a critical value. When inductance coils are used for this purpose, they are called *chokes*, because they choke off the current for frequencies higher than the critical value.

Exercise 7-7. Determine the current flowing in an inductance of 50 mh when a voltage of 75 v at 400 cycles is impressed upon it. Draw the phasor diagram.

Power in an inductance coil

The power flowing into the circuit at any instant is equal to the product of the current and the voltage. This is plotted

in Fig. 7-12 and is a sine wave of double frequency. The mathematical development is as follows:

$$p_{inst} = ei = E_{max} \cos (2\pi ft) I_{max} \sin (2\pi ft)$$

$$= E_{max} I_{max} \sin 2\pi ft \cos 2\pi ft.$$

Since

$$\sin X \cos X = \tfrac{1}{2} \sin 2X,$$

$$p_{inst} = \frac{E_{max} I_{max}}{2} \sin 4\pi ft,$$

or, using effective values,

$$p_{inst} = EI \sin 4\pi ft.$$

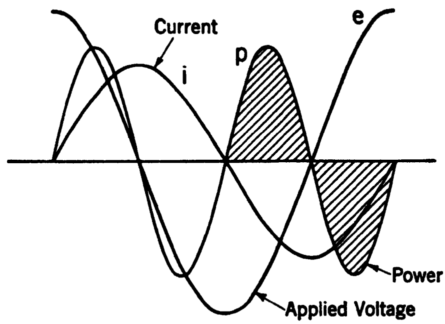

FIG. 7-12. Current, Voltage, and Power in an Inductive Circuit.

When the instantaneous power is averaged over a complete cycle, the average is found to be zero. This of course assumes that the resistance is negligible. The physical interpretation of this zero average power is that the power source is storing energy in the magnetic field of the coil during the time the current is increasing in magnitude and gives a positive loop of power. When the current in the coil is decreasing, this energy is returned to the circuit, and a negative power loop just equal to the positive loop is obtained. This energy storage in the magnetic field, which tends to maintain the current flow, is analogous to kinetic energy in the field of mechanics.

Resistance and inductance in series

The relationship between the current and the impressed voltage has been determined when the circuit consisted of resistance alone and inductance alone. For resistance, the voltage is equal to the product of the instantaneous current and the resistance. For the inductance coil, however, the voltage is a sine wave which is 90° ahead of the current in time phase. If a resistance is connected in series with an inductance coil, the current flow in each will be the same. The voltage across the resistance will be in phase with the current, and the voltage across the inductance coil will lead the current by 90°. This is shown in Fig. 7-13, where the sine wave i is the current, e_r the voltage drop in the resistance, and e_L the voltage impressed on the inductance coil. The voltage impressed on the combination is the instantaneous sum of e_r and e_L, which is shown

as e. This voltage wave leads the current by an angle θ as shown in the diagram. In the phasor portion of Fig. 7-13, the same information is obtained. In this type of representation, however, it is evident that the angle θ by which the voltage leads the current is the angle whose tangent is X_L/R.

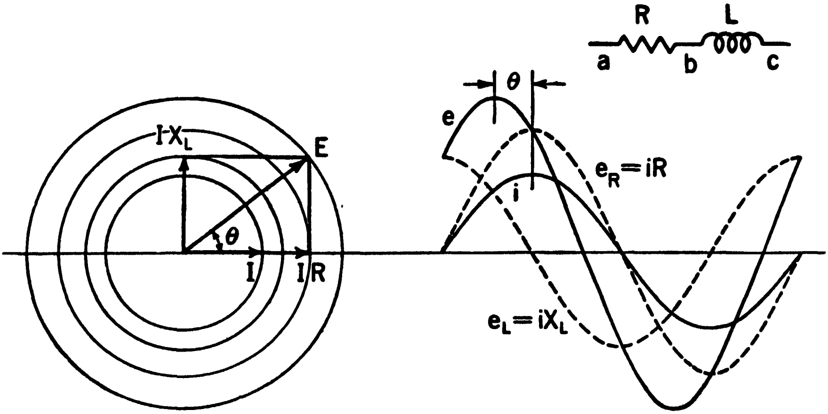

FIG. 7-13. Alternating Current and Voltage with Resistance and Inductance in Series.

Note. In an inductance coil there is always some resistance in the winding; therefore, any practical inductance will have both resistance and reactance and may be represented as shown in Fig. 7-13 with resistance and reactance in series. In this case it would be impossible to measure the IR drop and the IX_L drop separately. It is possible, however, to measure the resultant voltage and to determine the angle θ. The coil may be represented either by a resistance and inductance in series, as indicated above, or by a resistance and inductance in parallel. The series representation is much more common and will be used exclusively in this text.

Impedance and phase angle

In Fig. 7-13 the voltage across the resistance is equal to IR, and the voltage across the inductance is IX_L, but no single quantity by which the current may be multiplied to obtain the total impressed voltage has been discussed. It is desirable that such a quantity or circuit characteristic should be available for circuit calculations.

The diagram in Fig. 7-14 shows how such a quantity may be obtained. R is drawn along the horizontal axis, while X_L is drawn vertically upward. The vector sum of these quantities is a new quantity (known as *impedance*) by which the current may be multiplied to get the total impressed voltage. Similarly,

when the voltage is divided by the impedance, the current is obtained.

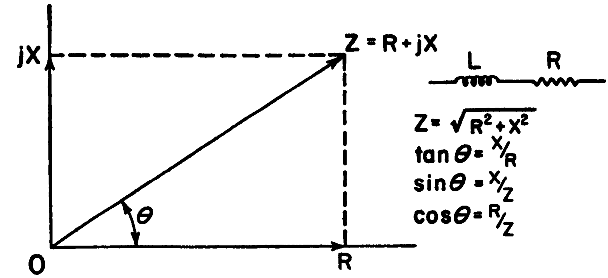

FIG. 7-14. Addition of Resistance and Reactance to Obtain Impedance.

The magnitude of the impedance in this simple circuit is

$$Z = \sqrt{R^2 + X^2},$$

and in more complex series circuits this becomes

$$Z = \sqrt{(\Sigma R)^2 + (\Sigma X)^2}.$$

This simply says that the impedance of several circuit elements in series is equal to the square root of the sum of squares of the summation of the resistances and the summation of the reactances.

The phase angle of the impedance is the angle θ whose tangent is X/R. When an inductive reactance is in the circuit, the angle θ is positive and the voltage leads the current. Thus the impedance is sometimes said to be a special kind of multiplier (vector-operator) which multiplies the magnitude of the current to produce the magnitude of the voltage and whose angle is added to that of the current vector to produce the voltage vector. Similarly, when the voltage is divided by the impedance, the magnitude of the current is the quotient of the voltage and impedance, and the direction of the current vector is obtained by subtracting the impedance angle from the angle of the voltage phasor.

There are several formal methods for doing this. For most elementary work it is sufficient to be able to determine the magnitude and phase relationships of the current and voltage without developing the specialized terminology ordinarily used by electrical engineers.

Example. Determine the voltage required to force a current of 20 ma through an inductance coil having a resistance of 50 ohms and

an inductance of 10 mh. The frequency is 2000 cps. What is the angle by which the voltage leads the current?

Solution: (1) Determine the reactance.

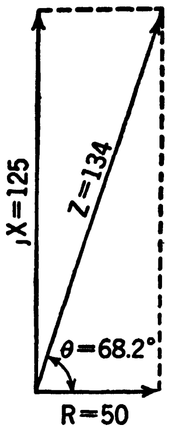

Fig. 7-15. Impedance Diagram.

$$X = 2\pi fL = 2\pi \times 2000 \times \tfrac{10}{1000} = 40\pi = 125 \text{ ohms.}$$

(2) Determine the impedance.*

$$Z = \sqrt{R^2 + X^2} = \sqrt{50^2 + 125^2} = \sqrt{2500 + 15{,}625}$$

$$= \sqrt{18{,}125} = 134 \text{ ohms.}$$

$$\tan \theta = \frac{X}{R} = \frac{125}{50} = 2.5$$

$$\theta = 68.2°.$$

(3) Determine the magnitude of the voltage.

$$E = IZ = 0.020 \times 134 = 2.68 \text{ v.}$$

An a-c voltage of 2.68 is necessary to force 20 ma through the above coil. The voltage will lead the current by a phase angle of 68.2°. (*Ans.*)

Exercise 7-8. What 60-cycle voltage will be required to cause a current flow of 20 amp through an inductance coil of 20 mh, if it has a resistance of 4 ohms? Determine the phase angle.

Exercise 7-9. If a 2000-cycle generator supplies 350 v to a coil having an inductance of 5 mh and a resistance of 25 ohms, what current results? What is the angle by which the current lags the voltage?

The impedance of a circuit of several elements

The sum of the voltages in a series circuit is not necessarily limited to two elements but may be of any number. In circuits of this type the voltages are added instantaneously, just as was demonstrated in Fig. 7-13. All of these voltage drops may be divided into parts due to resistance and parts due to reactance. Since this is possible, the impedance of the entire circuit may be obtained by the vector addition of the impedances of individual parts. This addition may be done graphically, or it may be done, as indicated previously, by adding all of the

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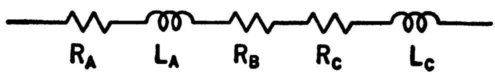
* Many students find that a carefully drawn diagram will give results of engineering accuracy. The use of a diagram drawn to scale is always a desirable check on arithmetical calculations even if it is not used for the original solution. (See Fig. 7-15.)

resistances and all of the reactances and from these obtaining the total or equivalent impedance.

Example. Determine the current taken from a 60-cycle 220-v a-c line when coil A, resistance B, and coil C are connected across it, in series. Coil A has a resistance of 0.3 ohms and an inductance of 2 mh. Resistance B has a magnitude of 1.2 ohms. Coil C has a resistance of 0.7 ohms and an inductance of 5 mh.



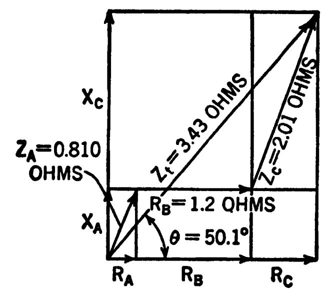


FIG. 7-16. Impedance Diagram for Example.

Solution: (1) Determine the reactance of the coils.
Coil A:

$$X_A = 2\pi fL = 2\pi \times 60 \times 0.002 = 0.24\pi = 0.754 \text{ ohms.}$$

Coil C:

$$X_C = 2\pi fL = 2\pi \times 60 \times 0.005 = 0.60\pi = 1.88 \text{ ohms.}$$

(2) Determine total impedance.

$$X_{\text{total}} = X_A + X_C = 2.63 \text{ ohms.}$$

$$R_{\text{total}} = R_A + R_B + R_C = 0.3 + 1.2 + 0.7 = 2.2 \text{ ohms.}$$

$$Z_{\text{total}} = \sqrt{R^2 + X^2} = \sqrt{2.2^2 + 2.63^2} = \sqrt{4.84 + 6.92}$$

$$= \sqrt{11.76} = 3.43 \text{ ohms.}$$

(3) Determine current and phase angle.

$$I = \frac{E}{Z} = \frac{220}{3.43} = 64.2 \text{ amp.}$$

$$\tan\theta = \frac{X}{R} = \frac{2.63}{2.2} = 1.19.$$

$$\theta = 50.1°.$$

Exercise 7-10. A 10,000-ohm resistance is in series with an inductance coil having a resistance of 2000 ohms and an inductance of 10 mh. This circuit is connected across a 75-v line having a frequency of 150 kc. Determine the current and phase angle.

Exercise 7-11. Two coils and a resistance are connected in series across a 110-v 60-cycle power line. Coil A has a resistance of 3 ohms and an inductance of 15 mh. Coil B has a resistance of 7 ohms and an inductance of 5 mh. The resistance C has a magnitude of 2 ohms. Determine the current and equivalent impedance.

Resistances and inductances in parallel

When a-c circuits are connected in parallel, the same general type of solution is used as was used for resistances in parallel. That is, the current flowing through each part of the parallel circuit is determined and the total current is the sum of the individual parts. The currents will, in general, not be in phase, and therefore the phasors representing them must be added with the proper phase relationship.

The equivalent impedance may be determined by dividing the voltage by the current. The phase angle of the impedance is determined by the angle the total current phasor makes with the voltage. The equivalent resistance is the product of the impedance and the cosine of the angle, while the equivalent reactance is the product of the impedance and the sine of the angle.

Example. It is desired to know the total current flow, the phase angle of the total current, and the equivalent impedance when coil C of the preceding illustrative example is connected in parallel with a

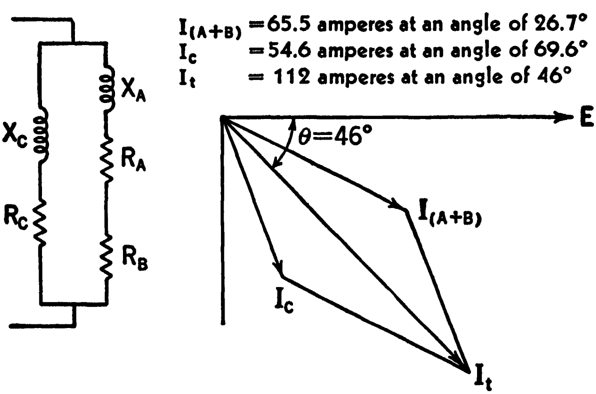


FIG. 7-17. Phasor Diagram for Parallel Circuit of Example.

series circuit composed of coil A and resistance B, as indicated in the circuit diagram of Fig. 7-17. The line voltage is 110 v at 60 cycles.

(1) Determine the current in coil A and the resistance B, using the values obtained in the preceding example.

$$Z_{A+B} = \sqrt{(R_A + R_B)^2 + X_A^2} = \sqrt{1.5^2 + 0.754^2}$$
$$= \sqrt{2.25 + 0.57} = \sqrt{2.82} = 1.68 \text{ ohms.}$$
$$\tan \theta_{A+B} = \frac{X}{R} = \frac{0.754}{1.50} = 0.503.$$
$$\theta_{A+B} = 26.7°.$$
$$I_{A+B} = \frac{E}{Z} = \frac{110}{1.68} = 65.5 \text{ amp,}$$

which lags 26.7° behind the voltage.

(2) Determine the current in coil C.

$$Z_C = \sqrt{R_C^2 + X_C^2} = \sqrt{0.7^2 + 1.88^2}$$
$$= \sqrt{0.49 + 3.54} = \sqrt{4.03} = 2.01.$$
$$\tan \theta_C = \frac{X}{R} = \frac{1.88}{0.7} = 2.68.$$
$$\theta_C = 69.6°.$$
$$I_C = \frac{E}{Z} = \frac{110}{2.01} = 54.6 \text{ amp,}$$

which lags 69.6° behind the voltage.

(3) Determine the total current by adding the individual currents.*

(Graphical method)

$$I_{A+B} = 65.4 \text{ amp at an angle of } 26.7°.$$
$$I_C = 54.6 \text{ amp at an angle of } 69.6°.$$
$$I_t = 112 \text{ amp at an angle of } 46°.$$

(By analytical method)

(4) Begin by determining the portion of currents in phase with the voltage.

$$I_{A+B} \cos \theta_{A+B} = 65.4 \cos 26.7° = 65.5 \times 0.893 = 58.5 \text{ amp.}$$
$$I_C \cos \theta_C = 54.7 \cos 69.6° = 54.7 \times 0.349 = 19.0 \text{ amp.}$$
$$I_{\text{in phase}} = 58.5 + 19.0 = 77.5 \text{ amp.}$$

* This can be done graphically as in Fig. 7-17 or by determining the portion of the current in phase with the voltage and 90° behind the voltage. These parts can then be added to find the portions of total current in phase with and 90° behind the voltage. The total current can then be determined from these by the use of right-triangle analysis.

(5) Next, determine the portion of the currents 90° behind the voltage.

$$I_{A+B} \sin \theta_{A+B} = 65.4 \sin 26.7° = 65.5 \times 0.45 = 29.4 \text{ amp.}$$
$$I_C \sin \theta_C = 54.6 \sin 69.6° = 54.7 \times 0.938 = 51.3 \text{ amp.}$$
$$I_{90° \text{ lag}} = 29.4 + 51.3 = 80.7 \text{ amp.}$$

(6) From the two individual currents, determine the total current flow.

$$I_{\text{total}} = \sqrt{I^2_{\text{in phase}} + I^2_{90° \text{ lag}}}$$
$$= \sqrt{77.5^2 + 80.7^2} = \sqrt{6006 + 6510}$$
$$= \sqrt{12{,}516} = 111.9 \text{ amp.}$$
$$\tan \theta_{\text{total}} = \frac{I_{90° \text{ lag}}}{I_{\text{in phase}}} = \frac{80.7}{77.5} = 1.04.$$
$$\theta_{\text{total}} = 46.2°.$$

(7) Determine the equivalent impedance.

$$Z_{eq} = \frac{E}{I_{\text{total}}} = \frac{110}{111.9}$$
$$= 0.984 \text{ ohms.}$$
$$R_{eq} = Z \cos \theta = 0.984 \cos 46.2° = 0.984 \times 0.692$$
$$= 0.682 \text{ ohms.}$$
$$X_{eq} = Z \sin \theta = 0.984 \times 0.722$$
$$= 0.711 \text{ ohms.}$$

Exercise 7-12. Determine the equivalent impedance of the circuit shown in Fig. 7-18.

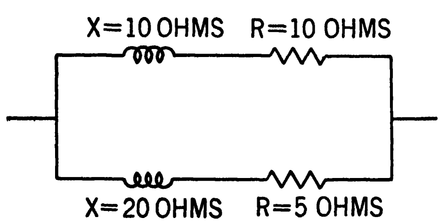


FIG. 7-18. Circuit Diagram for Exercise 7-12.

Exercise 7-13. A resistance of 100 ohms is connected in parallel with a reactance coil having an inductance of 25 mh and a resistance of 20 ohms. What total current will flow when 40 v at 1000 cycles are impressed on the circuit?

Exercise 7-14. Three circuits are connected across a 120,000-cycle 20-v source. Circuit A consists of a resistance of 2000 ohms. Circuit B consists of a coil having an inductance of 5 mh and a resistance of 100 ohms. Circuit C consists of a coil having an inductance of 1.2 mh and a resistance of 50 ohms. Determine the total current flowing and the equivalent impedance.

Power and power factor

It has been shown that the average power absorbed by an inductance coil with negligible resistance is zero. Since most coils do not have negligible resistance, some power is absorbed. The amount of this power is

$$P_{\text{average}} = I^2R.$$

In other words, the only energy that is supplied to an inductance coil and that is not returned to the circuit on the next quarter cycle is that which is converted into heat by the resistance of the winding.

Since IR is the component of the voltage which is in phase with the current, as shown in Fig. 7-13, it can be specified as

$$IR = E \cos \theta, \text{ where } \cos \theta = \frac{R}{Z}.$$

Using this relationship, the average power can be determined from a knowledge of the current, the voltage, and the angle between them. This is shown as follows:

$$P_{\text{average}} = I^2R = (IR)I = EI \cos \theta \text{ watts.}$$

In direct current and in alternating current with resistance only, the average power is equal to EI. The factor by which EI must be multiplied to obtain true average power is called the *power factor* and, as is seen above, this is equal to $\cos \theta$, where θ is the angle between the phasors of the current and the voltage. This angle is often called the *power-factor angle.*

The product of the current and voltage (EI) is called the *apparent power*. Since it does not measure real power, the units are not called watts but *volt-amperes*. Most a-c machinery is built to operate at a specified voltage, and the safe current is limited by the heating effects of the current; therefore the machines are often rated in volt-amperes of capacity.

Power factor may be expressed as a percentage; thus a 50-per cent power factor is actually 0.5. Since power companies can supply more power with the same equipment if the power

factor is high, their rates are sometimes arranged so that energy costs are less with a high power factor than with a low one.

Exercise 7-15. A power line supplies 20 amp to a welding transformer. The voltage is 220 and the wattmeter indicates 3.0 kw. What is the power factor?

Exercise 7-16. A variable reactance has been designed to control the current flow to the lights in a theater. If the resistance of the lights is 10 ohms, and the inductance coil in series with the lights on one of its points has a reactance of 20 ohms and a resistance of 2 ohms, determine the power factor. What is the voltage across the lamps when 120 volts is impressed?

The electric capacitor

When two conducting plates are placed close to, but insulated from, each other, they form what is known as a capacitor. An electric charge can be stored on these plates and will be retained as long as the plates of the capacitor are perfectly insulated from each other. One of the most fundamental of all electrical concepts is the repelling action of charges of the same polarity and the attractive forces between charges of opposite polarity. If, then, the charged plate, having an excess of electrons (or negative charge), is connected to the charged plate having a deficiency of electrons (or positive charge), the repelling and attractive action of the charges will cause electrons to flow through the conductor. This flow of electrons constitutes an electric current that is forced through the conductor against the resistive drop and, thus, causes the development of heat. This energy had been stored in the capacitor as potential energy as a result of the repelling force of the electric charge. The pressure or potential resulting from this repelling force is proportional to the charge. The magnitude of the potential is also dependent upon a proportionality constant that is determined by the size of the capacitor plates, the distance between them, and the insulating material. This constant is called the *capacitance* and is indicated by the symbol C. Expressed mathematically, the relation is

$$Q = EC$$

Q being measured in coulombs, E in volts, and C in farads.

The unit of capacitance is called the *farad* and may be defined as *that capacitance which will permit the storage of one coulomb of charge at a potential of one volt.*

Dielectric constant

The above analysis is based entirely upon the effect of the charges within the conductors and so is valid for conductors in a high vacuum, such as, for instance, the interelectrode capacitance of vacuum tubes. Experimental studies show that the same analysis is also valid for conductors in air. When most liquid or solid insulators are placed between the plates of the capacitor, however, it is found that the capacitance is considerably increased; this increase permits additional energy storage. The mechanism of this storage is indicated in Fig. 7-19. The positive charge on the upper plate attracts the negative electrons in the molecules of the dielectric. These electrons are bound so tightly to the molecule that they cannot flow as do the electrons in metals. The force of attraction exercised by the upper plate and the repulsion exercised by the lower plate combine to produce a strain in the molecules of the dielectric similar to the strain in a spring that has been stretched. The extent to which the effect of the repelling action of the charges in the capacitor plates can be neutralized by the strained condition of the dielectric is dependent upon the physical characteristics of the dielectric. An index of this ability of a dielectric to change the capacity of a capacitor is known as the *dielectric constant.* It may be defined as *the ratio of the capacitance of a capacitor, with the dielectric being considered, to the capacitance of the same capacitor if air or a vacuum were used as the insulating medium.* A list of

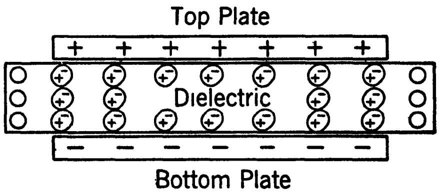


FIG. 7-19. Tension in Dielectric.

TABLE 7-2
DIELECTRIC CONSTANTS OF INSULATING MATERIALS

Material	*Dielectric Constant*	*Material*	*Dielectric Constant*
Air	1	Polystyrene	2–6
Glass	6–9	Bakelite	5–15
Porcelain	5–7	Paper	2–2.6
Steatite	5–6	Paraffin	2–2.5
Mica	6–7	Mineral Oil	2–2.5

the dielectric constants of several common insulators is given in Table 7-2. In many cases varying methods of manufacture or varying quality cause a range of values to be given.

Capacitance of a capacitor

Many of the commercial capacitors are flat-plate or rolled-foil capacitors, and the capacitance of these may be determined from the equation

$$C = 2248 \frac{AK}{d} \times 10^{-16} \text{ fd.}$$

In this equation A is the area (in square inches) of the dielectric under stress, d is the thickness (in inches) of the dielectric, and K is the dielectric constant, as specified in Table 7-2. The farad is such a large unit of capacitance that it is almost never used in practice. The microfarad (μf) and the micromicrofarad ($\mu\mu$f) are the units used most extensively. These units must, however, be converted to farads when substituting in equations, unless the equations are converted so that the smaller units may be used directly. The value of

$$1\mu\text{f} = 10^{-6} \text{ fd,} \qquad \text{or} \qquad 1\mu\mu\text{f} = 10^{-12} \text{ fd.}$$

It follows from the equation for capacitance that capacitors, when connected in parallel, have a capacitance equal to the sum of the capacitances of each individual capacitor. However, when capacitors are connected in series, the equivalent capacity must be obtained by use of the equation

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots.$$

This equation is analogous to the equation for the equivalent resistance of several resistors connected in parallel as developed in Chap. 1.

Exercise 7-17. Determine the capacitance of a paper and metal-foil capacitor where there are 21 sheets of foil connected to one terminal and 20 connected to the other. Each sheet has an effective area 2 in. wide and 10 in. long. The capacitor is made by the 41 sheets of foil, which are separated by 40 double sheets of paper, each double sheet having a thickness of 0.016 in.

Exercise 7-18. A variable air capacitor has 10 rotating plates, each having an external radius of 3 in. The 11 fixed plates have an internal radius of 1.0 in. Both fixed and movable plates are made of material

which is 0.05 in. thick, are spaced on 0.25-in. centers, and cover an arc of 180 deg. Determine the approximate maximum capacitance.

Relation between voltage and current in a capacitor

Since the instantaneous charge on a given capacitor is proportional to the voltage at any instant, the charge will increase and decrease as the voltage increases and decreases. In Fig. 7-20 the voltage has been plotted as a sine wave and the charge as another sine wave of different magnitude but in phase with the voltage. If the charge on the capacitor is continually changing, the conductor by which the connection to the capacitor is made must have a flow of electrons to and from the capacitor. The current in the circuit is, therefore, the rate at which

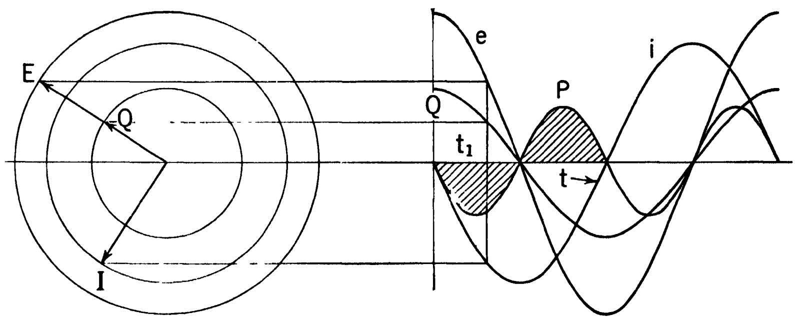


FIG. 7-20. Current, Voltage, and Power in a Capacitive Circuit.

the charge on the capacitor is being increased or decreased. The current is flowing from the line to the capacitor when the voltage is rising and going through the zero value. The current is therefore positive, and, since the voltage is changing most rapidly at this time, the magnitude of the current is a maximum. When a sine wave of current has a positive maximum value 90 deg before the positive maximum of the voltage wave, it is said to lead the voltage by 90 deg.

The same result may be obtained from a slightly different approach. It has already been learned that the rate of change of a sine wave is another sine wave that leads the original wave by 90 deg. Since the current is proportional to the rate of change of the voltage wave, it follows that the steady-state condition for current to a capacitor with a sinusoidal voltage impressed is a sine wave leading the voltage by 90 deg. This confirms the previous reasoning, and the waves are shown in Fig. 7-20. It is seen that this current is opposite in time phase

to the current in an inductance, which lags 90 deg behind the voltage.

The instantaneous power in a capacitor is also plotted in Fig. 7-20 and is found to be a sine wave having zero average power and double the frequency of the voltage wave. The same was true in the case of the inductance coil shown in Fig. 7-11. A comparative study, however, will show that for the quarter-cycle, when the voltage is going from positive maximum to zero, the power input is negative in the capacitor circuit but positive in the circuit containing the inductance coil. In the following quarter-cycle it is positive in the capacitor circuit and negative in the inductive circuit. This is an important characteristic of these circuit elements, because it permits periodic power transfer from one element to the other and is the basis for the oscillations of tuned circuits used so extensively in high-frequency equipment.

Capacitive reactance

In the preceding paragraph it was determined qualitatively that the current from a capacitor was equal to the *rate of change* of the charge. It remains to determine the numerical relationships that involve the frequency. In the discussion on the rate of change of a sine wave it was found that the maximum value for the rate of change is $2\pi f$ times the maximum value of the original sine wave. The maximum rate of change of Q is, therefore, $2\pi f$ times Q_{max}, and this is likewise the maximum value of the current. Stated mathematically, this relation is

$$I_{max} = 2\pi f Q_{max}.$$

It is known that the charge is equal to the product of the voltage and capacitance; thus

$$Q_{max} = CE_{max}.$$

Substituting this value in the above equation

$$I_{max} = (2\pi fC)E_{max}, \qquad \text{and} \qquad I_{eff} = (2\pi fC)E_{eff},$$

where $2\pi fC$, the admittance, is a constant giving the relation between the absolute values of the current and the voltage. Since the reactance of a capacitive circuit is that value by which the current must be multiplied in order to obtain the voltage, the reactance of a capacitor may be specified as $1/(2\pi fC)$. In

mathematical form this statement is

$$E = IX_C = \frac{I}{2\pi fC}.$$

Therefore

$$X_C = \frac{1}{2\pi fC}.$$

Since both the capacitance and frequency are in the denominator, it follows that the impedance is decreased with increase of frequency and is decreased also with an increase in capacitance. This is the opposite of the reactance of an inductance, which increases both with an increase of frequency and of inductance.

Resistance and capacitance in series

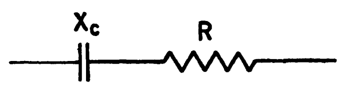
When resistance is connected in series with a capacitor, a situation is obtained similar in many ways to that existing in the case of a resistance in series with an inductance. The voltage across the resistance is in phase with the current, *while the voltage across the capacitor lags* 90° *behind the current.* The total voltage, being the sum of the two component voltages, also lags behind the current. This angle of lag, or the angle by which the current leads the voltage, has a tangent, the value of which is X_C/R.

Example. A capacitor of 1 μf is connected in series with a 1000-ohm resistance across a 500-cycle 12-v line. Determine the current, the equivalent impedance, and the power-factor angle.

Solution: (1) Determine the reactance of the capacitor.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 500 \times 10^{-6}} = \frac{1000}{\pi} = 318.5 \text{ ohms.}$$

(2) Determine the equivalent impedance.



$$Z = \sqrt{R^2 + X_C^2}$$
$$= \sqrt{1000^2 + 318.5^2}$$
$$= \sqrt{1,000,000 + 101,500}$$
$$= \sqrt{1,101,500}$$
$$= 1050 \text{ ohms.}$$

$$\tan\theta = \frac{X_C}{R} = \frac{318.5}{1000} = 0.318.$$

$$\theta = 17.7°.$$

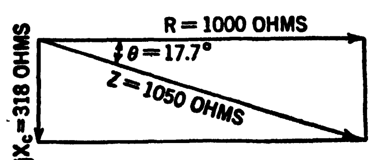


FIG. 7-21. Vector Diagram, Capacitive Circuit.

3. Determine the current.

$$I = \frac{E}{Z} = \frac{12}{1050} = 0.0114 \text{ amp} = 11.4 \text{ ma.}$$

This current will lead the voltage by 17.7°.

Exercise 7-19. What current will be taken from a 200-v source by a 0.001-μf capacitor in series with a 125-ohm resistance if the frequency is 600 kc? Determine the phase angle and draw the vector diagram.

Resistance, inductance, and capacitance in series

When resistance, inductance, and capacitance are connected in series, the total voltage is, as before, the instantaneous or vector sum of the voltages across the individual elements of the circuit. When an analysis is made of these voltages, it is observed that the voltage across the capacitor is directly opposed to the voltage across the inductance. The voltage across the combination may therefore be less than the voltage across either element by itself. The reactance of such a series circuit is the difference between the reactance of the inductance coil and the reactance of the capacitor. Since these two reactances tend to neutralize each other, it has become common practice *to assign one a positive value and the other a negative value. Since the inductive reactance is drawn vertically upward with respect to the resistance as reference, it is considered as positive and the capacitive reactance is considered as negative.* The power-factor angle θ on this basis is considered positive when the voltage leads the current and negative when the voltage lags behind the current. The methods of making the computations are the same as for the resistance and inductance in series, and the addition of impedances may be made graphically or algebraically.

Example. A capacitor of 50 μf, an inductance coil having a resistance of 5 ohms and an inductance of 0.08 h, and a resistance of 6 ohms are connected in series across a 110-v 60-cycle circuit. Determine the equivalent impedance, the current, the power-factor angle, and the voltage across each circuit element.

Solution: (1) Determine the impedance Z.

$$X_{\text{capacitor}} = -\frac{1}{2\pi f C} = -\frac{1}{2\pi \times 60 \times 50 \times 10^{-6}}.$$

$$= -\frac{10^6}{6000\pi} = -53.0 \text{ ohms.}$$

$$X_{coil} = 2\pi fL = 2\pi 60 \times 0.08 = 30.1 \text{ ohms.}$$

$$X_{total} = -53.0 + 30.1 = -22.9 \text{ ohms of capacitive reactance.}$$

$$R_{total} = 5 + 6 = 11 \text{ ohms.}$$

$$Z_t = \sqrt{R^2 + X^2} = \sqrt{11^2 + 22.9^2} = \sqrt{121 + 525}$$

$$= \sqrt{646} = 25.4 \text{ ohms.}$$

$$\tan\theta = -\frac{X}{R} = -\frac{22.9}{11} = -2.08. \qquad \theta = -64.4°.$$

(2) Determine the current.

$$I = \frac{E}{Z} = \frac{110}{25.4} = 4.33 \text{ amp.} \qquad \cos\theta = \text{power factor} = 0.43.$$

(3) Determine the voltages across each circuit element.

$$E_{capacitor} = IX_C = 4.33 \times 53.0 = 229 \text{ v.}$$

$$Z_{coil} = \sqrt{R^2 + X^2} = \sqrt{5^2 + 30.1^2} = \sqrt{25 + 906}$$

$$= \sqrt{931} = 30.5 \text{ ohms.}$$

$$E_{coil} = IZ_{coil} = 4.33 \times 30.5 = 132 \text{ v.}$$

$$E_{resistance} = 4.33 \times 6 = 26 \text{ v.*}$$

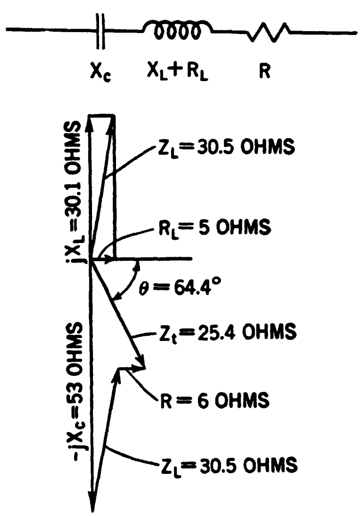


FIG. 7-22. Vector Diagram—Resistance, Inductance, and Capacitance in Series.

* In Fig. 7-22, which shows the graphical summation of impedances, it will be noted that each impedance was determined individually. Many engineers prefer to add the resistances and reactances separately in order to obtain the final result, as was done in the analysis. The vector method permits a more complete understanding of the voltages between different points on the circuit.

Note. It should be noted in the example above that the voltages across the coil and across the capacitor are both greater than the line voltage. This situation is common in high-frequency circuits, and the voltage may be many times the impressed voltage in circuits where the inductive and capacitive reactances are equal and the resistance is small.

Exercise 7-20. A resistance of 20 ohms is connected in series with an inductance of 100 μh and a capacitance of 0.05 μf across a 10-v 100-kc line. Determine the current and equivalent impedance.

Exercise 7-21. An inductance coil having a resistance of 0.3 ohms and an inductance of 1.5 μh is connected in series with a 2-ohm resistor and a 10-μf capacitor across a 5-v 40-kc line. Determine the difference in phase between the voltage across the coil and the voltage across the resistance. What is the difference in phase between the voltage across the coil and the impressed voltage?

Resistance, inductance, and capacitance in parallel

When parallel circuits are encountered which involve capacitive reactance in one circuit and inductive reactance in the other, each circuit is solved by itself and each current is obtained. The total current is then found by the vector addition of the individual currents. This addition may be done graphically or by determining the sum of the parts of the current in phase with the voltage and the algebraic sum of those parts that are 90 degrees out of phase. The equivalent impedance is determined, as in other circuits, by dividing the voltage by the current. The tangent of the power-factor angle is determined from the ratio of out-of-phase current to in-phase current.

Example. If the capacitor and the 6-ohm resistance of the preceding illustrative example are connected in series across the 110-v line, and if the coil is also connected across the line to make a parallel circuit, determine the equivalent impedance, the current, and the power-factor angle.

Solution: (1) Reactances may be taken from the preceding example.

$$Z_C = \sqrt{R^2 + X^2} = \sqrt{6^2 + 53^2} = \sqrt{36 + 2810}$$
$$= \sqrt{2846} = 53.4 \text{ ohms.}$$
$$\tan \theta_C = -\frac{X}{R}$$
$$= -\frac{53}{6} = -8.83.$$

$$\theta_C = -83.5°.$$

$$Z_L = \sqrt{R^2 + X^2} = \sqrt{5^2 + 30.1^2} = \sqrt{25 + 906}$$
$$= \sqrt{931} = 30.5 \text{ ohms.}$$

$$\tan \theta_L = \frac{30.1}{5} = 6.02.$$

$$\theta_L = 80.5°.$$

(2) Determine parts of currents in phase with voltage and 90° out of phase.

$$I_C = \frac{E}{Z_C} = \frac{110}{53.4} = 2.06 \text{ amp.}$$

$$I_L = \frac{E}{Z_L} = \frac{110}{30.5} = 3.61 \text{ amp.}$$

In-phase current:

$$I_{\text{in phase}} = I_C \cos \theta_C + I_L \cos \theta_L = 2.06 \cos(-83.5°) + 3.61 \cos(80.5°)$$
$$= 2.06 \times 0.112 + 3.61 \times 0.165 = 0.231 + 0.596$$
$$= 0.827 \text{ amp.}$$

Out-of-phase or quadrature current:

$$I_{\text{out of phase}} = I_C \sin \theta_C + I_L \sin \theta_L$$
$$= 2.06 \sin(-83.5°) + 3.61 \sin(80.5°)$$
$$= 2.06(-0.993) + 3.61 \times 0.984$$
$$= -2.05 + 3.56$$
$$= 1.51 \text{ amp.}$$

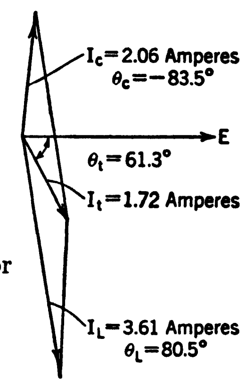


FIG. 7-23.

(3) Determine total current and power-factor angle.

$$I_{\text{total}} = \sqrt{0.827^2 + 1.51^2} = \sqrt{0.685 + 2.28}$$
$$= \sqrt{2.96}$$
$$= 1.72 \text{ amp.}$$

$$\tan \theta_t = \frac{1.51}{0.827} = 1.83.$$

$$\theta_t = 61.3°.$$

(4) Determine the equivalent impedance.

$$Z_t = \frac{E}{I} = \frac{110}{1.72} = 64 \text{ ohms.}$$

Note. In the example above it should be observed that each of the individual branch currents is larger than the total or resultant current. When the resistance is low and the inductive reactance of one branch is equal to the capacitive reactance of the other branch, the branch currents may be many times the resultant current.

Exercise 7-22. If a welding transformer is drawing 25 amp from a 120-v line and has a power factor of 0.7, what will the line current be when a capacitor of 300 μf is connected across the terminals of the transformer? The frequency is 60 cycles.

Exercise 7-23. Determine the current drawn from a 240-v 250-kc source by a parallel circuit containing a capacitor of 300 $\mu\mu$f in one branch and in the other branch a resistance of 250 ohms in series with an inductance of 1.0 mh. Also determine the maximum instantaneous power input (a) to the capacitor, (b) to the inductive circuit, and (c) into the combined circuit.

Exercise 7-24. A single-phase induction motor draws 4 amp from a 120-v 50-cycle line. The wattmeter reading is 300 watts. Determine the capacitance of a capacitor which would bring the power factor up to 90 per cent.

A-c circuits with series and parallel branches

The analysis of a-c circuits has already covered series circuits and parallel circuits involving resistance, inductance, and capacitance. In many practical circuit problems there are parallel branches in series with other circuit elements. The analytical tools for carrying out such a computation have all been covered. The procedure is to determine the equivalent impedance of the parallel branches and add that impedance to the series element to obtain the total impedance. Usually the voltage across the parallel circuit is not given. *In this case, a voltage equal to unity may be assumed so that the equivalent impedance of the parallel circuit may be determined.*

In order to illustrate this method of procedure and to show the completeness with which the series circuit may be analyzed by using the above methods, the following example will be solved.

Example. A series-parallel circuit is shown in Fig. 7-24 with values of circuit constants as follows:

$$C_1 = 0.001\ \mu f$$
$$R_1 = 100 \text{ ohms}$$
$$R_2 = 100 \text{ ohms}$$

$$L_2 = 100\ \mu\text{h}$$
$$L_3 = 80\mu\text{h}$$
$$R_3 = 50 \text{ ohms}$$

Determine the current in each portion of the circuit when 10 v at a frequency of 750 kc is impressed across the terminals. Draw a vector diagram of the currents and voltages and determine the voltage between the points *a* and *b*.

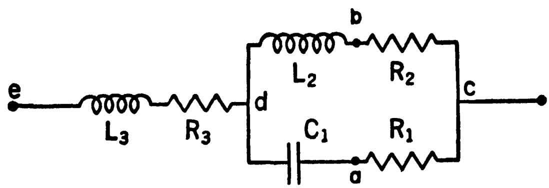


FIG. 7-24.

Solution: (1) Determine the reactance of all circuit elements.

$$X_{C_1} = \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 750{,}000 \times 0.001}$$
$$= -212 \text{ ohms.}$$
$$X_{L_2} = 2\pi fL = 2\pi \times 750{,}000 \times 100 \times 10^{-6}$$
$$= 472 \text{ ohms.}$$
$$X_{L_3} = 2\pi fL = 2\pi \times 750{,}000 \times 80 \times 10^{-6}$$
$$= 377 \text{ ohms.}$$

(2) Assume 1 v across the parallel branches ($E_{CD} = 1$ v) and determine the equivalent impedance of the parallel circuit.

$$Z_1 = \sqrt{R_1^2 + X_1^2} = \sqrt{100^2 + 212^2}$$
$$= 234.$$
$$\theta_1 = \tan^{-1}\frac{X}{R} = \tan^{-1} 2.12 = -64.7°.$$
$$I'_1 = \frac{E}{Z_1} = \frac{1.0}{234} = 0.00426 \text{ amp at an angle of } 64.7°.$$
$$Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{100^2 + 472^2}$$
$$= 482.$$
$$\theta_2 = \tan^{-1}\frac{X}{R} = \tan^{-1} 4.72 = 78°.$$
$$I'_2 = \frac{E}{Z_2} = \frac{1.0}{482} = 0.00207 \text{ amp at an angle of } -78°.$$

The portion of total current that is in phase with the voltage is

$$
\begin{aligned}
I_{eq} \cos \theta_{eq} &= I'_1 \cos \theta_1 + I'_2 \cos \theta_2 \\
&= [(0.00426 \times 0.427) + (0.00207 \times 0.208)] \\
&= 0.00182 + 0.00043 = 0.00225.
\end{aligned}
$$

The portion of the total current that is in quadrature with, or 90° behind the voltage is

$$
\begin{aligned}
I_{eq} \sin \theta_{eq} &= I'_1 \sin \theta_1 + I'_2 \sin \theta_2 \\
&= [0.00426 \times 0.907] + [0.00207 \times (-0.98)] \\
&= 0.00387 - 0.00203 = 0.00184.
\end{aligned}
$$

The equivalent current with 1 v across the parallel circuit is

$$I'_{eq} = \sqrt{0.00225^2 + 0.00184^2} = 0.00291 \text{ amp.}$$

$$\theta_{eq} = \tan^{-1} \frac{0.00184}{0.00225} = \tan^{-1} 0.82 = 39.2°.$$

The impedance is

$$Z_{eq} = \frac{1}{0.00291} = 346 \text{ ohms.}$$

$$
\begin{aligned}
R_{eq} &= Z_{eq} \cos \theta_{eq} = 346 \cos (-39.2°) \\
&= 346 \times 0.773 = 268 \text{ ohms.} \\
X_{eq} &= Z_{eq} \sin \theta_{eq} = 346 \sin (-39.2°) \\
&= 346 \times (-0.630) = -218.
\end{aligned}
$$

(3) Determine the total impedance of the circuit and the total current flow.

$$
\begin{aligned}
R_{total} &= R_{eq} + R_3 = 268 + 50 \\
&= 318 \text{ ohms.} \\
X_{total} &= X_{eq} + X_3 = -218 + 377 \\
&= 159. \\
Z_{total} &= \sqrt{R^2_{total} + X^2_{total}} = \sqrt{318^2 + 159^2} \\
&= 356 \text{ ohms.} \\
\theta_{total} &= \tan^{-1} \frac{X}{R} = \tan^{-1} \frac{159}{318} = 26.6°. \\
I_{total} &= \frac{E}{Z_{total}} = \frac{10}{356} = 0.0281 \text{ amp,}
\end{aligned}
$$

or 28.1 ma at an angle of 26.6° lagging behind the voltage.

(4) The first portion of the phasor diagram may now be drawn with the impressed voltage as the reference vector and the current of 28.1 ma lagging 26.6° behind the voltage.

(5) Determine the voltage across the parallel circuit.

$$\begin{aligned} E_{cd} &= I_{\text{total}} Z_{\text{eq}} \\ &= 0.0281 \times 346 \\ &= 9.7 \text{ v.} \end{aligned}$$

From part (2) of the solution the voltage across the parallel circuit lags behind the total current by an angle of 39.2°. This voltage phasor may now be added to the phasor diagram, using the point c as the origin.

(6) Determine the voltage E_{de}.

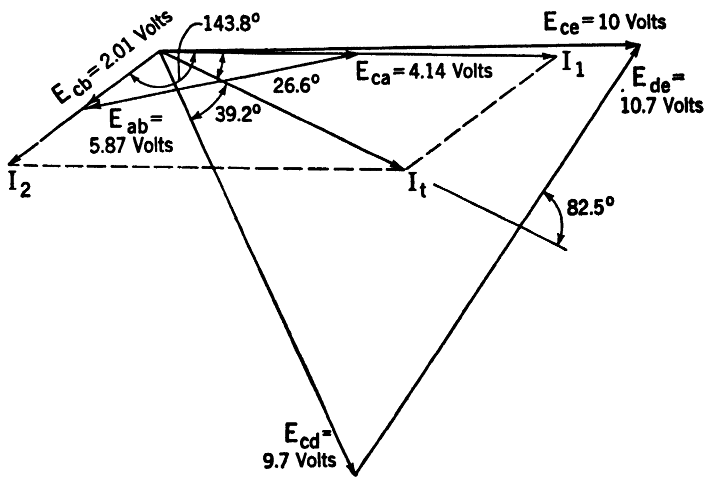


FIG. 7-25. Vector Diagram for the Example of a Series-Parallel Circuit Analysis.

Since the voltage across the terminals is E_{cd} plus E_{de}, the phasor voltage E_{de} may be obtained by completing the triangle in the diagram, E_{de} being drawn as indicated, having a magnitude of 10.7 v, and leading the total current by 82°. This may be computed as follows:

$$\begin{aligned} Z_3 &= \sqrt{R_3^2 + X_3^2} = \sqrt{50^2 + 377^2} = 380 \\ E_{de} &= I_{\text{total}} Z_3 = 0.0281 \times 380 \\ &= 10.7 \text{ v.} \\ \theta_3 &= \tan^{-1} \tfrac{377}{50} = \tan^{-1} 7.5 = 82.5°. \end{aligned}$$

(7) Determine the current in each of the parallel circuits.

The current I_1' was 0.00426, leading the voltage by 64.7° when 1 v was impressed. When 9.7 v are impressed, $I_1 = 9.7 \times 0.00426$

= 0.0414 amp, leading that voltage by 64.7° or lagging behind reference voltage by 26.6° + 39.2° − 64.7° = 1.1°.

Similarly, I_2 = 9.7 × 0.00207 = 0.0201 amp, lagging 78° behind E_{cd}, or 78° + 65.8° = 143.8° behind the impressed voltage E_{ce}.

(8) Determine the voltage between points *a* and *b*.

$$E_{cb} = I_1R_1 = 4.14 \text{ v at an angle of } 1.1° \text{ lagging.}$$

$$E_{ca} = I_2R_2 = 2.01 \text{ v at } 143.8° \text{ lagging.}$$

$$E_{ab} = E_{ac} + E_{cb} = -E_{ca} + E_{cb}.$$

The vertical component of E_{ab} is

$$-4.14 \sin(-1.1°) + 2.01 \sin(-142°) = 0.07 - 1.19 = -1.12 \text{ v.}$$

The horizontal component of E_{ab} is

$$-4.14 \cos(-1.1°) + 2.01 \cos(-143.8°) = -4.14 - 1.62 = -5.76 \text{ v.}$$

The magnitude of E_{ab} is

$$E_{ab} = \sqrt{5.75^2 + 1.12^2} = 5.87 \text{ v.}$$

The angle is in the third quadrant and is equal to

$$\theta = \sin^{-1} - \frac{1.12}{5.87} = -169°.$$

The solution of the above example is long, but the solutions of complex a-c circuits are not short and simple. The foregoing method of determining the current and voltages in a series-parallel circuit will be effective in the solution of most circuits met in engineering practice.

Concepts of resonance

The phenomenon of resonance in an electric circuit is of particular interest in radio, but it is also found in some electrical measuring instruments. It is desirable, therefore, that the student should obtain an elementary concept of the physical relations involved.

The name *resonance* comes from the characteristic of some objects to respond to or echo back sound. Usually these objects respond only to the particular pitches to which they are *tuned.* The responses are of such high frequency and small amplitude that it is impossible to observe them without special instruments. A similar phenomenon may be observed in the pendulum clock. Here the pendulum swings back and forth, shifting the kinetic energy of the pendulum due to the motion at the center of the swing to the stored potential energy of raised posi-

tion at the end of the swing. The slight impulse from an escapement wheel is all that is needed to keep this pendulum swinging back and forth. The amplitude of the vibration or movement is much greater than could be produced by a single impulse from the escapement wheel, but the regular impulse of the escapement is sufficient to replace the losses of energy in the friction and air resistance of the pendulum.

Another manifestation of the same type of energy transfer is observed in electric circuits which have both inductance and capacitance. Both of these circuit elements were found to store energy for one quarter cycle and to return it on the next quarter cycle. Since the capacitor is storing energy when the inductance is returning energy, it is possible for the capacitor and inductor to pass large amounts of energy from one to the other with the outside circuit supplying only the losses. This will operate most satisfactorily when the energy absorbed by the capacitor is just equal to the energy which is being discharged by the inductor and vice versa. In a series circuit the current is the same in both circuit elements. Since the voltage across the inductor increases with frequency, and the voltage across the capacitor decreases with frequency, there must be some frequency at which the two voltages are equal and at which the energy storage of the capacitor is equal to that of the inductor. This is called the *resonant frequency.* The voltage across the inductor as well as that across the capacitor may be several times the voltage of the line.

When the coil and capacitor are connected in parallel, the voltage across each will be the same so that the currents must balance for the resonant condition. If the ratio of reactance to resistance is high, as it is in most radio circuits, the resonant frequency will be approximately the same for both series and parallel connections.

Series resonant circuits

If reference is made to the impedance diagram of Fig. 7-22, it will be seen that the equivalent impedance is considerably less than the impedance of either the capacitor or the coil by itself. Furthermore, if a variable capacitor were used, and if the capacitance of the variable capacitor were increased, the capacitive reactance would decrease. This might be continued until the capacitive reactance would be just equal to the inductive reactance. The circuit impedance would then be the

resistance only, which in this case is 11 ohms. Under these conditions the circuit would be in resonance.

A series circuit is said to be in resonance when the inductive reactance in the circuit just neutralizes the capacitive reactance so that the equivalent impedance is due entirely to resistance. This condition may be obtained in practice by varying the capacitance of the capacitor, as was just indicated, or by varying the inductance of the coil, or by varying the frequency. In any of the conditions above indicated the resistance of the circuit may usually be assumed to remain constant and the

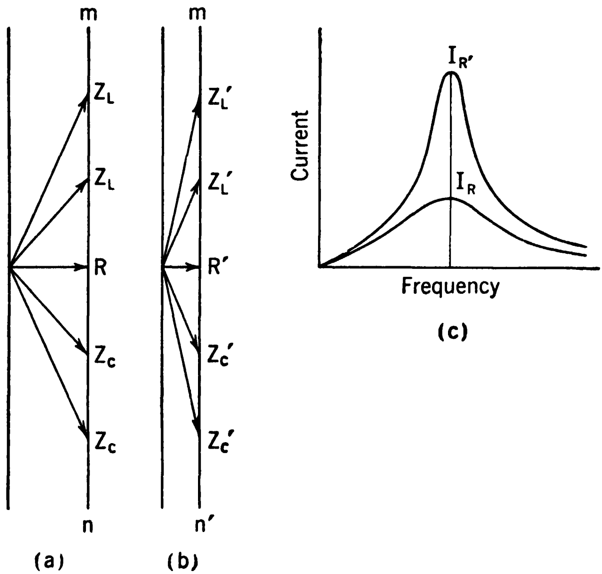


FIG. 7-26. Series Resonance—Variable Frequency.

circuit impedance would, therefore, always have a constant resistance component. This would mean that regardless of how the reactances were shifted the circuit impedance would always fall somewhere on the line *mn* in the diagram of Fig. 7-26(a). If the inductive reactance were the larger, then the reactance would be located above the axis, as indicated by one of the impedances labeled Z_L. If the capacitive reactance were the larger, the impedance would be as indicated by one of the Z_C vectors. If they were equal, the impedance would become R. The manner in which the current in this circuit varies with frequency (if the applied voltage is constant) is shown in Fig. 7-26(c). The current rises to a maximum at the resonant fre-

quency, as shown by I_r on the curve, but it does not rise very high nor is the peak very sharp. In Fig. 7-26(b) the same circuit is shown with considerably less resistance.

It is observed that the current will rise to much greater magnitudes as resonance is reached and also that a slight variation in reactance, due to change of frequency, will make a much greater proportional change in impedance, so that the curve is much steeper when the resistance is small.

Since in radio circuits it is necessary to select the one desirable frequency out of all of the many radio signals that are energizing the antenna, it is necessary to have circuits that will pass one frequency and keep the response to the others small. For this reason it is desirable to have circuits with a large ratio of reactance to resistance.

CHAPTER 8

Polyphase Alternating-Current Circuits

Three-phase concepts

Although single-phase a-c circuits are important in many measuring devices, heating and welding applications, electric illumination, and other similar uses, the majority of industrial power systems are three-phase.

Such a system of voltages may be produced by a single a-c generator with coils spaced at 120 deg, as shown in Fig. 8-1.

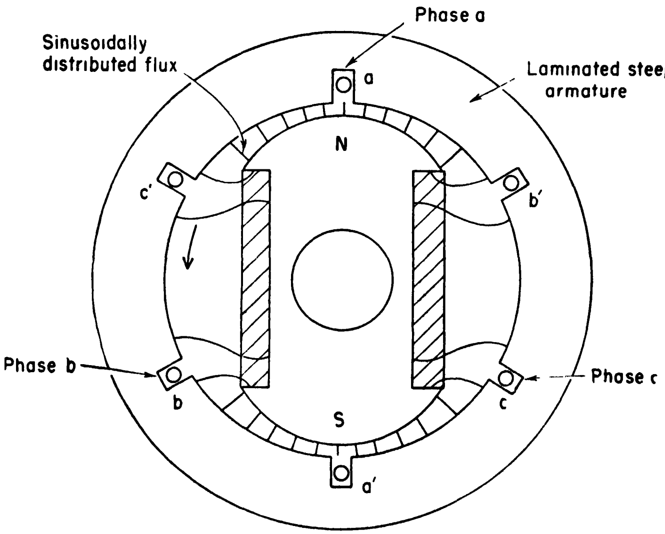
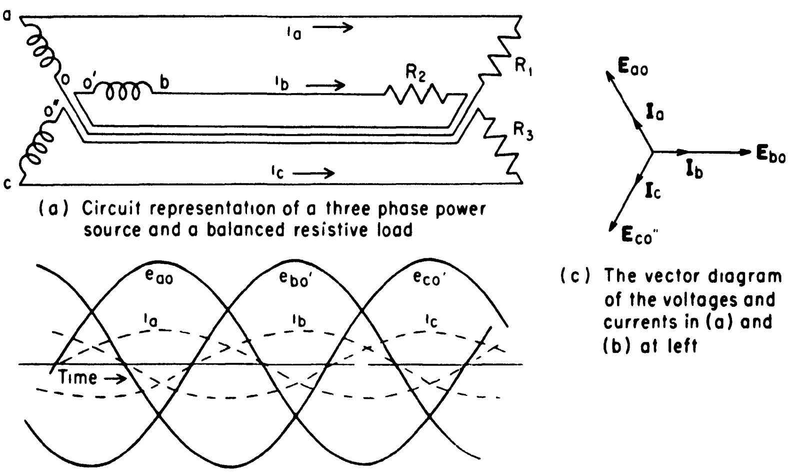


FIG. 8-1. An Elementary, Two-Pole, Three-Phase, A-C Generator. Coils are spaced at 120 deg.

In this elementary generator a rotating electromagnet is so designed as to give a sinusoidally varying magnetic flux around the armature, as shown by the spacing of the flux lines. As the coils *aa'*, *bb'*, and *cc'* are successively cut by the air-gap flux, the polyphase voltages shown in Fig. 8-2 are generated.

Each coil or phase may be considered as a single-phase generator, and they may be connected as shown in Fig. 8-2(a). Each

phase is displaced from the other two by 120 deg in time phase. In a 60-cycle system, the phases come to a maximum in a regular sequence of $\frac{1}{180}$ of a second apart, as is indicated in Fig. 8-2(b). The positive voltage indication of e_{ao} indicates that the point a is more positive in potential than point o. When this voltage becomes negative on the diagram, it indicates that the point a of the circuit is negative with respect to o. When a is



(a) Circuit representation of a three phase power source and a balanced resistive load

(b) Instantaneous voltages and currents in the above circuits

(c) The vector diagram of the voltages and currents in (a) and (b) at left

FIG. 8-2. Three-Phase Power Source with Independent Circuits to Balanced Independent Loads.

positive with respect to o, the current will flow in the direction of the arrow marked i_a, so that i_a on the diagram is positive when e_{ao} is positive. In order to simplify the analysis, a resistor R_1 is assumed connected across the conductors a and o; so the current in the circuit will be in phase with the voltage.

The voltage source between o' and b gives a voltage $e_{bo'}$ which is of the same magnitude as e_{ao} but reaches its maximum value $\frac{1}{180}$ sec or 120 degrees later in time phase than e_{ao}. The circuit is connected to R_2, which is a resistor of the same value as R_1. A current i_b will result, which is of the same magnitude as i_a, but which is in phase with $e_{bo'}$.

Similarly, there is a voltage source between o'' and c to give a voltage $e_{co''}$ that is of the same magnitude as e_{ao} but reaches its maximum $\frac{2}{180}$ sec or 240 deg after e_{ao}. The resistor R_3 connected across the circuit will have a current i_c equal in magnitude to i_a, but which is in phase with $e_{co''}$.

The voltage e_{ao} will then follow $e_{co''}$ by $\frac{1}{180}$ sec or 120 deg, and so the sequence will be continuous, each phase following the other by 120 deg. The currents will similarly follow each other by 120 deg if the impedance of the load is the same in each phase.

The phasor diagram of the voltages and currents are shown in Fig. 8-2(c). The instantaneous power in each phase is pulsating, as demonstrated in Chap. 7. The average power will

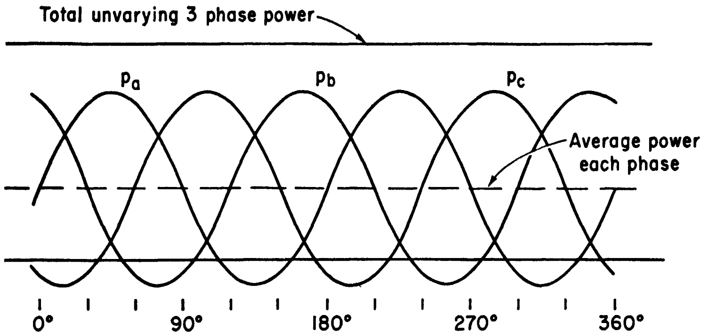


FIG. 8-3. Three Balanced Single-Phase Loads on Three Phases Produce Individual Instantaneous Power That Varies, But a Total Power That Is Constant.

depend not only upon the current and voltage but also upon the power factor. When the instantaneous power of three phases having balanced currents and voltages* are plotted, as in Fig. 8-3, it is found that the instantaneous sum of the power in all three phases is constant. This constant value is equal to three times the average power of each phase. The sequence of phases, therefore, provides a continuous flow of power from the generator to the load. This is important because three-phase windings make possible the manufacture of better and cheaper motors and generators. The construction and theory of these machines are covered in subsequent chapters. The present chapter is concerned with the relation of these voltages and currents combined in various ways.

Three-phase four-wire circuits

If the wires marked o, o', and o'' in Fig. 8-2 are connected together at both ends, that is, at the power source and at the load, the voltage relationships of the individual phases will not

* Currents and voltages in a three-phase system are said to be balanced when they are of the same magnitude in each phase and are displaced 120 deg in time.

be changed. This then results in a three-phase four-wire system, as is shown in Fig. 8-4. The currents in the phase wires a, b, and c of Fig. 8-2 will not be changed. The current in the return wire will, however, become the vector sum of the currents in the phase wires. In the case of the balanced circuit of Fig. 8-2 both the vector and instantaneous sums of these currents are zero; hence no current will flow in the neutral wire.

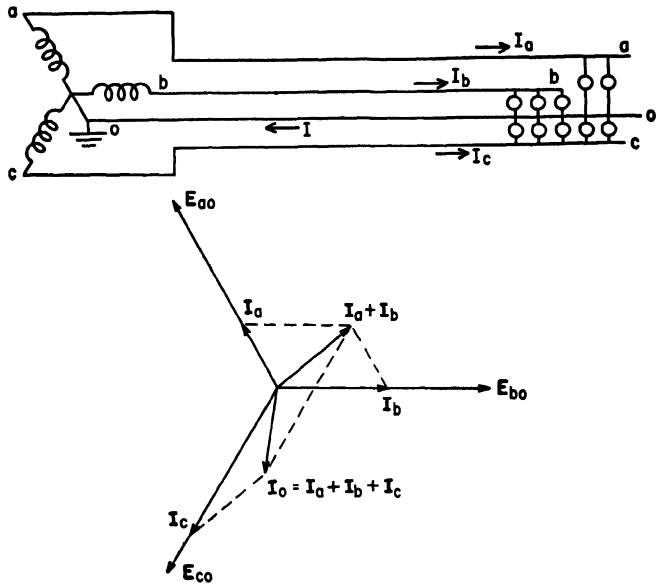


FIG. 8-4. Three-Phase, Four-Wire Circuit with an Unbalanced Lighting Load.

If the loads on the individual phases are not balanced, as shown in Fig. 8-4, where two lights are connected across phase a, three across phase b, and five across phase c, then the current in the neutral wire is the vector sum of the phase currents. This is shown in the phasor diagram of Fig. 8-4. In spite of the badly unbalanced loads, the current in the neutral is quite small, so that it is usual to use a neutral wire that is no larger than the phase wires. This permits a considerable saving in copper as well as in power loss in the neutral wire.

This system of power supply is used extensively in industrial areas because it is possible to supply both lighting and motor loads from the same system. In this arrangement the voltage of each phase is 120, and it is possible to supply the lights from any phase wire to the neutral. A balanced load is

not required but is desirable since it reduces power loss in the neutral and gives better system voltage. To calculate the voltage drop in any phase, it is necessary to add the drop in the phase wire vectorially to that in the neutral. To get the terminal voltage, this drop must then be subtracted vectorially from the source voltage.

Exercise 8-1. In a three-phase four-wire line that is 200 ft long, all wires are #8 copper. (a) If phase *a* supplies a load of 35 amp in lights by itself, what will be the voltage at the lights if the supply voltage is 120 v per phase? (b) What will be the voltage at the lights when each phase supplies a load of 35 amp in lights?

When a three-phase four-wire power system is used to supply a motor as shown in Fig. 8-5, the motor winding is connected to the phase wires only. Since the currents are balanced their sum is always zero, and there is no need of the neutral wire.

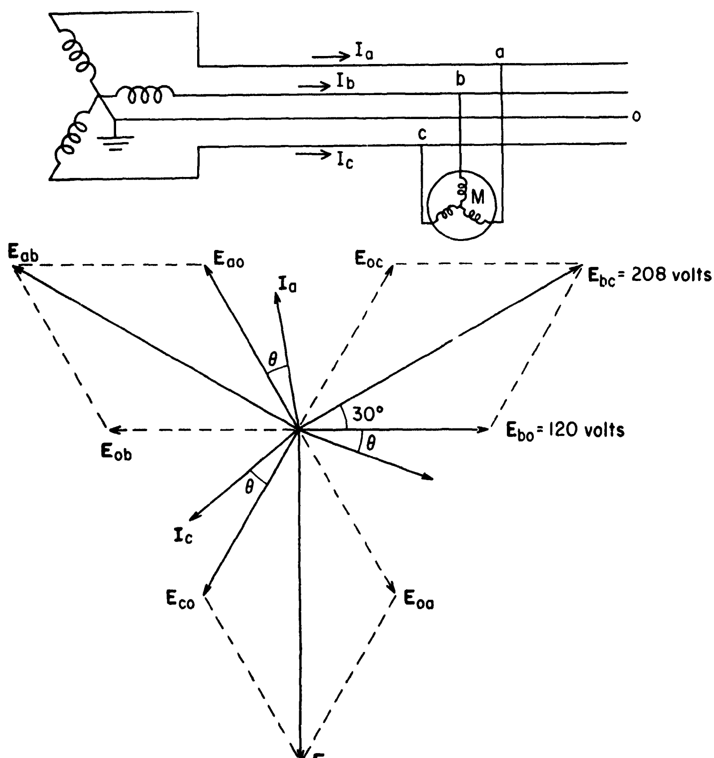


FIG. 8-5. A Three-Phase, Four-Wire Power System Supplying a Three-Phase Motor.

The motor windings have inductance; so these currents will lag behind the phase voltages producing them by some power-factor angle, such as θ.

The voltages across the terminals of a Y-connected motor will be greater than the 120 v of each phase. In the phasor diagram it is shown that the voltage from a to b is the sum of $\boldsymbol{E}_{ao}$ and $\boldsymbol{E}_{ob}$. From the geometry of the diagram the base angles of the isosceles triangle, having $\boldsymbol{E}_{bc}$ as a base and $\boldsymbol{E}_{bo}$ as one of the sides, are observed to be 30 deg. From this it follows that

$$\begin{aligned} E_{bc} &= \sqrt{3}\, E_{bo} \\ &= \sqrt{3} \times 120 \text{ v} \\ &= 208 \text{ v}. \end{aligned}$$

The three wires a, b, and c may therefore be treated as a three-phase source having a line-to-line voltage of 208. The line currents will lag behind the line-to-line voltages by an angle equal to the power-factor angle plus 30 deg. This relationship between the phasor line current and phasor line voltage often causes confusion for beginning students, and a clear understanding of the development of this diagram is highly recommended.

When both lighting and polyphase motors are supplied from the same circuit, the current in the neutral wire is the same as with the lighting alone. In order to measure the power in this system, it is necessary to measure the power supplied by each phase. This means that three wattmeters must be used, and they will be connected with the current coil in the line wire and with the potential coil from line to neutral.

Three-phase three-wire circuits

Balanced load. One form of three-phase three-wire circuit is obtained by the type of connection shown in Fig. 8-5. It is noted that no connection is made to the neutral wire, and so it may be omitted. The connection forms a Y; so it is called a Y-, or wye-, connected circuit. In most commercial circuits using the Y-connected source, the voltage is quite large. The system voltage is normally specified on the basis of line-to-line (ordinarily referred to as line) voltage. The phase voltage or voltage to ground will then be the line voltage divided by $\sqrt{3}$. Thus, for a 60,000-volt line the phase voltage or the voltage to ground is but $60{,}000/\sqrt{3} = 34{,}600$. It is necessary to insulate only for the line-to-ground voltage in most cases, and so a very

considerable saving in insulator cost is obtained with this system.

The power in this system is the sum of the powers in the phases and, since they are equal, it may be written

$$P = 3E_{bo}I_b \cos \theta.$$

Since it is usual to define power in terms of line voltage and since $E_{bc} = \sqrt{3}\, E_{bo}$, then

$$P = \sqrt{3}\,\sqrt{3}\, E_{bo}I_b \cos \theta$$
$$= \sqrt{3}\, E_{bc}I_b \cos \theta.$$

Omitting the subscripts since both current and voltage are line values,

$$P = \sqrt{3}\, EI \cos \theta.$$

An alternate method of connecting the phase voltages is in the form of a ring or mesh. Since the diagram of this type of circuit is similar to the Greek letter Δ, such circuits are said to be delta-connected. It is noted in Fig. 8-6 that the vector sum of the voltages around the circuit is zero; therefore no current will flow in the windings of the power source when the load is

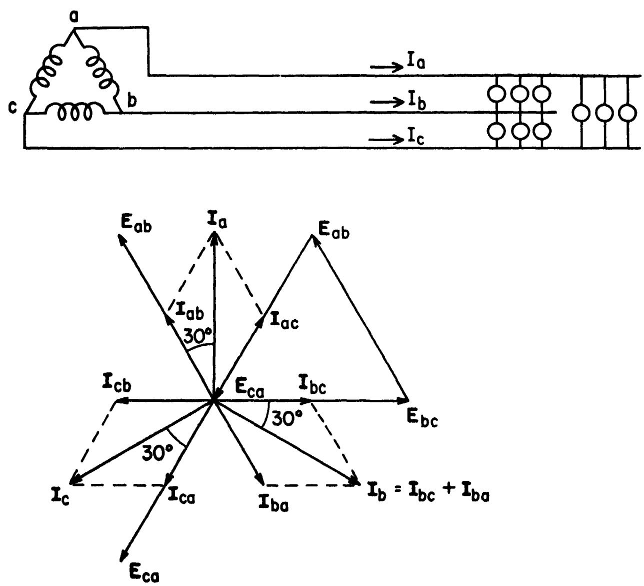


FIG. 8-6. A Three-Phase, Three-Wire, Delta-Connected System of Power Supply.

disconnected. Also the line voltage is equal to the phase voltage.

When a load is connected to this system, the currents will be in phase with the voltages if the load is resistive. With the lighting load shown in Fig. 8-6 currents are in phase with the voltage. The line currents, however, will be the vector sum of the two individual phase currents connected to the line as shown in the phasor diagram. For instance, the current in line b is the sum of the currents flowing from b to c and from b to a. In the diagram this is shown as I_{bc} plus I_{ba} (the current I_{ba} is the reverse or negative value of I_{ab}), which gives a value I_b. This current lags 30 deg behind E_{bc}, the line voltage. In the Y-connected circuit of Fig. 8-5 it was observed that the line current lagged the line voltage by an angle of 30 deg plus the power-factor angle. This conforms to the conclusion with the Δ circuit, since in Fig. 8-6 a unity-power-factor load is assumed.

Power in the Δ circuit is the sum of the powers in the phases.

$$P = 3E_{bc}I_{bc}.$$

Since a unity-power-factor load has been assumed in this diagram, in order to make the equation applicable to balanced loads at other power factors, it will be necessary to insert the power factor; thus

$$P = 3E_{bc}I_{bc} \cos \theta.$$

From the diagram $I_b = \sqrt{3}\, I_{bc}$

so

$$P = \sqrt{3}\, E_{bc}I_b \cos \theta$$

$$= \sqrt{3}\, EI \cos \theta.$$

This is the same as for the Y-connected circuit. It may be assumed, therefore, that the three-phase power in a balanced three-phase three-wire circuit is

$$P = \sqrt{3}\, EI \cos \theta$$

where E is the line-to-line voltage, I is the line current, and θ is the true or phase power factor.

Exercise 8-2. Three loads consisting of a 100-ohm resistance and a 173.2-ohm inductive reactance are connected in Y and fed from a balanced 2300-v line. Determine (a) the line current, (b) the total power taken by the load, and (c) the total KVA.

Exercise 8-3. How much current will be drawn from a 230-v three-phase line by a 25-hp motor at full load if it has an efficiency of 0.85 and operates at 0.87 power factor?

Exercise 8-4. What size wire is needed to feed a 10-kva three-phase oven at 230 v? What voltage drop can be expected if the power run is 350 ft? What effect would this voltage drop have upon the energy supplied to the oven?

Three-phase three-wire unbalanced loads

In a three-phase circuit it is possible to connect a single-phase load across any pair of the three wires, provided the voltage of the line corresponds to the rated voltage of the load device. The line currents will then be the vector sum (or difference) of the two phases being supplied by the line wire. The numerical solution of unbalanced polyphase circuits is beyond the scope of this book.

Polyphase circuits other than three-phase. Although three-phase circuits are by far the most important of the polyphase arrangements, there are other combinations that are sometimes used. For instance, one large city uses a distribution system where four phases are spaced at 90 deg in time phase with a common neutral wire. Since the two alternate phases are in opposition, it is possible to obtain both from the same transformer, and so such a system is sometimes called a two-phase system.

Another instance of multiple phases is in the supply to large industrial rectifiers where the supply is divided into six phases spaced at 60 deg in time phase.* It is even possible to obtain a greater multiplicity of phases, but these will not be treated at length in this text.

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* See the discussion on rectifiers in Chap. 14.

CHAPTER 9

Alternating-Current Measurements

Measurement of current and voltage

Many of the fundamentals of electrical measurement learned in connection with direct currents apply to alternating currents as well. On the other hand, changes must be made for many of the measuring instruments and for the interpretation of their readings. The first difficulty is that a-c current will produce zero average torque on the permanent-magnet moving-coil type of instrument, which is used so extensively for d-c measurements. The second is that power measurements must be made with a wattmeter, as the product of current and voltage is no longer a measure of power because of power factor. A third is that polyphase circuits introduce specialized problems of measurement. There are other problems also, but these are the most important and most common.

As in direct current the measurement of alternating current and voltage is fundamentally a measurement of current. A

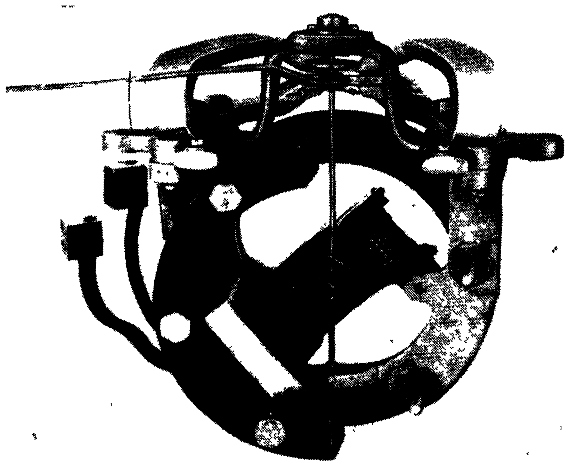


FIG. 9-1. Inclined-Coil Attraction Type of Meter. (Courtesy of General Electric Co.)

dynamometer type of meter will operate equally well on direct or alternating current because the magnetic field reverses at the same time as the current in the moving coil. Thus a voltmeter of this type has the moving coil connected in series with the field coil and the resultant torque is proportional to the square of the current flow and to the square of the voltage.

When this type of instrument is used for current measurement, it is often necessary to shunt most of the current around the moving coil. This shunt must have an impedance that has the same ratio of inductance to resistance as the moving coil, so that the current in the moving coil will be in phase with that of the field coil. Dynamometer-type instruments are excellent for power frequencies, but are expensive; hence they are not usually used, except for standard meters and other high-accuracy applications.

A more common type of instrument used for current and voltage measurement is known as the iron-vane type. Although it is made in many different forms, they all have a common characteristic in that the movable element is a soft-iron vane attached to a shaft and a pointer which are mounted in jeweled bearings. The restoring torque is provided by a spring. The current being measured flows through a single fixed coil that produces a magnetic field of a magnitude proportional to the magnitude of the current. One form of this meter has one or two iron vanes that are inclined on the shaft. The coil is inclined in such a way that, as the magnetic field becomes stronger, the shaft rotates to bring the vanes more nearly in line with the coil axis. This is shown in Fig. 9-1, and the meter is known as the inclined-coil attraction type of meter.

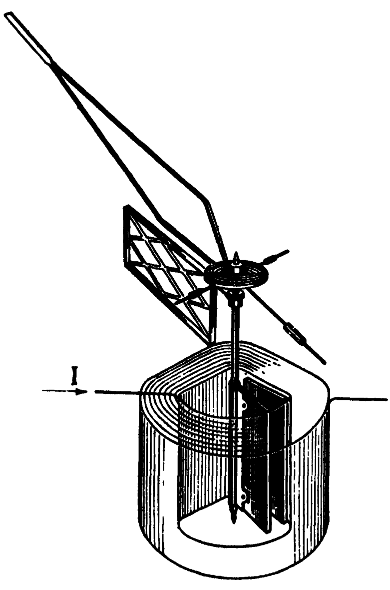


FIG. 9-2. Iron-Vane Type of Meter. (Courtesy of Weston Electrical Instrument Corp.)

A second form uses a fixed and a movable soft-iron plate, as shown in Fig. 9-2. This construction produces corresponding

north poles on one end and south poles at the other end when the current flows in the coil. The polarity of both plates is the same, and so they repel each other. The rapid reversal of the polarity occurs simultaneously in both plates, and so the repelling action always produces torque in the same direction.

When such meters are used as voltmeters, the coil is composed of a large number of turns of fine wire. When they are used as ammeters, a smaller number of turns of larger wire are used. The torque is approximately proportional to the square of the current; therefore, the scale is usually compressed at the lower values. There are special designs of iron-vane meters which permit adjustment of scale distribution so that an almost uniform scale is obtained.

Although the iron-vane type of meter is very satisfactory for usual power frequencies, it is not suitable for higher frequencies. For measurements above power frequencies, therefore, either rectifier-type meters or thermocouple-type meters are used.

In the case of the rectifier-type instrument a standard permanent-magnet moving-coil type of meter is used with a full-wave rectifier. The circuit arrangement of such meters for both current and voltage measurement is shown in Fig. 9-3. The rectifier unit is shown at the left of the diagram. Current ratings of such meters are usually limited to a few milliamperes. The energy loss in the rectifier type of meter is only a fraction of that in the iron-vane meter, and so it is used occasionally in preference to that type of meter, even at a lower frequency. Although the scale is made to read rms volts, it is subject to errors when nonsinusoidal waves are used, since the torque is proportional to the average value of rectified current. Rectifier meters, using copper oxide or selenium rectifiers, are not usually satisfactory for frequencies above the audio range. Rectifier-type meters using crystals as the rectifier elements may be used at very high radio frequencies.

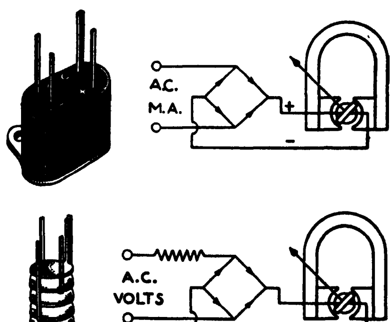


FIG. 9-3. Rectifier Type of Meter. (Courtesy of Weston Electrical Instrument Corp.)

In the thermocouple meter the current flowing through a resistor is used to heat a small thermocouple that, in turn, forces

a small d-c current through a sensitive permanent-magnet moving-coil type of instrument. Figure 9-4 shows a diagrammatic sketch of such an instrument. The current flowing through the resistor between A and B heats up the wire and the hot junction of the thermocouple at E. The cold junctions are at C and D. Since the meter depends only upon the heating effect, it is particularly well adapted to high frequencies. The current ranges are not limited as in the case of the rectifier instruments. The rectifier and thermocouple meters are used most extensively for communication and radio measurements.

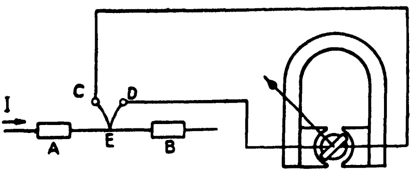


FIG. 9-4. Thermocouple Type of Meter. (Courtesy of Weston Electrical Instrument Corp.)

Measurement of single-phase power

The power flow into a circuit is dependent upon the instantaneous product of the current and voltage. This is shown in Fig. 9-5 for unity, 0.866 lagging, and 0.50 lagging power factor.

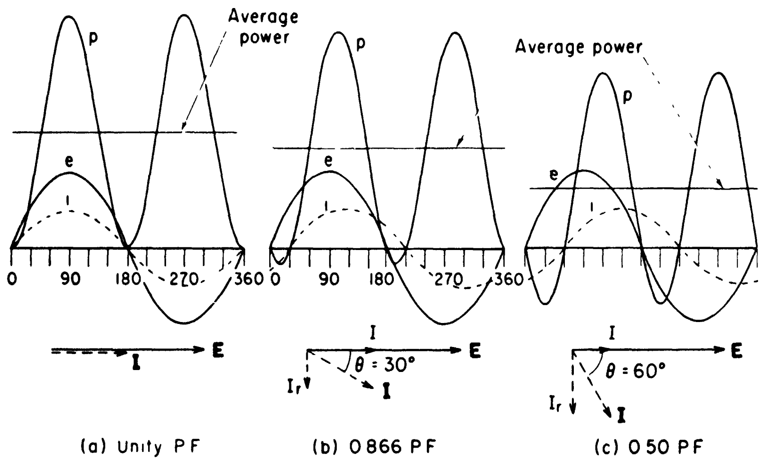


FIG. 9-5. Instantaneous Values of Current, Voltage, and Power at Different Power Factors.

The torque on a dynamometer meter, in which the current coil produces the magnetic field and the current in the movable coil is proportional to the voltage, is directly proportional to the product of the current and voltage. Such a meter therefore has

an instantaneous torque proportional to the instantaneous power. Since the inertia of the element is relatively large, the deflection of the element will be proportional to the average torque and average power.

To measure the power in a single-phase three-wire system, it is necessary to use two wattmeters, one for each side of the line. It is possible to mount both meter elements on the same shaft, in which case the total torque is the sum of the average torques of the individual elements.

Measurement of power in three-phase three-wire systems

Power in a three-phase three-wire system can also be accurately determined, regardless of wave form, power factor, or degree of unbalance, by the use of only two wattmeters; hence it is customary to measure it in this way. The connections for both Y- and Δ-connected loads are shown in Fig. 9-6.

In order to prove that the reading of W_1 plus W_2 will give the true power input to the load, an analysis of the instantaneous

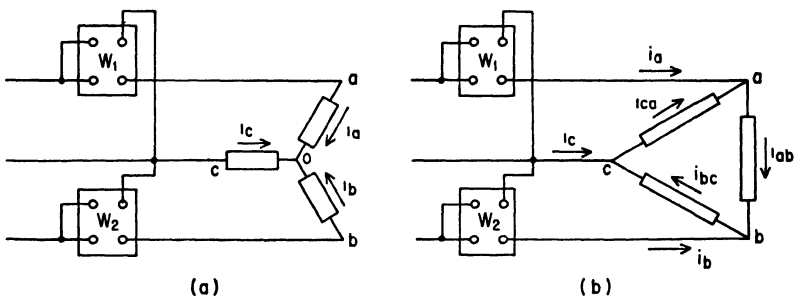


Fig. 9-6. Connections for Measuring Power in a Three-Phase, Three-Wire System by Use of Two Wattmeters.

torques on the wattmeters will be made, placing no restrictions whatever as to the character or magnitude of the load impedances. For instance, in Fig. 9-6(a), to obtain a badly unbalanced load, the impedance between a and o may be a low value of resistance, the impedance between b and o may be a capacitor, and an inductance coil may be connected between c and o. Similarly, no restrictions are placed on the character of impedances or the extent of unbalance in the Δ-connected load.

In the case of the Y-connected circuit of Fig. 9-6(a), i_a, i_b, and i_c are the instantaneous currents flowing in the line and in the individual phase impedances. The instantaneous voltages

across the phases are e_{ao}, e_{bo}, and e_{co}. The total instantaneous power is therefore

$$P = e_{ao}i_a + e_{bo}i_b + e_{co}i_c.$$

The voltage applied to the potential coil of W_1 is e_{ac} and to W_2 is e_{bc}. The instantaneous torques on the wattmeters are equal to the product of the instantaneous currents and voltages, therefore

$$W_1 = e_{ac}i_a, \qquad \text{and} \qquad W_2 = e_{bc}i_b.$$

From the diagram it is seen that

$$e_{ac} = e_{ao} + e_{oc} = e_{ao} - e_{co}$$
$$e_{bc} = e_{bo} + e_{oc} = e_{bo} - e_{co}.$$

Also it may be seen that

$$i_a + i_b + i_c = 0,$$

from which

$$i_c = -(i_a + i_b).$$

It follows that

$$\begin{aligned} W_1 + W_2 &= e_{ac}i_a + e_{bc}i_b \\ &= e_{ao}i_a + e_{co}i_a + e_{bo}i_b - e_{co}i_b \\ &= e_{ao}i_a + e_{bo}i_b - e_{co}(i_a + i_b) \\ &= e_{ao}i_a + e_{bo}i_b + e_{co}i_c \\ &= P. \end{aligned}$$

Since each wattmeter will average the instantaneous power, then the algebraic sum of the readings of W_1 and W_2 will give the true average power, regardless of the power factor or degree of unbalance.*

A similar development using the Δ-connected load of Fig. 9-6(b) demonstrates the correctness with such a load. The instantaneous power is

$$P = e_{ab}i_{ab} + e_{bc}i_{bc} + e_{ca}i_{ca}.$$

The wattmeters will read

$$W_1 = e_{ac}i_a, \qquad \text{and} \qquad W_2 = e_{bc}i_b.$$

From the diagram

$$\begin{aligned} i_a &= i_{ab} - i_{ca} \qquad \text{and} \qquad i_b = i_{bc} - i_{ab} \\ &= i_{ab} + i_{ac}. \end{aligned}$$

* As will be described later, this is an algebraic sum since with balanced loads, and power factors below 0.5, one of the wattmeters will give a negative indication.

Then

$$\begin{aligned} W_1 + W_2 &= e_{ac}i_a + e_{bc}i_b \\ &= e_{ac}(i_{ab} + i_{ac}) + e_{bc}(i_{bc} - i_{ab}) \\ &= e_{ac}i_{ac} + e_{bc}i_{bc} + i_{ab}(e_{ac} - e_{bc}). \end{aligned}$$

From the diagram it is seen that

$$e_{ac} - e_{bc} = e_{ac} + e_{cb} = e_{ab}.$$

Therefore,

$$W_1 + W_2 = e_{ac}i_{ac} + e_{bc}i_{bc} + e_{ab}i_{ab} = P.$$

The two wattmeters W_1 and W_2 will therefore give the instantaneous power of the total circuit, and since they will each average these instantaneous values over a cycle, the resultant readings of the two meters will give the true average power supplied to the load.

Two wattmeters on balanced loads. Although the two wattmeters will correctly measure the power on unbalanced currents and voltages in a three-phase three-wire system, the majority of power-measurement problems deal with balanced loads, such as are encountered in polyphase motors. Since most students meet these problems in the laboratory, it is desirable that they be discussed briefly.

In the first place wattmeters are marked on the terminals to give an indication as to which potential terminal should be connected to the wire that passes through the current coil of the wattmeter. This marking is usually in the form of a $\pm$ mark on the potential terminal that is connected to the coil side of the potential circuit.

If this marked terminal is connected to the current coil of the wattmeter, as shown in Fig. 9-6, and if the source of power is connected to the same side of the wattmeter for both W_1 and W_2, then the meters are correctly connected. If both read downscale, then the direction of current flow through the wattmeters should be reversed.

Difficulty is often experienced in the laboratory because (after connecting the wattmeters properly on an induction motor test) one of the meters will be positive and the other negative. This usually occurs when the motor has no load.*

* It is possible to check the correctness of the sign of the low-reading wattmeter by moving the potential connection of this wattmeter from the line containing no wattmeter to the line of the high-reading wattmeter. If this change causes the low-reading wattmeter to reverse, the meter was properly connected and its reading must be subtracted from that of the high-reading wattmeter.

It is wise, therefore, to apply a reasonable load to the motor to see if under load both meters do not give positive readings before changing connections.

The phenomenon of a reversed wattmeter reading at low power factor may be explained by reference to Fig. 9-7, where it is shown that the current in the line lags behind the line voltage by an angle of 30 deg plus the power-factor angle. The voltage vector E_{ac} is shown in a dashed line, and the current and voltage vectors of W_1 and of W_2 are tied together with an

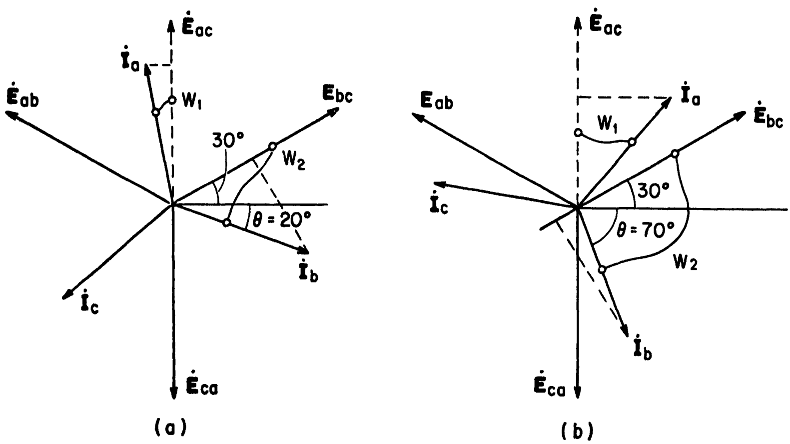


FIG. 9-7. Phasor Diagrams of Balanced, Three-Phase Loads and Their Associated Wattmeter Readings.

irregular line. In this diagram the power factor is high, the power-factor angle being only 20 deg. It is observed, however, that the phase angle between the current and voltage of W_2 is 30 deg + 20 deg, or 50 deg, so that it registers considerably less than W_1.

When the power-factor angle increases, the current vector drops farther behind the voltage vectors, and the difference in readings increases until at a power-factor angle of 60 deg the current of W_2 will lag the voltage by an angle of 90 deg and a zero reading will be obtained. The power factor at 60 deg lag is 0.5, and with power factors greater than 0.5 both meters will read positive, whereas with power factors less than 0.5 negative readings will be obtained with one of the meters, as shown in Fig. 9-7(b). In this figure a power-factor angle of 70 deg is assumed, which places the current in W_1 lagging the voltage by 70 deg − 30 deg = 40 deg and the current in W_2 lagging the

voltage by 70 deg + 30 deg = 100 deg. When the angle of lag is greater than 90 deg, a negative reading results since the cosines of angles greater than 90 deg are negative.

At low power factors it is necessary to reverse the connections on one of the wattmeters to get a positive reading and thus determine the magnitude of the indication. When this is done, the true wattmeter reading is a negative value. When reversing the wattmeter, it is proper to reverse the current connections

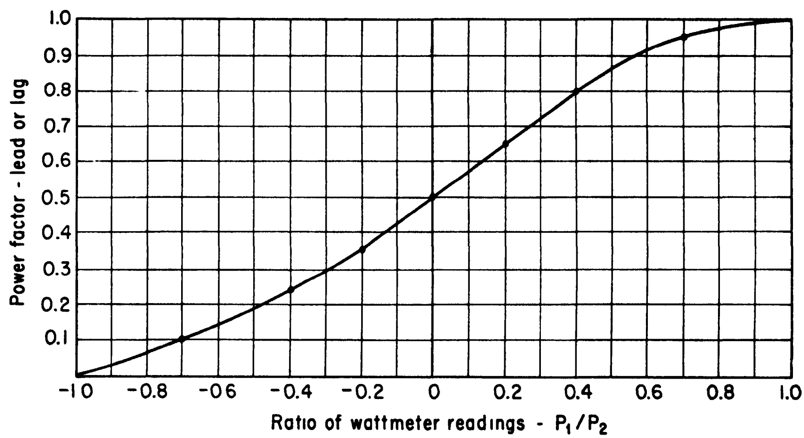


FIG. 9-8. Chart for Power-Factor Determination from Single-Phase Wattmeter Readings.

so as to retain the coil of the potential circuit at the same approximate potential as the current coil.

The ratio of the wattmeter readings is a very good indication of the power factor of motors and other *balanced* loads in industrial plants. These ratios have been determined for various power-factor angles and plotted in Fig. 9-8. This curve will be found quite useful in testing both in the laboratory and in industrial plants.

Exercise 9-1. Determine the two single-phase wattmeter readings for a 5-hp 220-v three-phase 60-cycle induction motor taking 10 amp at 0.8 power factor. Draw the vector diagram.

Exercise 9-2. A centrifugal pump is driven by a 25-hp 440-v three-phase 60-cycle 1150-rpm squirrel-cage induction motor. The power input is measured by two single-phase wattmeters, one of which reads 5000 w and the other of which reads 100 w. What is the power factor? What conclusions would you make as to pump output?

Exercise 9-3. The input to a 15-hp 220-v three-phase motor is to be measured using two single-phase wattmeters. The full-load efficiency of the motor is 88 per cent and the line current is 38 amp. Assuming full load on the motor, what is the reading of each wattmeter, the total power input, and at what power factor is the motor operating?

Although the use of single-phase wattmeters to measure three-phase power is often preferable in the laboratory, the necessity of adding the readings is not satisfactory for most industrial metering. It is usual, therefore, for two wattmeter elements to be mounted on the same shaft to function as a polyphase wattmeter. The resultant torque is the sum of the torques on both individual elements and so measures the total power.

Power-factor measurement

Power factor in single-phase circuits can be read directly on a scale by use of a power-factor meter, the principle of which is shown in Fig. 9-9b. In this meter the moving element has two coils that are crossed at approximately 90 deg. The main magnetic field is supplied by the current, whereas the voltage supplies both crossed coils. One of the coils has current in phase with the voltage, while the other coil has current that lags 90 deg behind the voltage because of the series inductance. If the current and voltage are in phase, then the axis of coil *A* will line up with the axis of the current coil. If, however, the current lags 90 deg, the axis of coil *B* will line up with the axis of the current coil. For intermediate conditions the crossed coils will assume a position depending upon the relative magnitudes of the torques of the two crossed coils. This meter does not have a spring, as no restoring torque is necessary. Numerous variations of this principle are used for both single-phase and polyphase power-factor meters.

The Var-meter. An alternate interpretation and measurement method of power factor is by the use of the Var-meter. Power has been shown to be the product of the voltage and the component of the current that is in phase with the voltage, which is I_p, as shown in Fig. 9-5. A measure of the quadrature component of I would give a good measure of power factor, or rather the deviation of the power factor from unity. This measurement is made by the use of an ordinary wattmeter mechanism with a voltage applied that is 90 deg from the normal voltage.

Since most circuits are polyphase, this quadrature voltage is easily obtained by a pair of small autotransformers. The product of the quadrature component of current I_r and the voltage E is known as *reactive voltamperes, or vars.* This method of measuring the deviation from unity power factor is very accu-

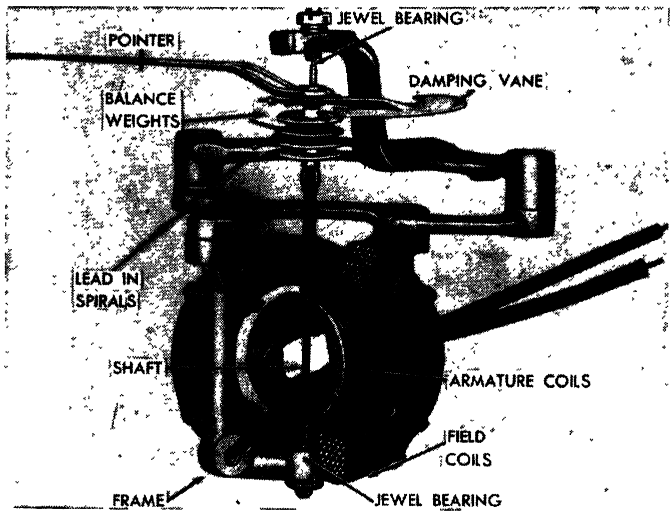


FIG. 9-9a. Single-Phase Crossed-Coil Power-Factor Meter. Cutaway view.

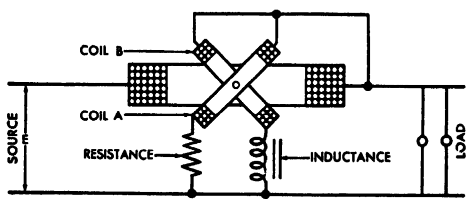


FIG. 9-9b. Diagram of Single-Phase Crossed-Coil Power-Factor Meter.

rate at high power factors, and so this method is gaining favor in operating practice.

Exercise 9-4. In an a-c circuit the meter readings are as follows:
Voltage = 230 v
Current = 14 amp
Power = 2400 w
Determine (a) the power factor and (b) the vars.

A-c bridges—general

The Wheatstone bridge with d-c voltage impressed was studied in Chap. 3. By applying Kirchhoff's laws using vector

values, it is possible to develop a similar bridge for use on alternating current. It is, of course, necessary to have a sensitive a-c detector to replace the galvanometer. In Fig. 9-10 the a-c voltage impressed on the bridge is in the audio frequency range and the sensitive detector is a pair of headphones. The equations are the same as in the Wheatstone bridge, except that the quantities are all phasors. Thus

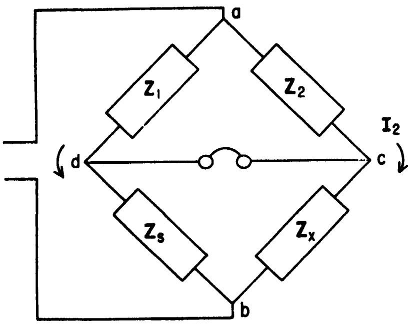


FIG. 9-10. General Wheatstone Bridge Diagram.

$$\frac{I_2 Z_x}{I_2 Z_2} = \frac{I_1 Z_s}{I_1 Z_1},$$

and

$$Z_x = \frac{Z_2}{Z_1} Z_s.$$

The above phasor equation is a general one. In the following paragraphs several special cases of the general form will be discussed.

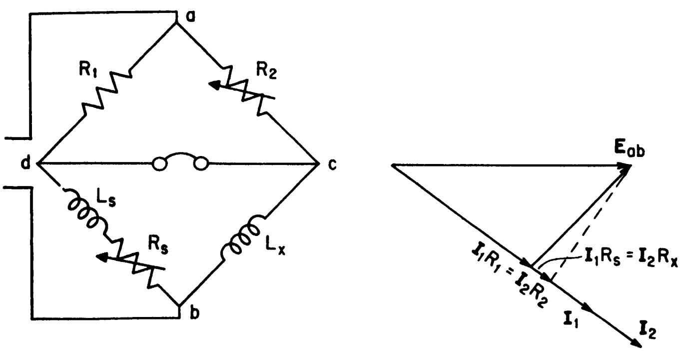


FIG. 9-11. An Inductance Bridge Using a Variable Bridge Arm.

Inductance bridge. If in the circuit of Fig. 9-10 Z_1 and Z_2 are resistors, then the balance equation becomes

$$Z_x = \frac{R_2}{R_1} Z_s.$$

Such a bridge, in which the unknown impedance is an inductance coil, is shown in Fig. 9-11. The vector diagram for the current and the voltage drops around the $R_2 - L_x$ side of the circuit is also shown in the figure.

Since resistors have a zero impedance angle, the ratio of R_2 to R_1 becomes a real number and the power factor of $\boldsymbol{Z}_x$ must be equal to $\boldsymbol{Z}_s$. This is not the usual condition; hence it is necessary to insert a small resistance in the circuit of either $\boldsymbol{Z}_s$ or $\boldsymbol{Z}_x$ in order to adjust the power factor until they are equal. In the diagram it is assumed that the power factor of the standard is the lower one and the resistance is inserted in that branch of the circuit.

In the phasor diagram the voltage drop I_2R_2 determines the potential of point c. The potential of point d is determined by I_1R_1; and since potential of points c and d must be the same at balance, then I_1 must be in phase with I_2. Not only must it be in phase, but the magnitude of I_1R_1 must equal I_2R_2. This requires that a balance be obtained in both magnitude and phase angle.

If R_2 is equal to R_1, then the bridge becomes a direct comparison bridge, and a variable standard inductor may be used to balance for the magnitude of the inductance while the variable resistor R_s is used to balance for the phase angle. The procedure is normally to adjust the inductor L_s until a minimum tone is obtained in the headphones and then to adjust R_s until a new minimum is reached. After this sequence is followed several times, it is usually possible to obtain a satisfactory balance.

Sometimes fixed standards of inductance are used, in which case it is necessary to vary the ratio of R_2/R_1 to obtain a balance. The R_s power-factor correction is still required.

Capacitance bridge. It is possible to use the same bridge for comparing the capacitance of capacitors. Since variable standard capacitors are more common than variable standard

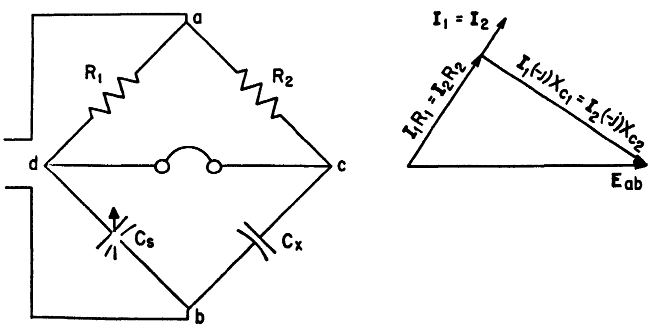


FIG. 9-12. Capacitance Bridge Using a Variable Standard Condenser.

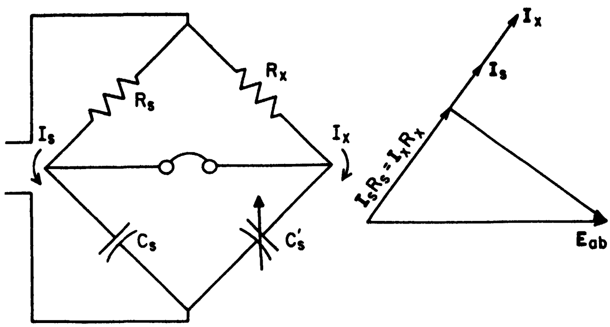


FIG. 9-13. Resistance Measurement by Means of Capacitance Bridge.

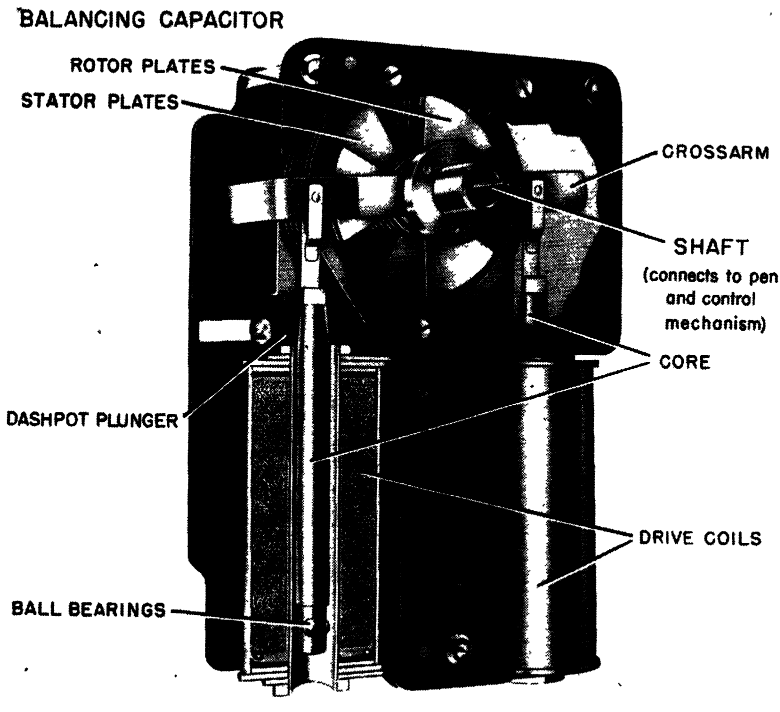


FIG. 9-14. A Dynapoise Drive Unit.

inductors, the use of a direct comparison bridge with R_1 equal to R_2 is most common. In most cases the use of a power-factor correction resistor is not needed because the losses in capacitors are very small. The diagram of such a bridge with its associated vector diagram is given in Fig. 9-12. The vector diagram for the

drops in the standard side of the bridge, will be identical with that of the unknown side.

A rather unusual application of this bridge is made by the Foxboro Company for temperature (and other types of industrial) measurement. In this case a variable capacitor is used to measure the variation of resistance in a coil located in the temperature well. The diagram of this circuit is shown in Fig. 9-13, and the meter itself is shown in Fig. 9-14. An electronic amplifier detects an unbalance and rebalances the circuit. Variations in the magnitude of the capacitor setting are recorded directly on the circular chart.

CHAPTER 10

Transformers

Use and characteristics of transformers

The transformer, which is to be studied in this chapter, is an electrical unit that makes a major contribution to the usefulness of the a-c power system. With it the pressure-flow or voltage-current relations in a circuit may be changed with very little power loss and at relatively low equipment cost. Transformers consist of primary and secondary coils of wire wound on a common magnetic core. These coils are electrically insulated from one another, but the power transfer is accomplished through the common magnetic field in the laminated steel core. The efficiencies of transformers usually range from 96 per cent for small units (of about 1 kw) to 99 per cent or even higher in some of the very large units.

With transformers, power generated at hydroelectric power sites at comparatively low voltage is stepped up in voltage and transmitted over high-tension power lines to cities and industrial plants. It is then stepped down by similar transformers to voltages that can be advantageously used in the city or plant. It is quite common to have several steps of voltage reduction in order to obtain the most economical power distribution and utilization system.

Fundamental transformer theory

Mutual inductance was defined in Chap. 4 as the circuit characteristic whereby a rate of change of current in the primary circuit would produce a voltage in the secondary circuit. In a transformer the normal alternation of current in the primary causes a change of flux that, in turn, produces the secondary voltage.

A simple form of transformer is shown in Fig. 10-1. Here a coil of wire with N_p turns is wound on a common magnetic core with a second coil having N_s turns. The first coil, which is nor-

mally connected to the power source, is called the primary, and the second is termed the secondary. It may be assumed, as a first approximation, that the flux is restricted to the magnetic core, and therefore all of the flux that links the primary also links the secondary. When an a-c voltage is applied to the primary coil, an a-c current flows that causes a flux in the transformer core that varies in magnitude sinusoidally with time. As explained in Chap. 7, the flux set up in an inductance (the transformer primary is an inductance) produces a voltage that

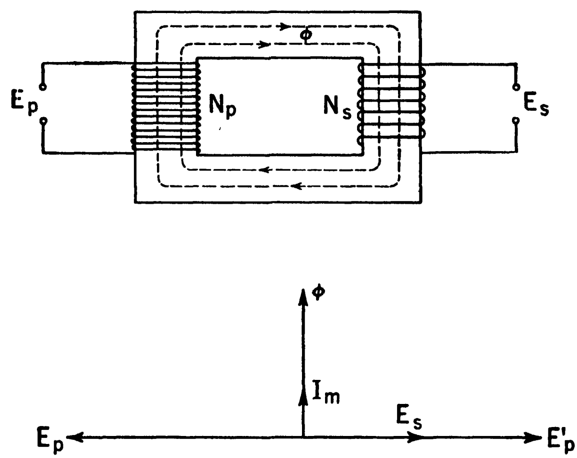


FIG. 10-1. Elementary Transformer Diagram—No Load.

is equal to and opposes the impressed voltage. Since the flux links both primary and secondary coils, the voltage per turn is the same in each coil and the internal voltages of the coils are proportional to the number of turns. Expressed mathematically this gives

$$\frac{E_p}{E_s} = \frac{N_p}{N_s}.$$

Transformer phasor diagram, no load. The analysis of electric circuits is greatly aided by the use of phasor diagrams, and these diagrams are similarly very helpful in the analysis of transformers and other electrical machinery. Not only is a simple iron-core transformer shown in Fig. 10-1 in diagrammatic form, but there is also supplied a phasor diagram of the voltages, the flux, and the exciting current. The impressed voltage E_p causes a current I_m to flow. Since the inductance is very

high, this current lags almost 90 deg behind the voltage and causes a flux which also lags 90 deg behind the primary voltage in time phase. This flux, as indicated in the diagram, sets up a voltage E'_p in the primary coil (which is equal and opposite to the impressed voltage) and a voltage E_s in the secondary coil. Since there are twice as many turns in the primary as in the secondary coil, the secondary voltage is only one-half of that of the primary; and the transformer is a 2:1 step-down transformer. This diagram assumes no secondary current and, therefore, represents the no-load condition.

Transformer phasor diagram, with load. If a load is connected to the secondary coil of the transformer, as indicated in

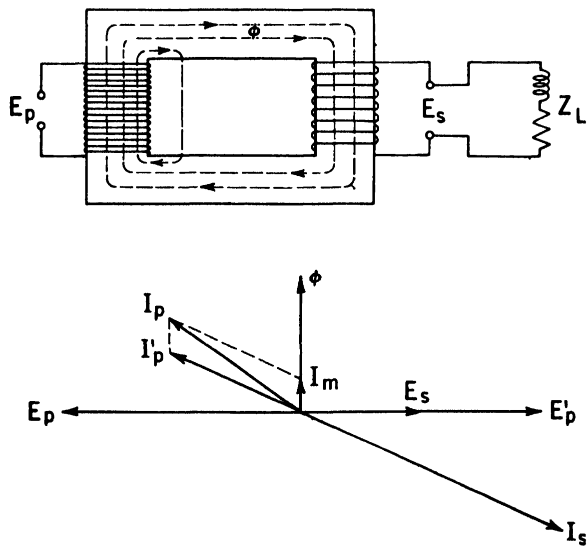


FIG. 10-2. Elementary Transformer Diagram—Lagging Load.

Fig. 10-2, a current will flow, with a magnitude and power factor determined by the impedance of the load. When this current flows in the secondary coil, it produces magnetomotive forces that tend to change the flux and disturb the previous balance of primary voltages. If resistance and reactance drops within the transformer are neglected (in normal transformers they amount to only a few per cent), the voltage produced in the primary E'_p must, at all times, be equal and opposite to the impressed voltage E_p. Since the impressed voltage is assumed as constant, the primary voltage E'_p must also be constant; and the flux in the core must also remain constant not only in mag-

nitude but also in phase. In order for this to be true, a current must flow in the primary, which will neutralize the magnetomotive force of the secondary coil. In the case of the 2:1 transformer under discussion, only half as much current is required in the primary as in the secondary to produce equal ampere-turns.

A more general statement that is very useful in the analysis of transformers is that, *neglecting exciting current, the ampere-turns of the primary are always just equal and opposite to the ampere-turns of the secondary.* Stated mathematically, this is

$$N_p I'_p = N_s I_s,$$

and

$$\frac{I'_p}{I_s} = \frac{N_s}{N_p},$$

where I'_p is the primary load current and I_s is the secondary current, as shown in the vector diagram of Fig. 10-2. The total primary current then becomes the vector sum of the primary load current I'_p and the exciting current I_m.

It is noted that if the secondary current lags behind the secondary voltage, then the primary current will lag behind the primary impressed voltage. Similarly, if the load impedance is such that it draws a leading current from the secondary, then the primary current is also leading. The secondary load is therefore accurately reflected in the primary, except that the primary current is equal to the secondary current multiplied by the reciprocal of the ratio of primary to secondary turns. Furthermore, the product of the current and voltage in the secondary is equal to the product of the load current and voltage in the primary.

The transformer acts, therefore, as a device for changing the impedance of a load as it appears to the primary circuit. In the 2:1 transformer under discussion the current in the primary is only half that in the secondary, but the primary voltage is twice that of the secondary. Thus, the secondary load impedance must be multiplied by the square of the turns ratio in order to determine the equivalent impedance to the primary circuit. Thus,

$$Z_p = \left(\frac{N_p}{N_s}\right)^2 Z_s.$$

Example. A 10-ohm resistor is connected across the secondary of a 2400 to 240-v transformer. What impedance does this circuit present to the primary or 2400-v circuit?

Solution:

$$Z_p = \left(\frac{N_p}{N_s}\right)^2 Z_s = 10^2 \times 10 = 1000 \text{ ohms.}$$

Exercise 10-1. A load composed of a resistance of 8 ohms and an inductive reactance of 12 ohms is connected to the secondary of a 2300/230-v transformer. What impedance is presented to the 2300-volt line by the transformer and its load?

Exercise 10-2. A 22,000/6600-v transformer has 1000 turns on the high-tension winding. (a) How many turns will be on the low-tension winding? (b) What cross-section must the iron core have in order that the maximum flux not exceed 100,000 lines /in.^{2}?

In the foregoing analysis it was assumed that all of the flux linking the primary also linked the secondary. This is not entirely true since the opposing magnetomotive forces of the two coils cause some flux to pass across the air gap, as indicated in the top diagram of Fig. 10-2. A more accurate analysis will be given as soon as the construction of transformers has been discussed.

Construction of transformers

The objective of the design and construction of ordinary transformers is to have the two windings interlaced as intimately as possible, consistent with insulation requirements and with cooling. These windings are provided with a closed magnetic path composed of laminated sheet steel. This path or core has sufficient cross-section so that only a small magnetizing current is required and low iron loss results. Many of the smaller transformers achieve this objective by wrapping the primary and secondary in form-wound coils, as shown in Fig. 10-3. These are then joined magnetically by wrapping a long sheet of steel in a compact spiral to form a short and efficient magnetic circuit. The spiral-wound core is shown in Fig. 10-3 after it has been formed and annealed but before it has been wrapped into the coils. After these transformers have been assembled, they are placed in a weatherproof tank, which is filled with oil to improve insulation and cooling. A careful inspection of the coil group in the figure will reveal wooden spacers, located at the ends of the coils, which produce openings in the winding through which the oil may circulate. The electrical connections are brought out of the case through porcelain insulators known as bushings. In these small distribution transformers

the primary or high-voltage connections are brought out on one side of the transformer case, while the secondary or low-voltage connections are brought out on the other side.

Different manufacturers use quite different forms of construction for these transformers. Some very excellent engineering has increased the efficiency and reduced the cost of trans-

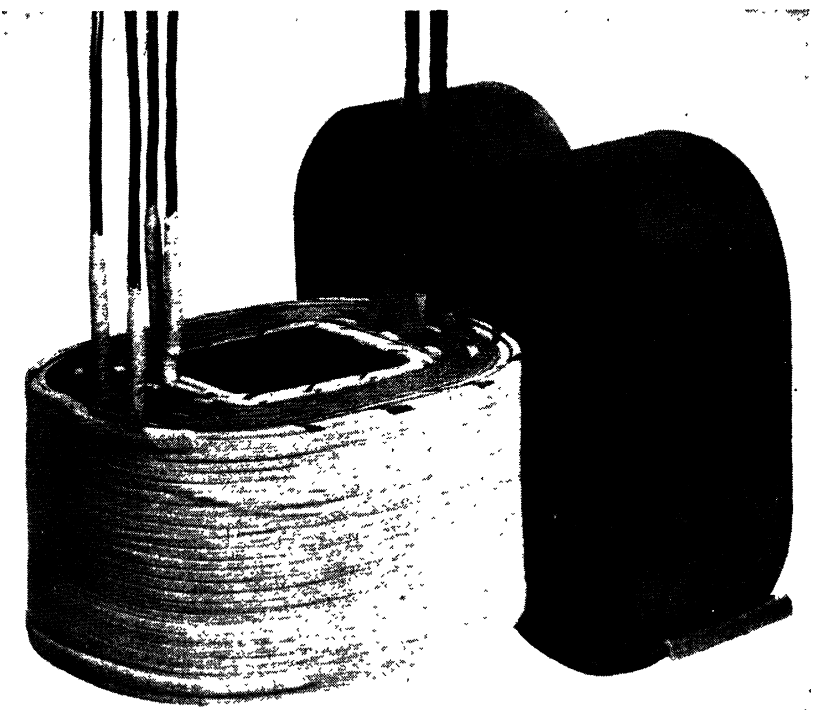
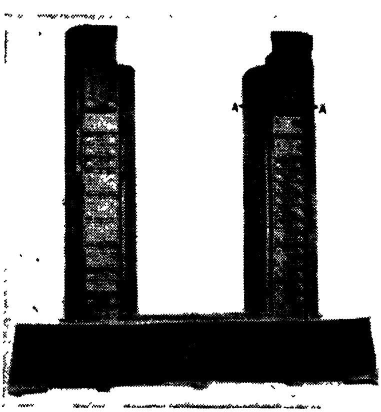


FIG. 10-3. Windings of G-E 10-kva Spirakore Transformer with Two Sections of Core before They Are Cut into Short Lengths.

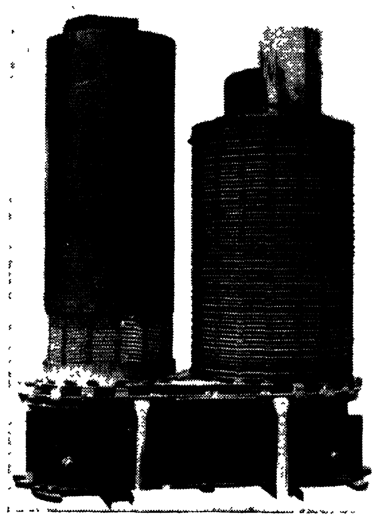
formers. The fundamental objectives illustrated above are the same in all designs.

In the larger, higher-voltage transformers the magnetic core is constructed of rectangular sheets of transformer steel. The corners are so arranged that the sheets are interleaved to form an essentially continuous magnetic circuit. The core is completely assembled except for the top. The low-tension coils are then wound on fiber cylinders, dipped in varnish, baked, and are then ready for assembly on the core (as shown in Fig. 10-4). One set of coils are usually for high voltage, so special attention

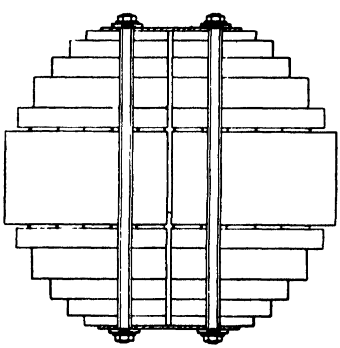
must be given to the problem of insulation. The high-tension winding is often made of a number of separate coils insulated from each other and, of course, from the secondary winding and the core. The separate high-tension coils are connected in series, but the separation of the coils protects the winding from



(a)



(c)



(b)

FIG. 10-4. Construction of a Large, Single-Phase Power Transformer. (a) Core before coils have been placed. (b) Cross-section of the core at A-A. (c) Low-tension windings on both legs and high-tension winding on the right leg.

lightning and switching surges that might otherwise puncture the insulation. In the drawing of Fig. 10-5, half of a large high-voltage transformer is shown with coils in cross-section. In this diagram the insulating cylinder, on which the low-tension coil is wound, may be seen. There is enough space between

this cylinder and the core to permit some flow of cooling oil. Since this flow is inadequate to remove all of the heat produced in the core, an oil duct is provided in the center of the core. The high-voltage cylinder is sufficiently large that oil flow may be obtained between it and the low-voltage coil, and is held in place by wood or fiber spacers located around the coil. These spacers

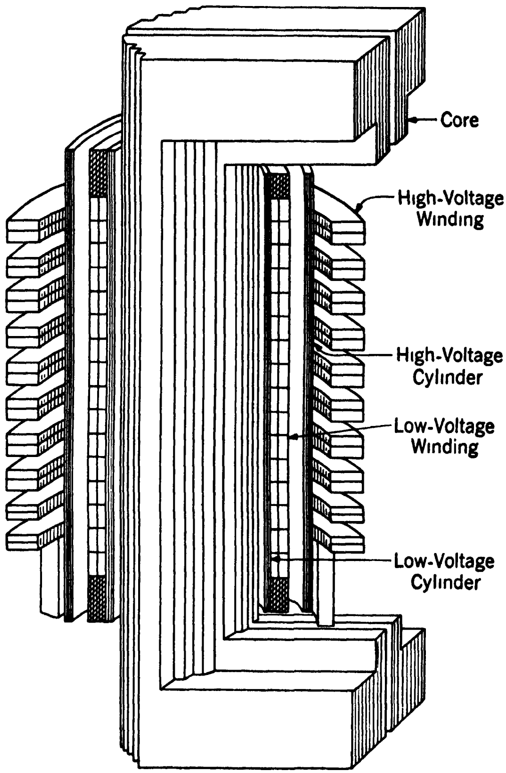


FIG. 10-5. Schematic Diagram Showing Arrangement of Core and Coils of a Power Transformer.

are placed vertically so as not to interfere with the oil flow. The individual high-voltage coils are then assembled on the outside of the high-voltage cylinder and carefully braced and tied in position so that heavy short-circuit currents will not move or twist them. After the secondary and primary coils have been assembled on both legs of the transformer, the top of the core is assembled and clamped. The series and parallel connections are made to the coils and the terminal leads are provided. The entire assembly is then placed in a large tank and filled with

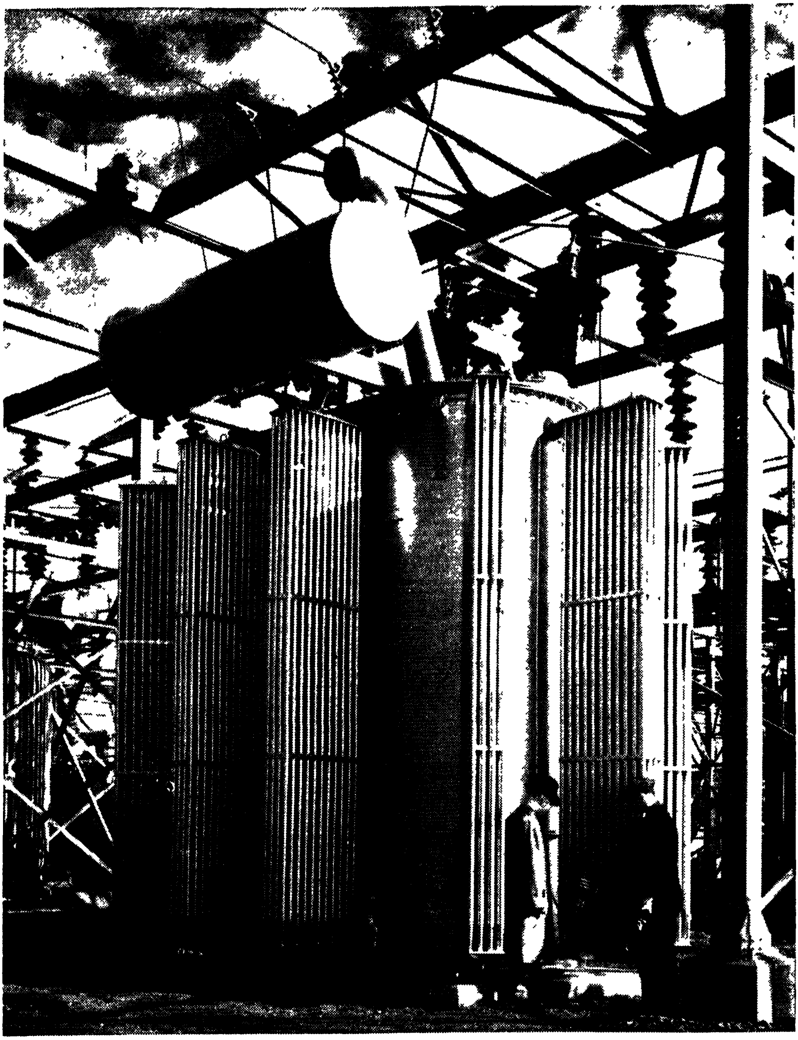


FIG. 10-6. A 12,000 kva, Three-Phase, High-Voltage Transformer.

insulating oil. The tank is usually provided with radiators arranged to cool the oil. The top of the tank is provided with porcelain bushings through which the high-tension and low-tension leads are brought out.

The oil-cooling system operates on a convection principle. The oil flows up through the coils and core as it is heated and then out to the radiators and down through them as it is cooled,

the energy of the losses being absorbed in heating the outside air. Figure 10-6 shows an exterior view of a large high-voltage transformer.

The cores for transformers are of several different forms. The small transformer of Fig. 10-3 uses one set of coils with a split magnetic circuit, wound with an exterior section on either side of the transformer. Such construction is called a *shell* type. The large power transformers shown in Fig. 10-4 and 10-5 have two sets of concentric coils mounted on the two legs of a single-circuit magnetic core. Such a transformer is called a *core*-type transformer. Each type has some advantages, although by properly designing the proportions of the core and coils it is possible to obtain efficient and satisfactory designs in both forms for almost all sizes of transformers.

As indicated above, oil is the most satisfactory cooling and insulating medium for general use. It has high dielectric strength, low viscosity, and does not sludge, therefore it is used on most outdoor transformers. Oil is combustible, however, and special fire protection must be provided where oil-filled transformers are located in buildings. Since such precautions are expensive where transformers are installed inside of buildings, it is common practice to use air-cooled transformers in small sizes and to substitute pyranol or a similar noncombustible liquid for the oil in the larger high-voltage transformers.

Leakage reactance

It has been seen that most transformers have a definite separation between primary and secondary coils to provide for insulation and flow of the cooling medium. It was also learned in a previous paragraph that the magnetomotive forces of the primary and secondary load currents were equal and opposite. Since these load currents are quite large, the magnetomotive forces are also large and cause a very considerable flux to exist between the coils. In Fig. 10-7 a cross-section diagram of the core and coils of the transformer of Fig. 10-5 is shown. The high-tension coils produce (at the particular instant shown) a magnetomotive force tending to send flux in a counterclockwise direction around the core. The secondary coils produce an equal and opposite magnetomotive force. These two opposing magnetomotive forces produce a net difference of magnetic potential in the space between the coils with resultant flux in the air space as shown by the flux lines indicated in the diagram

as ϕ_L. This flux varies with the total primary and secondary current and, therefore, is not in time phase with the main flux in the core, which is in phase with the exciting current of the primary winding. The variation of this leakage flux produces voltages in the primary and secondary windings, which may be considered as reactance drops. The magnitude of this flux is seen to depend upon the reluctance of the magnetic path between primary and secondary coils and upon the net magnetomotive

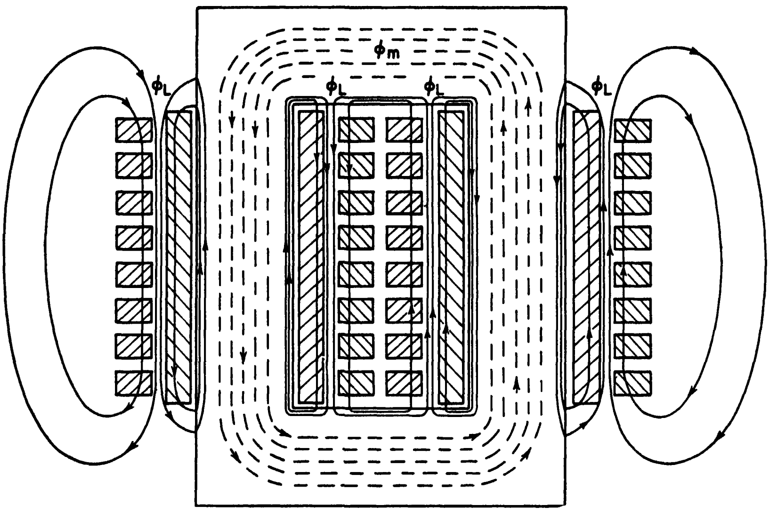


FIG. 10-7. Leakage Flux in a Transformer under Load. *Note:* ϕ_L indicates path of leakage flux. This is independent of the main flux ϕ_m and is in time phase with load currents rather than normal exciting current.

force acting on the path. It is noted that in the transformer of Fig. 10-7 only half of the total primary or secondary magnetomotive force acts across either leakage path. The same situation is obtained in the shell-type transformer by placing the high-tension winding between the two halves of the secondary winding.

The effect of the leakage flux and of the coil resistance on the phasor diagram is shown in Fig. 10-8. In this diagram different voltage and current scales are used on primary and secondary so that the primary and secondary induced-voltage phasors are of the same magnitude. The primary load-current phasor is the same magnitude as the secondary current phasor. This is important to a clear presentation of the diagram where fairly

large ratios of transformation exist. The voltage induced by the main transformer flux is assumed as the reference phasor.

The secondary terminal voltage must be less than the induced voltage by phasor voltages necessary to overcome resistance and leakage reactance. Similarly, the primary ter-

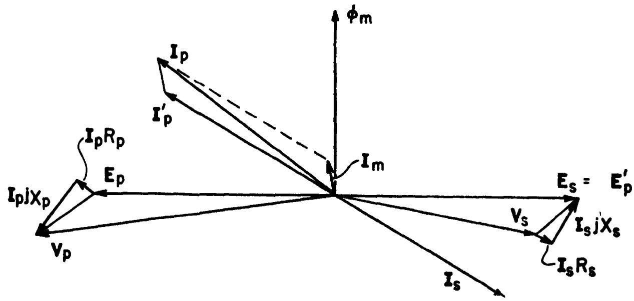


FIG. 10-8. Transformer Phasor Diagram with Lagging Load.

minal or impressed voltage must be greater than the primary induced voltage by the phasor voltage necessary to overcome the resistance and leakage reactance drops of the primary.*

In most transformers the leakage-reactance drops of primary and secondary will each be from 3 to 5 per cent of rated

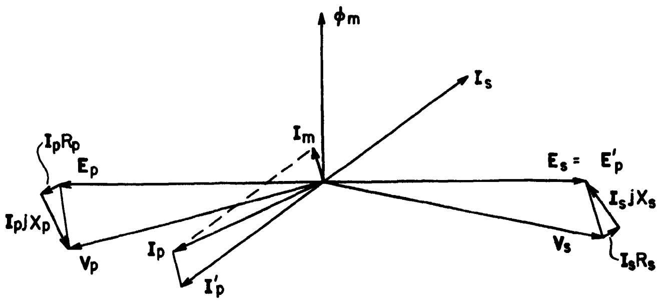


FIG. 10-9. Transformer Phasor Diagram with Leading Load.

voltage at full-load current, while the resistance drops will be from ½ to 1½ per cent of rated voltage. Thus, *the impedance triangles are very small as compared to the figure above,* which has from 15 to 20 per cent reactance drop in each. Furthermore,

* One other minor change is made in the diagram and that is in the magnetizing current phasor. It is given a small component in phase with the impressed voltage to supply the iron losses in the transformer, as will be discussed later.

the effect of these drops on terminal voltage will depend upon the power factor of the load current.

In the case of a leading current, as shown in Fig. 10-9, the reactance voltage drops will tend to make the *secondary voltage greater under load than at no load.* This characteristic is used in order to obtain a small amount of voltage adjustment on certain types of electrical machinery.

Exciting current

The flux in the core changes sinusoidally with time in order that a voltage, equal and opposite to the impressed voltage, may be induced in the primary winding. A magnetizing current in the primary winding produces a magnetomotive force sufficient to supply this sinusoidal flux. In a magnetic circuit composed entirely of iron the magnetomotive force necessary for a flux that is continually reversing may be obtained from the hysteresis loop. This magnetizing current is nonsinusoidal, rising to a peak as the maximum flux is approached.

If the voltage is increased, the maximum value of flux must also increase in the same ratio. Since it is customary to design transformers to operate with maximum flux densities just below the knee of the magnetization curve, it follows that a relatively small increase in voltage and flux will cause a large increase in exciting current. Transformers will, therefore, overheat if operated at more than 10 or 15 per cent above rated voltage. Transformers, however, may be operated at voltages lower than rated as long as the current rating of the transformer is not exceeded. (For instance, a 10-kva transformer may be operated at half-voltage as long as the normal current for 10 kva at rated voltage is not exceeded. It would not be possible to supply a load in excess of 5 kva at half voltage.)

The effect of frequency on transformer design and utilization is of particular interest to aeronautical engineers. Since the voltage induced in a transformer is proportional to the rate of change of flux, an increase in frequency will reduce the maximum flux and thus will make possible a reduction in the size of the transformer. Since weight is important, the frequency of 400 cps has been established as preferred for electrical systems in aircraft. (It is not possible to use 400-cycle transformers on 60 cps unless the voltage is reduced to 60/400 of the rated value.)

Transformer losses and efficiency

The losses of transformers are similar to losses of d-c machinery, except, of course, that there are no friction and windage losses. The iron losses are particularly important since they are constant and since transformers are normally connected to the power lines 24 hours per day.

The iron losses are of two types. The first is caused by hysteresis, a type of molecular or domain friction loss described in Chap. 2. This loss, which occurs on each reversal of the direction of the magnetism, is dependent upon the composition, rolling, and heat treatment of the iron. The loss per cycle for any particular material and core assembly will vary in accord with the maximum flux density. The hysteresis loss is proportional to the area of the hysteresis loop for any material, and the area of the loop has been found to vary approximately as the 1.6 power of the flux density. Thus, the hysteresis loss in transformers may be assumed to vary as the 1.6 power of the voltage. It will also vary directly as the frequency since the same energy loss occurs each cycle.

The second type of iron loss is the result of eddy currents. Figure 10-10 shows an end view of laminations, the thickness

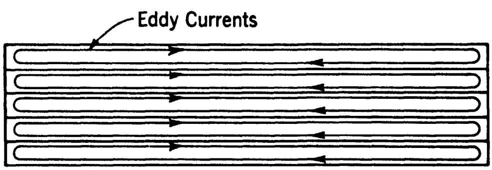


FIG. 10-10. Eddy-Current Paths in a Laminated Iron Core.

of which has been enlarged for purposes of demonstration. Magnetic flux perpendicular to the plane of the paper is assumed to be varying rapidly in magnitude, thus producing a voltage around the periphery of the lamination. This voltage causes a current to flow in the iron of the lamination as indicated by the lines with arrows in the diagram. Energy is transferred to heat by this current flowing through the resistance of the iron.

The manner of variation of this loss with thickness may be analyzed as follows. When the thickness of the lamination is reduced, for instance, to one-half of its former thickness, then the flux producing the voltage for the eddy current is also cut in half. The resistance of the path of the current is doubled

since the cross-section is reduced to one-half. The current will therefore be reduced to approximately one-fourth of its previous value. Since the power is equal to I^2R, the power for a single lamination is equal to $(\frac{1}{4})^2 \times 2 = \frac{1}{8}$ of the previous value. Since two laminations would be required for the same previous flux, the net eddy current loss for the same volume of iron would be $\frac{1}{4}$ of the previous value. It may be concluded that the eddy current loss varies approximately as the square of the thickness of the laminations for a fixed condition of flux and frequency.

The variation of eddy current loss with maximum flux density at a fixed frequency is again based on the fact that the current varies directly with voltage, which, in turn, varies directly with flux density. Since the loss varies as the square of the current, the loss may be said to vary as the square of the maximum flux density, or voltage, at any specified frequency.

If the flux density is held constant, the rate of change of flux varies directly with frequency. This change of flux causes the voltage and the eddy currents, so the loss may be said to vary as the square of the frequency, if the maximum flux density is held constant. The rapid increase in loss with frequency requires the use of very thin laminations at high frequencies.

It has been demonstrated that both types of iron losses vary with frequency and maximum flux density. Curves may be obtained from the manufacturers of electrical steel sheets, which give these losses per unit volume or unit weight for varying maximum flux density at different frequencies. The important characteristic of these losses from an operating point of view is that they are essentially constant with time in a given installation, since both impressed voltage (and thus flux density) and frequency are constant in the normal usage of transformers. The iron losses of power transformers are, therefore, normally assumed constant.

The above explanation of the iron losses of a transformer will hold with very minor variation for the iron losses in any type of electrical machine, whether it be alternator, induction motor, d-c motor, or d-c generator. The student should, therefore, understand that this explanation will not be repeated for each machine, but that when iron losses are referred to they are of the above types.

In addition to the constant iron losses in the transformer, there are also losses caused by the load current flowing through the resistance of the coils. Since the coils are made of copper

wire, they are usually referred to as copper losses and, of course, vary as the square of the load current.

Any machine that has both constant losses and losses that increase with the square of the load will have a maximum efficiency at the load where constant and varying losses are equal. Since the iron losses are continuous and the copper losses occur only during the time the transformer is loaded, it is usual for the designer to hold the iron loss low. The maximum transformer efficiency normally occurs, therefore, at one-half to three-quarters of full load.

Exercise 10-3. A 20-kva transformer has an iron loss of 250 w and a full-load copper loss of 500 w. Plot the efficiency against the percentage of full-load current.

Exercise 10-4. A 100-kva, 6900/230-v transformer has an iron loss of 900 w. The copper losses are divided equally between primary and secondary windings. If the full-load efficiency is 97.4 per cent, determine the resistance of both primary and secondary windings.

Exercise 10-5. A 50 kva 2300/230 volt 60-cycle single phase transformer has a core loss of 400 w and a full-load copper loss of 600 w. What is the efficiency when carrying 40 kw at 0.9 power factor?

Transformer rating

Transformers are rated on the basis of frequency, their primary and secondary voltages, and on the current that they will carry without overheating.

Nearly all industrial transformers are designed for and used on 60-cycle circuits. A reduction in frequency produces an increase in flux equivalent to a corresponding percentage increase in voltage.

The specification of voltage gives the voltage at which efficient use is made of the iron core. If a lower voltage is used, the flux density is lower and the transformer will operate satisfactorily as long as the current limits are not exceeded. It will not, however, put out its full power rating since the voltage is low. If, on the other hand, the transformer is used on a higher voltage, the exciting current and iron losses increase rapidly, thus causing overheating even with no load. Since transformers are usually designed fairly liberally, a voltage increase of 10 per cent above rated voltage will not cause difficulty, but voltage in excess of this amount may be expected to cause excessive heating.

The main limitation of transformer loading is that of temperature, which is determined by the losses of the transformer, the cooling system, and the temperature of the outside air (except where water is used as a cooling medium). In any particular transformer the cooling system is fixed, and so the temperature is primarily dependent upon the losses that are converted to heat within the transformer. The iron losses have been indicated as constant for constant-voltage transformers; hence the load limits are set by the copper losses that are, in turn, dependent upon load current. The load that any transformer will carry without overheating can, therefore, be specified in terms of the product of the rated voltage and the current. This gives an answer in terms of volt-amperes or kilovolt-amperes. Thus, a transformer is normally specified as having a certain kva capacity with specified primary and secondary voltages. The name plate will also show possible series or parallel coil connections, and taps to make slight adjustments in the voltage ratio; it may also specify the equivalent leakage reactance.

Because transformers are rated for continuous load, they may usually be overloaded without damage for short periods of

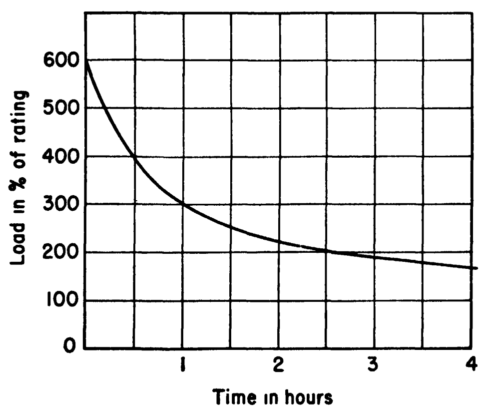


FIG. 10-11. Length of Time for Transformers to Come to Limiting Temperature with Various Loads.

time, provided they have not previously been operating at full load. On cold days the rated temperature rise above the air will still leave the operating temperature appreciably below the danger point. In operating transformers the actual temperature of the transformer is the danger criterion rather than the

instantaneous load. The load that may be carried by one type of transformer for short periods is shown in Fig. 10-11.

Exercise 10-6. How large a transformer would be required to supply a 230-v, 40-amp load at 70 per cent power factor?

Exercise 10-7. A 20-kva, 4100/230-v transformer is normally carrying one-half of the rated load on a construction job. It is proposed to load it to 200 per cent for an hour and a half. As supervising engineer, would you approve this proposal? What factors not included in the problem might affect your decision?

Parallel operation of transformers. The requirements for parallel operation of transformers are as follows:

(1) The voltage rating of the two transformers should be approximately the same.

(2) The ratio of the primary to secondary voltage should be exactly the same in both transformers.

(3) The impedance drops of the two transformers should be about the same when carrying rated load.

Thus, a 2300/230-v transformer might be operated in parallel with a 2200/220-v transformer. They would each carry the proper portion of the load if their impedances were inversely proportional to their ratings. Usually, if the first two conditions are satisfied, the two transformers may reasonably be expected to operate satisfactorily in parallel.

The manner of making such parallel connections is shown in the diagram of Fig. 10-12. Since standards of polarity have been observed only during recent years, care should be taken to check the polarity or directions of voltage before connecting them. If they are connected physically identically but are of a different polarity, a bad short circuit will result. It is wise, therefore, to test the voltage around the secondary loop with a voltmeter before connecting the two transformers in parallel. If a voltage equal to twice the

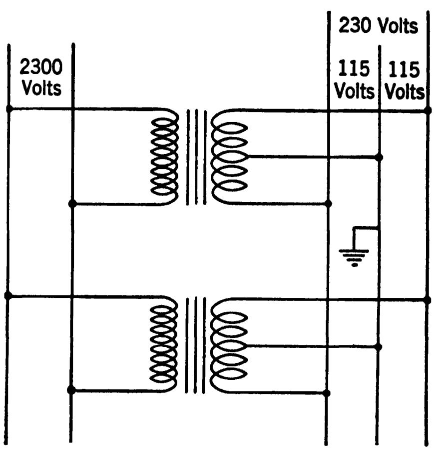


FIG. 10-12. Connections for Parallel Transformers.

rated secondary voltage of the transformer appears, a reversal of one pair of leads externally will correct the difficulty.

Polyphase transformer connections

Three-phase power is commonly supplied to industrial users at potentials of from 2000 to 11,000 v. These voltages must be stepped down to lower voltages for distribution within the industrial plant. This step-down is provided by transformers, and several connections may be used.

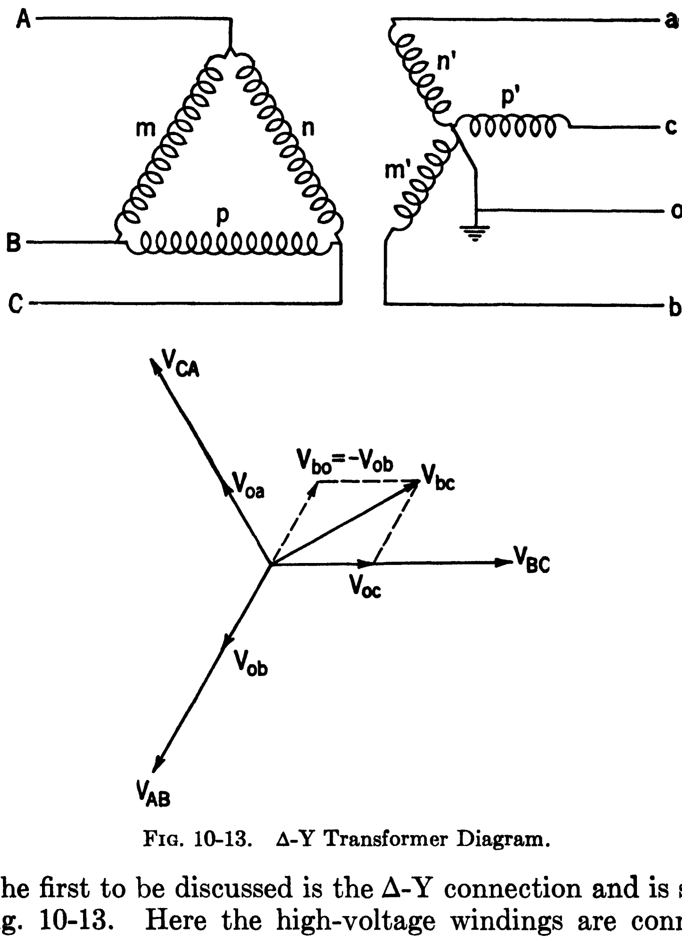


FIG. 10-13. Δ-Y Transformer Diagram.

The first to be discussed is the Δ-Y connection and is shown in Fig. 10-13. Here the high-voltage windings are connected across the high-tension lines. One secondary lead from each transformer is connected to the common, or ground wire, and the other leads supply the line voltages. Care must be taken

to connect the proper leads to the neutral, so that a balanced polyphase voltage is provided across the outside wires of the secondary circuit. A voltmeter test across these outside wires will determine whether or not they are correctly connected.

This secondary system is observed to be the three-phase four-wire system studied earlier. This is the most common

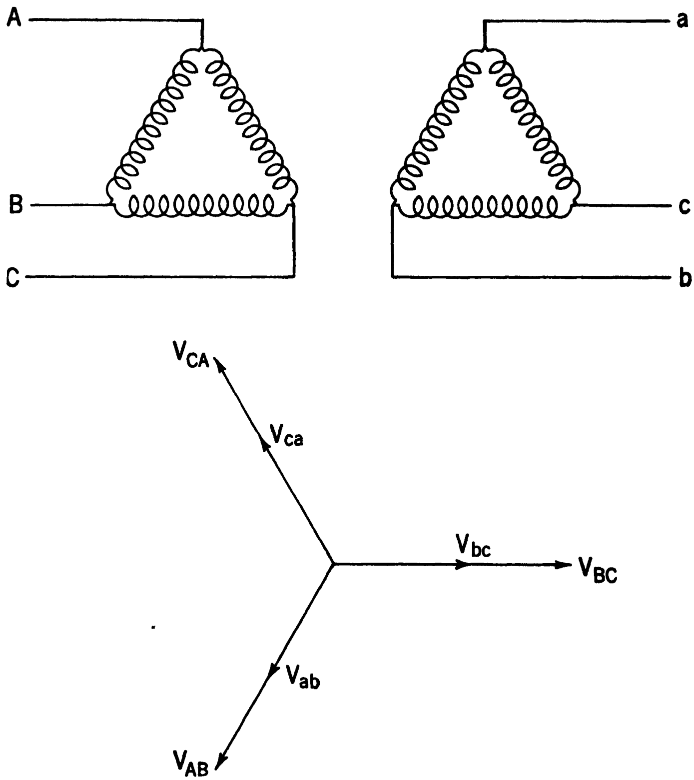


FIG. 10-14. Δ-Δ Transformer Diagram.

method of connecting transformers to supply this system. It is important to note that the voltages between phase wires are displaced 30 deg from the phase voltages of the primary. This is caused by the addition of the voltage V_{bo} and the voltage V_{oc} to obtain the line voltage V_{bc}.

The second common polyphase transformer connection is Δ-Δ, shown in Fig. 10-14. This is used most often where three-phase power is supplied to motors and other heavy industrial loads. In this transformer connection the phase voltages of the

primary and secondary are in phase, as will be observed from a study of the vector diagram.

It is not possible to operate a Δ-Y and a Δ-Δ transformer bank in parallel because of the shift of phase in the secondary voltages.

A third common connection is the open delta or V-V, which is identical with the Δ-Δ, except that one of the transformers is omitted. It is used as an emergency connection when one transformer has failed. It is also used where an increase in load is anticipated within a few years and where it is desired to plan for increased capacity, but the expense of the additional transformer is not immediately justified. The capacity of a V-V bank of transformers on balanced polyphase load is 87 per cent of the combined capacity of the two transformers or 58 per cent of the capacity of a completed Δ-Δ bank. This reduction in capacity is caused by the fact that the transformer current is not in phase with the voltage across it.

Polyphase transformers

Although single-phase transformers connected externally to form one of the above polyphase connections have been most extensively used in electric power distribution, economy of space, labor, and material has led to an increasing popularity of three-phase transformers. Particularly is this true in high-voltage transformers where the cost of a single bushing to get the lead through the transformer case may run as high as several thousand dollars. The substitution of three bushings for six is, therefore, a real saving.

Sometimes three single-phase transformers are mounted in one case and only the polyphase connections brought out. Usually, however, still further economy is obtained by using a common core on the three phases of the transformer, as shown in Fig. 10-15. Here each of the three phases of the transformer is mounted on one of the three legs of the core. The flux in one of the legs is returned through the other two. This is satisfactory as long as the voltages are balanced because the fluxes of the three legs have the same magnitude and are displaced 120 deg in time phase.

An alternate core design, known as the shell-type core, is shown in Fig. 10-16. In this case the normal flux is restricted to the three main legs, but any unbalance of voltages will cause flux to shift to the outside legs, which have no windings. The

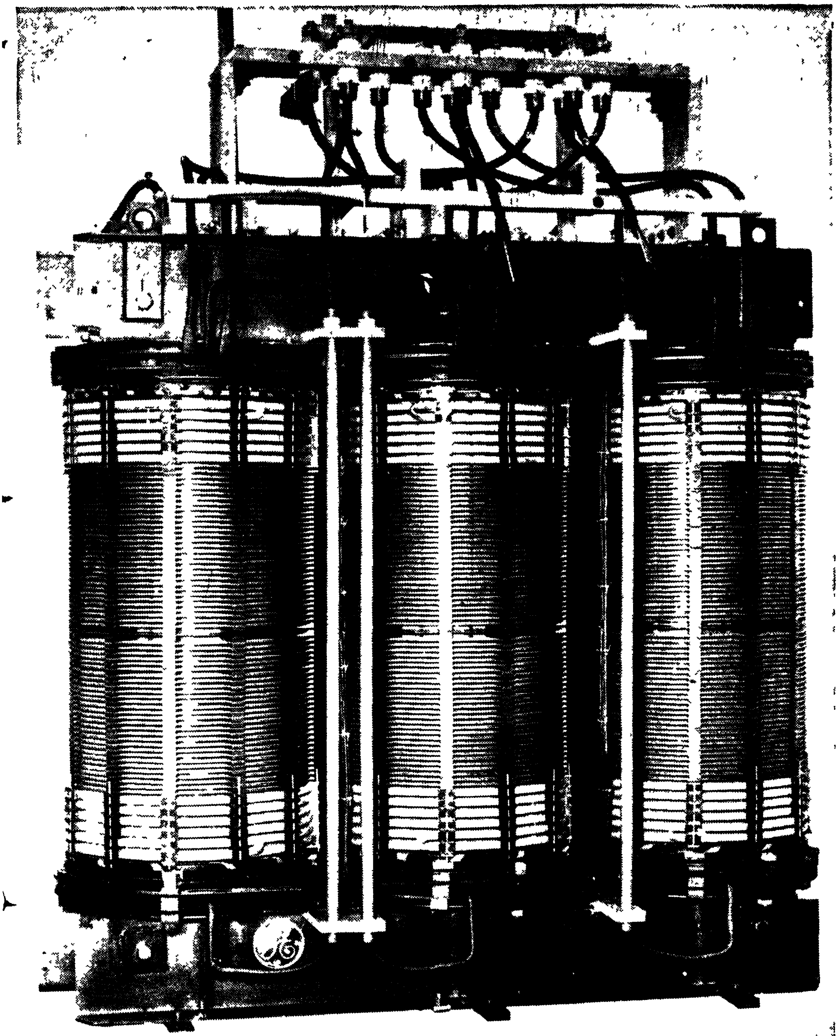


FIG. 10-15. A Three-Phase Power Transformer before Being Placed in the Tank.

chief advantage of this construction is that with it the transformer may be operated with a V-V connection under emergency conditions when one phase of the transformer has failed. To do this it is necessary to disconnect and short-circuit the windings of the faulty phase. This will permit no flux in that leg of the transformer and the flux of that phase will then pass through the external legs.

Polyphase transformers are especially advantageous for unu sual or complicated connections. It has been noted that th secondary voltages of a transformer are always approximatel in phase with the primary. The magnitude is proportional t the turns. By using these elementary vector voltages in com bination, it is possible to obtain any magnitude and phase o

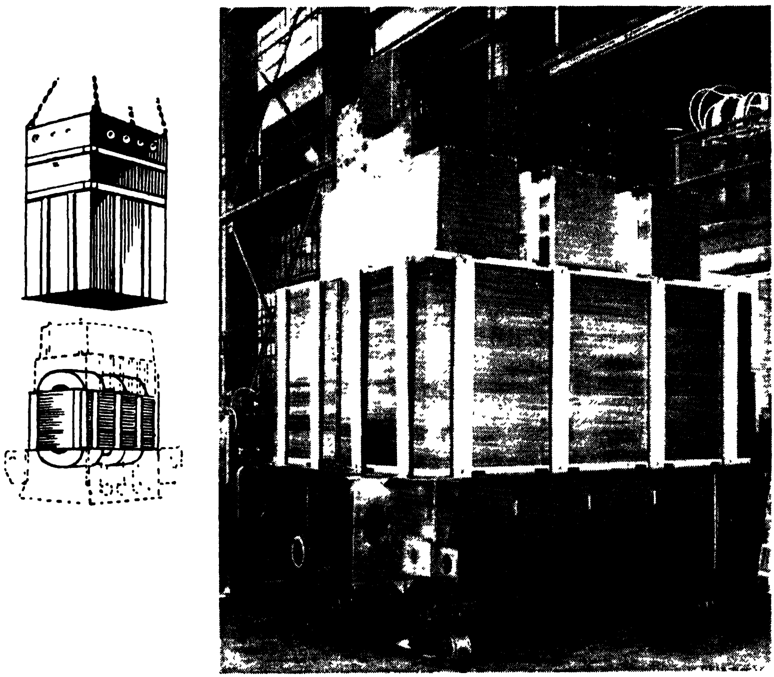


FIG. 10-16. Drawing and Photograph Showing the Construction of a Shell Type Three-Phase Transformer. The tank is in two parts: the upper portion lift off from the core and coils, which are assembled in the bottom portion of the tank (Courtesy Westinghouse Electric Corp.)

secondary voltage desired. Numerous complicated arrangements of polyphase circuits have been and are being used fo specialized purposes. Illustrative of the simpler forms of these connections are the six-phase transformer connections shown in Chap. 16 on rectifiers. The connection diagrams are self-explanatory, since if the midpoints of the three-phase secondaries are connected together, the terminal voltage will be of the same magnitude and will have 60 deg phase displacement. A polyphase transformer makes the secondary connections of such

a transformer much simpler since they can be made inside the case.

In modern standardized unit substations for industrial installations the use of polyphase transformers is almost universal. The chief disadvantage is that when an insulation failure occurs, the equipment is completely inoperative. With the development of new and better insulation materials and better designs, however, these failures have become so infrequent as to justify the risk of a shutdown.

CHAPTER 11

Alternating-Current Generators

Single-phase a-c generators

In the study of d-c generators it was found that the voltag was produced by having a conductor sweep through a magneti field. When a current flowed in the conductor, it produced force tending to oppose the relative motion of the conductor an the field. These same conditions exist in a-c generators excep that in most a-c generators the field poles, which are excited b direct current, are constructed as the rotating element of th machine, and the armature is stationary. Thus, the magneti field sweeps past the armature conductors to produce th voltage.

An elementary a-c generator with a two-pole rotor is show in Fig. 11-1. As the poles sweep past the conductors, voltage will be produced in a direction out of the paper at the top of th

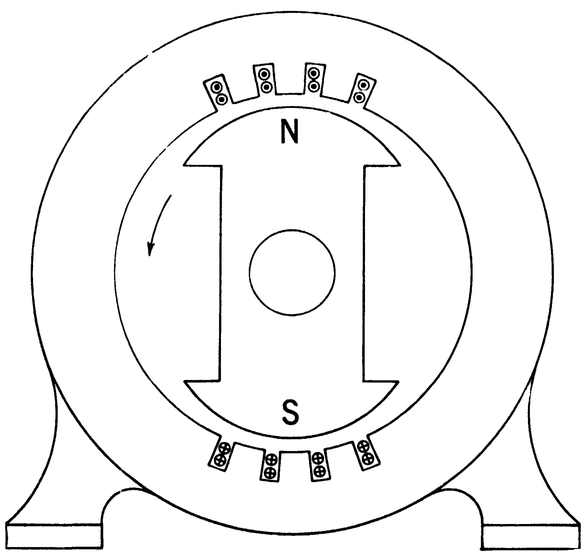


Fig. 11-1. Elementary Diagram of a Single-Phase Alternator.

machine and into the paper at the bottom. When these conductors are connected in series to form coils and the coils, in turn, are connected in series, considerable voltage is produced in the winding. The magnitude of the voltage will depend not only on the flux density of the field poles but also on the length of the armature, the peripheral velocity, and the number of turns in the armature coils.

One quarter of a revolution after the position shown in Fig. 11-1 the conductors are no longer under the poles, and so no voltage is generated. After half a revolution, the north pole is sweeping past the bottom conductors and the south pole is sweeping past the upper conductors, so that the voltage is maximum in the reverse direction. This is an alternating voltage, and if the distribution of flux density around the machine is made sinusoidal, the time variation of voltage in the winding will also be sinusoidal. The distribution of the winding in the four slots will tend to neutralize any irregularities of the flux wave if a sinusoidal distribution is not achieved.

If the terminals of the winding of the machine in Fig. 11-1 are connected to a resistor, a current will flow. This current flow in the generator is in such a direction as to oppose the motion of the field. Therefore, as the field rotates, it will experience a retarding force that comes to a maximum each time the poles sweep past the vertical axis and that drops to zero when the field sweeps past the horizontal axis. The retarding force or torque is proportional to the product of the current in the armature conductors and to the field strength. Since the field strength has been assumed to vary sinusoidally around the periphery of the machine and since the field is rotating at a uniform angular velocity, the field strength at the conductor positions varies sinusoidally with time, being zero when the pole

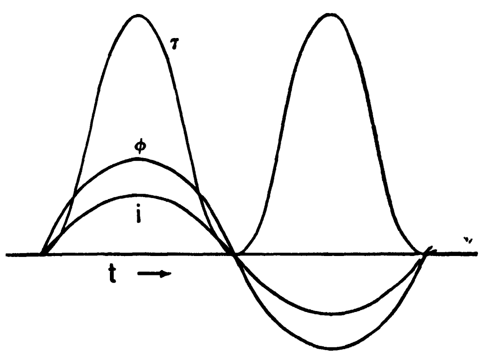


FIG. 11-2. Time Variation of Generator Torque with Single-Phase Resistance Load.

axis is horizontal. Since the current likewise is zero when the pole axis is horizontal, it will be in phase with the flux. The resultant torque is shown in Fig. 11-2 as the instantaneous product of the current and flux. This torque is identical in form with the power in a single-phase circuit, which was studied in Chap. 7. There it was demonstrated that this instantaneous power is the sum of a constant term plus the cosine of twice the normal phase angle. Thus the power input is measured by the product of the torque and angular velocity, and the power output is measured by the product of the current and voltage.

In small single-phase machines this rapid pulsation of torque produces annoying noise and vibration. In the larger single-phase machines the mass is so great as to prevent vibration at this frequency.

Polyphase a-c generators

In the machine shown in Fig. 11-1 only one-third of the total periphery is used for the armature winding. It is possible,

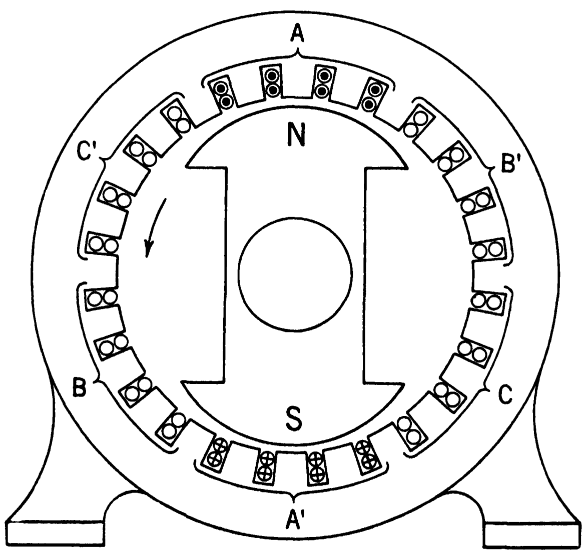


Fig. 11-3. Elementary Diagram of a Three-Phase Alternator.

therefore, to place two additional windings on the machine, each of which is similar to the one shown in Fig. 11-1. Such a machine, with three similar windings, is shown in Fig. 11-3 and each group of conductors is labeled. The winding in the vertical axis is designated as phase A. The winding designated as

phase B is 120 deg around the periphery, and phase C is 240 deg around the periphery. The voltages generated in these phases are shown in Fig. 11-4a and are the same three-phase voltages studied in Chap. 8.

If identical resistors are connected to each phase, balanced currents—that is, currents that are equal in magnitude and 120

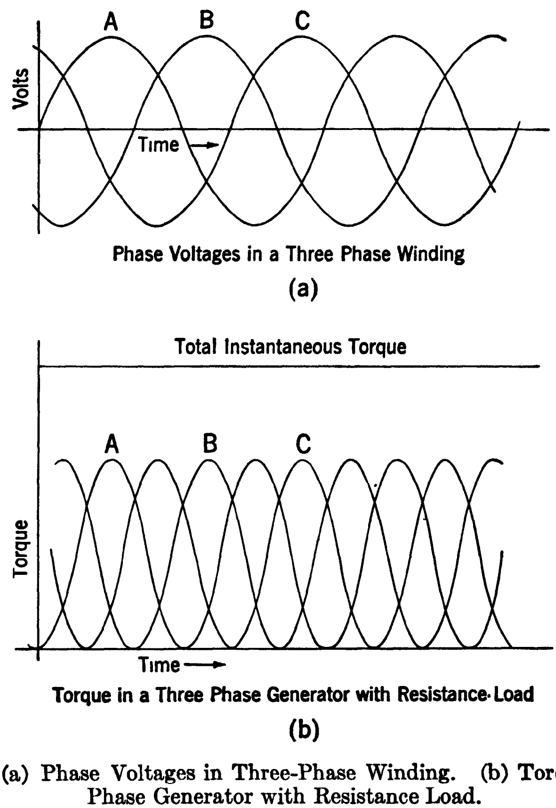


FIG. 11-4. (a) Phase Voltages in Three-Phase Winding. (b) Torque in Three-Phase Generator with Resistance Load.

deg apart in time—will result. For each phase, the torque on the generator will take the form shown in Fig. 11-2. The torque in each of the three phases is plotted in Fig. 11-4b. The sum of the torque in all three phases at any instant will be observed to be a constant. This constant value is three times the average value of each phase. A similar relation was developed for three-phase power in Chap. 8. This constant torque may be visualized in another way. When the rotor is on the axis of phase A, current in that phase is a maxi-

mum; hence the retarding torque is a maximum. As the rotor moves from the axis of phase A to that of phase B, the torque of phase A decreases but that of phase B increases. In this way each phase picks up its share of the retarding torque as the windings come under the influence of the field poles.

This constant torque of three-phase machines (either genera-

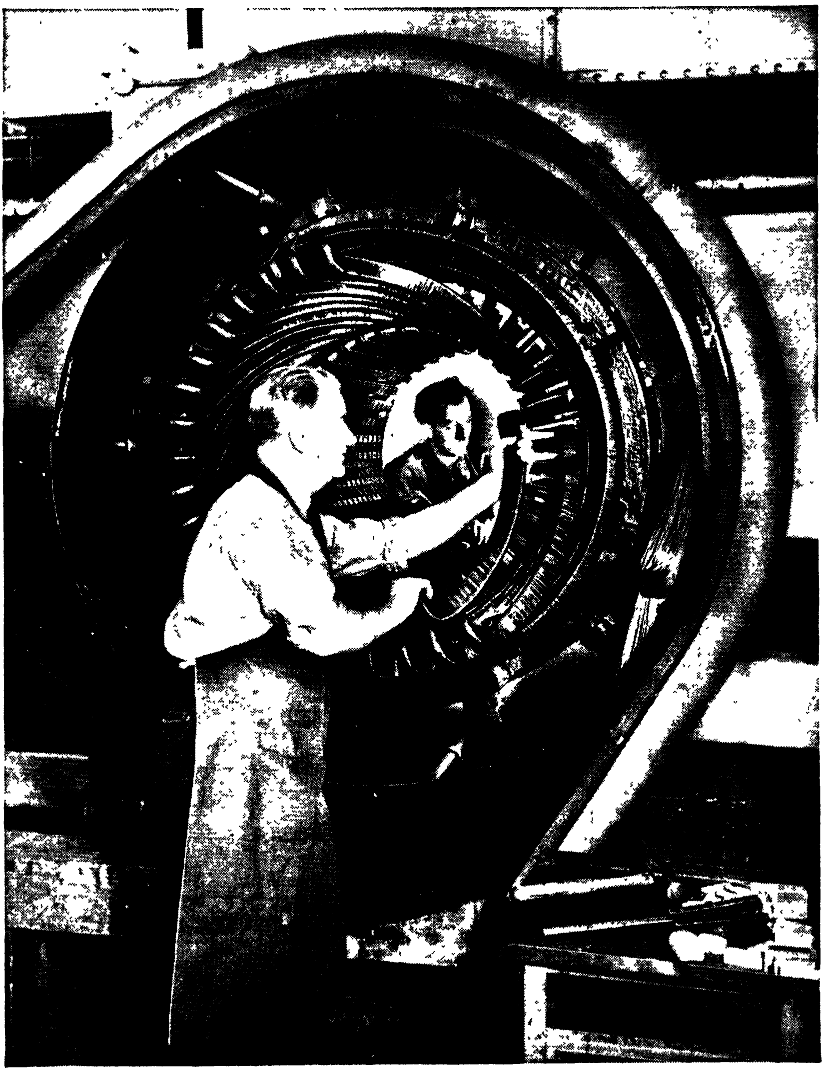


FIG. 11-5a. Coils Being Placed in Slots of Stationary Armature for a Turbine Generator.

tors or motors) is one of their important characteristics, because it permits greater capacity in the same size machine or the same capacity in a smaller and cheaper machine than with single phase. The propery of uniform torque is not limited to three-phase machines but is common to all polyphase machines. Since nearly all of the polyphase equipment in the United States is three-phase, discussion in this text will be limited primarily to three-phase equipment.

Three-phase generator connections. The three-phase windings in the machine of Fig. 11-3 are independent and may sup-

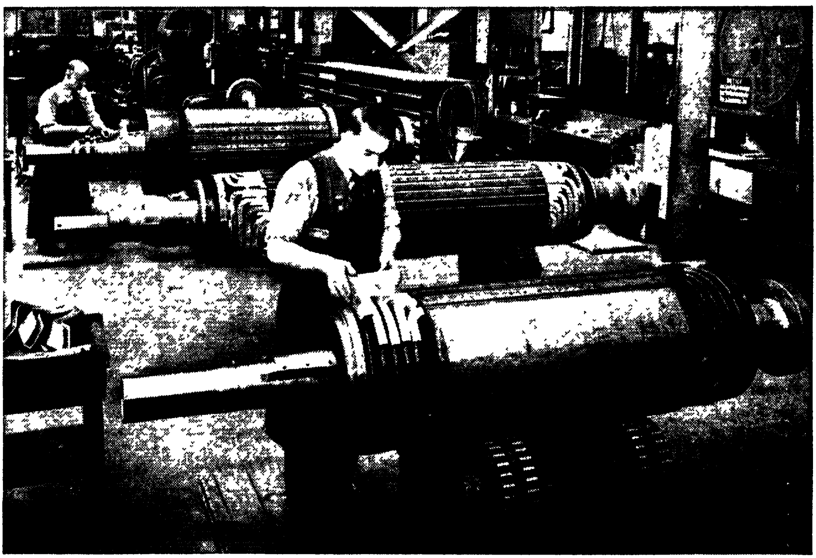


FIG. 11-5b. Aluminum Saddles Being Placed on End Windings of Revolving Field for Turbine Generator.

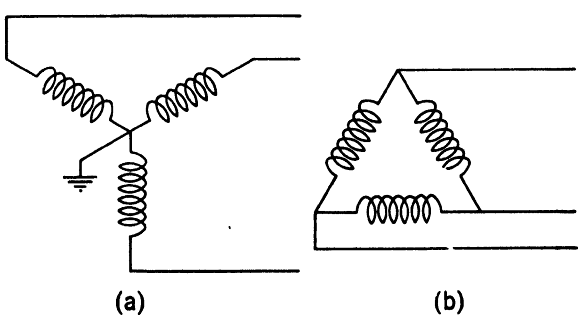


FIG. 11-6. Connections for Three-Phase Generators.

ply three independent single-phase systems. This, however, does not save copper in the distribution system, so the coils are always interconnected to form a three-phase generator. This can be done in either of two ways, as explained in Chap. 8. If one end of each coil is connected to a common terminal in such a manner that the voltages of the other terminals are 120 deg apart, as shown in Fig. 11-6a, the machine is said to be Y-connected. If the windings are connected so as to close on themselves, as in Fig. 11-6b, the machine is said to be Δ-connected. It should be observed that the proper phase relationships must be maintained in making this connection, or the sum of the voltages around the circuit will not be zero and a short circuit will result.

In large generators that supply transmission and distribution transformers, the Y-connection is usually preferred, as it provides an easy method of establishing ground potential, holds the insulation stress to a minimum, and makes it possible to use simple, yet sensitive, differential relays to disconnect the machine in case of insulation failure.

The windings of a-c machines, shown in Fig. 11-5, are somewhat less complicated than those of d-c machines. Each phase is simply an arrangement of coils that will cause the voltages to add. In the winding in Fig. 11-3, all conductors of one phase are located within a 60-deg phase belt. This requires that the coils have full pole pitch between coil sides. The length of the end connections can be reduced and copper saved by using slightly less than full pitch. In some slots, this places the coil sides of two adjacent phases in the same slot. A slightly reduced total voltage is obtained, but the wave form of the voltage is improved and smoother operation of the machine results. The magnetomotive force of the armature is also more nearly sinusoidal in its distribution around the periphery of the armature, so most machines have armature windings with a coil pitch that is less than the pole pitch.

Alternator calculations

To demonstrate these relationships, some of the more important voltage and torque computations will be made on a simple machine.

Example. In an a-c generator, such as is shown diagrammatically in Fig. 11-3, and the construction of which is shown in Fig. 11-5, there are 24 armature slots. The coils are full-pitch and are grouped in

three phases, as shown in Fig. 11-3. The field is two-pole, revolves at 3600 rpm, and has a sinusoidally distributed flux of a maximum air-gap value of 50,000 lines/in.[2] The internal diameter of the armature is 12 in., with an active length of 30 in.

(a) How many turns per coil are required to produce a phase voltage of 2400?

(b) What would the line voltage be if connected in Y?

(c) If one phase of the machine is carrying a single-phase unity-power-factor current of 100 amp, what is the maximum instantaneous torque?

(d) What is the constant torque for a three-phase load of 100 amp per phase (173 amp per line if Δ-connected)?

(e) What is the generated power?

Solution: As shown in the diagrams, each coil will have one side in the bottom of a slot and the other side in the top of a slot that is directly opposite. The voltages in these coil sides will be additive. Also, each slot will have two coil sides. For purposes of computation, it is assumed that each coil has one turn, and the instantaneous voltage under these conditions is calculated for phase A-A' when the pole is in the position shown in Fig. 11-3 and rotating at 3600 rpm.

(1) Determine the instantaneous voltage with one turn.

$$e = Blv \times 10^{-8}$$

where $l = 30$ in., $v = 12\pi \times 60 = 2260$ in./sec, $B = 50{,}000 \cos\theta$ lines/in.^{2}, and θ is the angle between the center of the slot and the pole axis. There are four slots that are $7\frac{1}{2}$ deg from the pole axis. Each has two coil sides or two conductors, so that the total voltage contributed by these four slots is

$$\begin{aligned} e' &= 50{,}000 \cos 7\tfrac{1}{2}^\circ \times 30 \times 2260 \times 8 \times 10^{-8} \\ &= 269 \text{ v.} \end{aligned}$$

There are also four slots that are $22\frac{1}{2}$ deg from the pole axis, and their contribution to the voltage is

$$\begin{aligned} e'' &= 50{,}000 \cos 22\tfrac{1}{2}^\circ \times 30 \times 2260 \times 8 \times 10^{-8} \\ &= 251 \text{ v.} \end{aligned}$$

The total maximum voltage is $269 + 251 = 520$ v with a single-turn coil.

(2) Determine how many turns are required for 2400 v effective.

Since 520 v is the instantaneous voltage under maximum conditions, the effective voltage is

$$\frac{520}{\sqrt{2}} = 367 \text{ v.}$$

The number of turns required is

$$\tfrac{2400}{367} = 6.54 \text{ turns.}$$

Since it is impossible to have a fractional number of turns, it will be necessary to use either six or seven turns and adjust the field flux to produce the required voltage.

Seven turns will be assumed. Since seven turns are used, it will be necessary to reduce the flux so that the product of flux density and turns is the same. Thus

$$7 \times B' = 6.54 \times 50{,}000$$

or

$$B' = 50{,}000 \times 6.54/7 = 46{,}600 \text{ lines/in.}^2$$

(3) The line voltage if connected in Y will be

$$2400\sqrt{3} = 4150 \text{ v} \qquad (\textit{Ans.})$$

(4) Determine the maximum instantaneous torque with 100 amp in a single phase.

With an effective current of 100 amp, the maximum value will be

$$100\sqrt{2} = 141 \text{ amp.}$$

This current will occur when the pole flux is along the plane of the phase. Each slot will have 14 conductors and each will carry 141 amp; therefore the current per slot producing force or torque is

$$14 \times 141 = 1980 \text{ amp.}$$

From Chap. 3

$$F = 8.84BlI \times 10^{-8}.$$

The force produced by the four slots $7\frac{1}{2}$ deg from the pole axis is

$$\begin{aligned} F' &= 8.84 \times 46{,}600 \times \cos 7\tfrac{1}{2} \text{ deg} \times 30 \times 1980 \times 4 \\ &= 970 \text{ lb.} \end{aligned}$$

The force produced by the four slots $22\frac{1}{2}$ deg from the pole center is

$$\begin{aligned} F'' &= 8.84 \times 46{,}600 \cos 22\tfrac{1}{2} \text{ deg} \times 30 \times 1980 \times 4 \\ &= 900 \text{ lb} \end{aligned}$$

The total force is $970 + 900 = 1870$ lb. The torque (maximum value per phase) is equal to the force times the radius, or

$$T = 1870 \times \tfrac{1}{2} = 935 \text{ lb-ft} \qquad (\textit{Ans.})$$

(5) The balanced three phase torque is

$$T_3 = 3 \times \tfrac{1}{2} \times 935 = 1400 \text{ lb-ft.}$$

(6) The power converted from mechanical to electrical energy for a balanced three-phase load is therefore

$$P = \frac{2\pi T_3 \times \text{rps}}{550}\ \text{hp}$$

$$= \frac{2\pi \times 1400 \times 60}{550} = 962\ \text{hp} \qquad (Ans.)$$

$$= 962 \times 0.746 = 718\ \text{kw} \qquad (Ans.)$$

Note. This computation may be checked by the electrical power equation, which is

$$P = \sqrt{3}\,EI = \sqrt{3} \times 4150 \times 100 = 720\ \text{kw}.$$

This equivalence of power will hold only when losses are neglected.

Exercise 11-1. In a steam-turbine generator, such as is shown in construction in Figs. 11-5a and b, the two-pole field rotates at 3600 rpm. The stator or armature has an internal diameter of 24 in. and a length of 54 in. There are 36 slots, and the coils are full-pitch. If the field flux is sinusoidal and has a maximum air-gap value of 45,000 lines/in.^{2}, how many turns per coil are required to produce a voltage of 13,200 if the machine is Y-connected? If the line current is 400 amp at unity power factor, determine the torque and electric power generated. Check, using the equation for the electric power output.

Frequency and speed

The machine shown in Fig. 11-3 has two poles, so that a complete revolution of the rotor in one second produces 1 cps

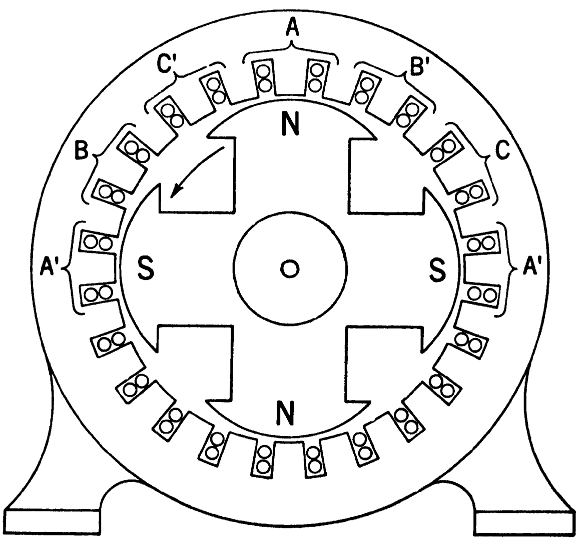


FIG. 11-7. Elementary Four-Pole Alternator.

of alternating current. Sixty revolutions per second, or 3600 rpm, would be required, therefore, to generate 60 cps. A four-pole generator is shown in Fig. 11-7. In this machine, only one-half revolution is required to complete the magnetic cycle with respect to the armature, and so for 60 cps the speed is only 1800 rpm.

The relationship among speed, frequency, and number of poles may be stated as follows:

$$f = \frac{p}{2} \times \frac{\text{rpm}}{60},$$

or

$$\text{speed in rpm} = \frac{120f}{p},$$

where f = frequency in cycles per second and p = number of poles. For high-speed prime movers, such as steam turbines, two- or four-pole generators are most common, whereas for the slower-speed water turbines many poles are required for the 60-cycle frequency. For instance, the 180-rpm water turbines at Boulder Dam require 40 poles to produce 60 cps.

Exercise 11-2. Determine the number of poles required to obtain 60 cps with each of the following prime movers: (a) Steam turbine: 1800 rpm. (b) Diesel engine: 450 rpm. (c) Water wheel: 277 rpm.

Armature magnetomotive forces

It has been assumed in previous discussions that with unity power factor the current reaches a maximum when the axis of the pole is in the center of the coil. This assumption is not correct since the air-gap flux is determined by the resultant of the magnetomotive forces of the d-c field winding and of the armature. In order to understand the operation of a-c generators and motors, it is necessary to determine the character of the armature reaction and the manner of its combining with the d-c field in a synchronous generator.

If the current in phase A of Fig. 11-3 is a maximum when the pole axis is vertical, as shown in the diagram, then the magnetomotive force of phase A is horizontal, producing a south pole on the right-hand side of the armature. As the rotor turns in a counterclockwise direction, successive phase belts will reach maximum currents. The south pole at the right side of the armature caused by phase A will be succeeded by a similar pole at the top of the armature caused first by phase C' and then by phase B. Thus the south pole caused by the armature currents

follows around the periphery of the armature at the same speed as the rotor, always lagging behind the north pole of the rotor. (Since the armature south pole attracts the rotor north pole and repels the rotor south pole, it is necessary to drive the rotor against this retarding force, which provides an alternate concept to the "force on a conductor" explanation of generator torque.)

The air-gap flux will be the resultant of the magnetomotive force of the main field winding and of the armature reaction, which in this case will produce a flux that reaches the axis of the phase belt later than the pole axis. In order for the current to reach maximum when the pole axis coincides with the center of the phase belt, the current must lead the voltage in time phase.

It is usual for the current in an a-c generator to lag behind the voltage in time phase; therefore, the current maximum is not normally reached in any phase until after the pole axis has passed some distance beyond the center of the phase belt. This causes the south pole produced on the armature to be much nearer the south pole of the rotor than the north pole and effectively reduces the magnitude of the resultant magnetomotive force. This will cause a decrease in the air-gap flux and in the terminal voltage.

In order for the voltage to be maintained it is necessary to increase the rotor magnetomotive force so that the resultant flux is the same as before, even though the armature mmf is partially opposed to the rotor mmf. Thus with an alternating-current generator supplying a load having a lagging power factor, the field current must be greater than with unity power factor.

By similar analysis, if the load has a greatly leading power factor, the current in any phase will reach a maximum before the pole axis reaches the center of the phase belt. Thus the south pole on the armature will shift forward and be much closer to the north pole of the rotor. This will cause an increase in resultant mmf and a corresponding increase in air-gap flux and terminal voltage. When the generator has a leading-power-factor load, it is necessary to reduce the field current of the rotor in order to maintain the terminal voltage constant.

To summarize, *the time variation of currents in the armature coils (which are fixed in position) of a polyphase alternator cause magnetic poles to be produced on the armature; these poles rotate*

*around the surface of the armature at synchronous speed.** If the currents are lagging behind the voltage in time phase, the poles not only tend to retard the motion of the rotor but also tend to reduce the magnitude of the air-gap flux. If the current is leading the voltage in time phase, the poles still tend to retard the motion of the rotor, but an increase in air-gap flux results.

The rotating magnetic field

A more detailed analysis of the currents and magnetomotive forces described in the previous paragraph is often helpful in understanding how the time-varying currents in the different phases produce the rotating magnetic field. In order to simplify the analysis and to give the student familiarity with the more common four-pole machines,† the alternator shown in Fig. 11-7 will be used as the basis for study.

The upper portion of the stator is shown unrolled or flattened out in the upper portion of Fig. 11-8. In this figure along the right side are shown the time variations of the three-phase currents. Vertical lines marked t_0, t_1, t_2, t_3, etc., show successive instants of time for the successive magnetomotive forces that are shown along the left portion of the figure.

The phase belts are marked as in Fig. 11-7 with A or A' to indicate that in one group of phase conductors the current is flowing in one direction, whereas in the next adjacent group of the same phase, the current is flowing in the opposite direction. In this diagram positive current in the time-variation diagram at the right indicates that current is flowing into the paper in A, B, or C, but flowing out of the paper in A', B', or C'.

At time t_0, as shown in (a) of the diagram, the current in A is a maximum positive, whereas the currents in B and C are one-half magnitude negative. In order to analyze the magnetomotive force, the neutral position n_0 will be assumed‡ between the two slots of A phase. The magnetomotive force will then be plotted against the space position around the air gap.

In progressing to the left, a slot containing two coil sides of phase A is first met. Since maximum current is flowing into

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* This effect is produced by any set of balanced polyphase currents flowing in an armature winding and is the basis of the rotating field of an induction motor, which will be studied later.

† Four poles are particularly common in induction motors.

‡ The complete construction procedure gives this neutral as assumed. For brevity it is omitted.

the paper in these coil sides, the mmf diagram will show an increase of two units in the space between phases A and C'. A slot of C' is next met and since the current in C is negative, the current in C' is positive; so the mmf diagram is again increased. This time the current in each coil side is one-half magnitude; so the two coil sides together give only one unit increase in mmf.

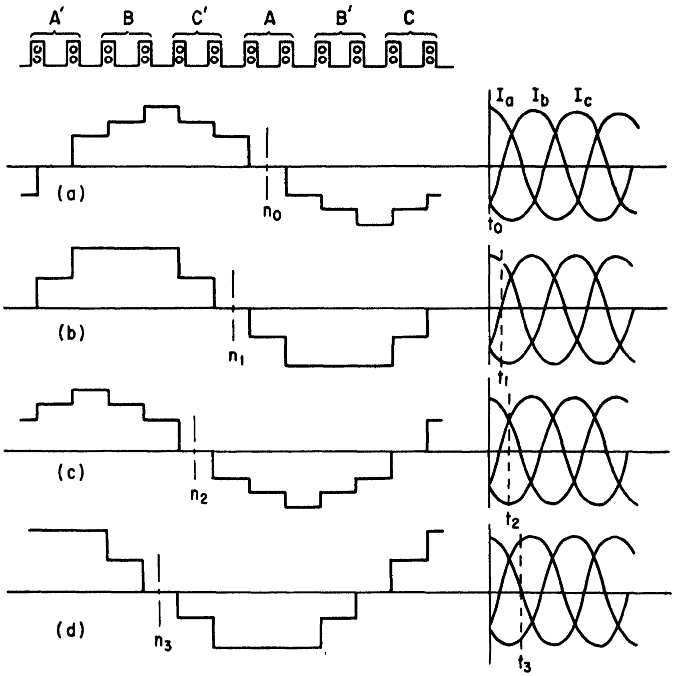

FIG. 11-8. Balanced Polyphase Currents Flowing in an Alternator Winding Produce a Magnetomotive Force That Rotates around the Armature but Is Essentially Constant in Magnitude.

When the second slot of C' is met, a second increase of one unit is obtained. When the first slot of B is reached, it is observed that with negative current the mmf decreases one unit since the B current is one-half magnitude. The second slot produces a second unit decrease in mmf. When the first slot of A' is reached, the mmf drops two units (A' current is negative) to bring it back to zero. When moving to the right of the neutral point, a similar curve in the negative direction will be obtained. It is noted that this step curve is a fair approximation to a sine wave and may be so assumed in elementary analysis.

In diagram (b) the time is 30 time degrees, or $\frac{1}{720}$ sec later, and is shown at t_1 on the diagram at the right. It is noted here that A is 0.86 positive, B is zero, and C is 0.86 negative. A neutral n_1 is assumed to be 30 deg to the left of n_0 and again the mmf analysis is made. The first slot of C' steps the mmf up 1.73 units and the second slot steps it up another 1.73 units. Since B has zero current, it produces no effect, while A' steps the mmf down 1.73 units for each slot and returns the mmf to zero. The negative portion of the curve is again to the right of n_1. The curve (b) is of a different shape from curve (a), but it will be noted that it has approximately the same equivalent sine wave. The maximum value of this sine wave will be below the maximum of (a) and above the maximum of (b), so that it will be slightly less than 4 and somewhat more than 3.46.*

In diagram (c) the neutral point n_2 is chosen 30 deg to the left of n_1 and the time t_2 is 30 deg later than t_1. The current in C is a maximum negative, while both A and B are one-half maximum positive. The analysis of slot mmf's gives a curve that is identical with (a) but displaced from (a) by 60 electrical space degrees, or one-third of the distance between the axis of adjacent poles.

In diagram (d) the time t_3 is 30 deg later than t_2 and 90 deg later than t_0. The current in A is zero, while that in B is 0.86 positive and that in C is 0.86 negative. The neutral is assumed at n_3, which is 30 electrical space degrees to the left of n_2 and the form of the curve is similar to the curve of (b).

The analysis of the change in wave form is beyond the scope of this text. The objective of the analysis here presented is to demonstrate how currents that are changing in magnitude in coils that are fixed in position in the machine can produce a magnetomotive force that is essentially constant in magnitude, but which is shifting in space at the same speed as the field poles of the alternator. These conclusions are more concisely and accurately given in the following statement: *When balanced three-phase currents are flowing in the armature of an alternator (or an induction motor), a magnetomotive force is produced that is fixed in magnitude and rotates around the armature at synchronous speed.*

The above is a general statement and may be qualified to

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* In most commercial machines, as explained previously, the coils have slightly less than full pitch, which gives an even closer approximation to a sine wave.

add that as long as the power factor is constant, the position of the armature-reaction magnetomotive force is fixed with respect to the field poles of an alternator. As discussed previously, the armature reaction with unity power factor causes the resultant flux to reach a maximum after the pole axis has passed a specified point on the armature. When the power factor of the load current is lagging, a component of the magnetomotive force opposes the main field flux. When the current leads the voltage by an angle that is greater than the angle by which the resultant flux lags the pole axis, a component of the armature magnetomotive force aids the main field flux.

Synchronous reactance

In developing the concept of synchronous reactance, which is necessary to an adequate understanding of the performance of synchronous motors as well as generators, it is desirable to first review the effect of inserting a reactance coil between an a-c voltage source and the load, as shown in Fig. 11-9. The effect of this coil can then be compared to the effect of the armature reaction in an alternator.

When a current I flows in a reactance coil, there must exist a voltage IX_L that leads the current by 90 deg, as shown in Fig.

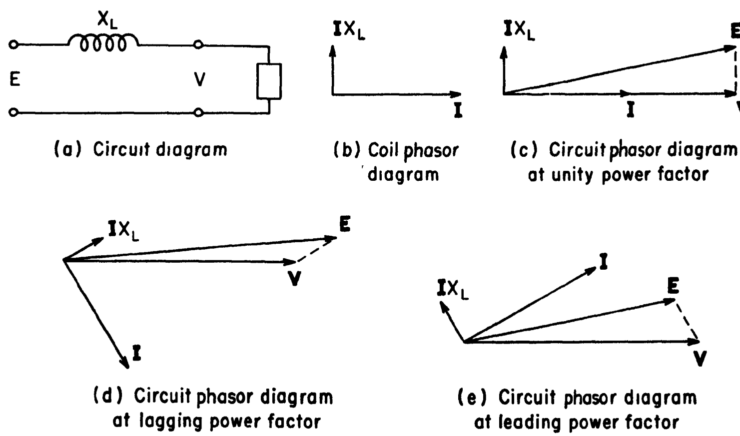

FIG. 11-9. The Variation in Source Voltage Necessary to Maintain a Constant Load Voltage when an Inductor Is between the Source and the Load.

11-9b. If the load current is in phase with the load voltage V, then the source voltage must be advanced in phase, as shown in Fig. 11-9c.

It was learned in the study of the armature reaction that current in phase with the load voltage caused the terminal voltage to lag behind the voltage produced by flux of the field alone, but was changed only slightly in magnitude. If the voltage developed by the field flux (the no-load voltage) is considered as a generated, or source, voltage, then the armature reaction produces an effect similar to that of an inductance coil, that is, the armature reaction causes the terminal voltage to lag behind the generated or source voltage in time phase.

Referring again to Fig. 11-9d, when the current lags behind the load voltage V, it is necessary to increase the source voltage in order that the IX_L drop be provided. In the study of armature reaction it was found that it opposed the field mmf when the current lagged behind the terminal voltage, and it would, of course, be necessary to increase the field current in order to maintain the same terminal voltage. This again is similar to the action of an inductance coil.

When the load current leads the voltage V, then the voltage IX_L required to overcome the coil reactance swings around so that a smaller source voltage E is adequate to maintain the constant voltage V. This action is similar to the armature reaction of the alternator, which aids the magnetomotive force of the field, thus requiring a smaller field current in order to maintain the constant terminal voltage.

Since the armature reaction produces an effect similar to that of a reactance, it is usually included in the analysis of alternators as a portion of an equivalent reactance, known as the *synchronous reactance.* The windings of the different phases have in addition to the effect of armature reaction a large amount of true inductive or leakage reactance, and this constitutes the remaining portion of the synchronous reactance.

This leakage reactance is caused by flux across the slots of the armature and around the end connections of the armature windings. In fact, it includes any flux caused by the current in the armature windings that does not cross the air gap. The computation of this flux is beyond the scope of this text, but the reactance caused by it is large in comparison to the resistance of the windings.

Since the magnitude of the synchronous reactance drop is usually about 0.6 to 0.8 of rated voltage when rated current is flowing, a large change of voltage would occur with variations

of load if constant field current were maintained. This change of voltage can be calculated from the phasor diagram of the alternator, as shown in Fig. 11-10.

The armature resistance is small, and hence the numerical difference between the *synchronous impedance* and *synchronous reactance* is usually less than the accuracy of the original determination. The synchronous reactance is therefore assumed as equal to the synchronous impedance.

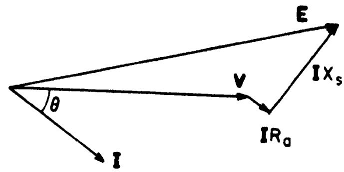

Fig. 11-10. Phasor Diagram of an Alternator with Lagging Load.

Alternator voltage control. Although alternator regulation has some interest to the engineer in comparing the characteristics of machines, it is seldom used in operation. The usual plan of operation is to change the field current as the load changes to maintain the generator terminal voltage constant. This may be done manually, but requires the constant attention of an operator. Usually it is done automatically by a voltage regulator.

Automatic voltage regulators take many forms depending upon the rapidity with which the field current must be made to respond and upon the development of the art at the time of installation of the unit. The most simple arrangement from a theoretical point of view is a motor-driven field rheostat in the alternator field. The motor is controlled by a contact-making voltmeter that causes an adjustment to be made if the voltage becomes either higher or lower than the predetermined limits.

It is quite usual for small d-c generators to be built on the same shaft with large alternators. These d-c generators supply the excitation of the alternator field and are called *exciters*. The alternator field current may be controlled by controlling the field current of the exciter by quick-acting rheostats. This exciter field current is so much less than the alternator field current that it is usually done in this way.

On large alternators the field current of the exciter is, in turn, controlled by another exciter. Such an exciter is called a *pilot exciter*. In the pilot exciter very small changes in its field current will cause very large changes in the field current of the alternator. Such an arrangement is a form of electromagnetic amplifier.

Efficiency and losses

Alternators have efficiencies comparable in small sizes with other electrical machines of the same capacity. The losses may be divided into groups, depending upon whether they are constant or variable. The constant losses are friction and windage (including brush friction and ventilation), and iron losses. The field losses are dependent upon the magnitude and power factor of the load and include the losses in the field rheostat as well as in the field itself. Where the field is supplied by an exciter, the exciter losses are also included. The armature losses include the true I^2R losses in the windings and the losses resulting from eddy currents or nonuniform distribution of current in the copper and additional core losses because of armature current. These latter losses are often called *stray-load* losses. They may be included in the I^2R losses if the R is increased to an effective resistance.

Efficiencies range from 90 per cent for 100-kw units to 97 per cent for turbine-driven alternators of 25,000-kw capacity. With the large high-speed alternators the windage losses are so large a proportion of the total that it has been advantageous to enclose them and use hydrogen for cooling. With hydrogen the efficiencies have gone as high as 98.5 per cent.

CHAPTER 12

Alternating-Current Motors

Rotating field of a polyphase induction motor

Most electric power utilization occurs either in lights, in heating for industrial or domestic purposes, or in the development of mechanical energy. The greatest usefulness, from an industrial point of view, is in supplying the mechanical energy to turn the wheels of industry. Although d-c motors, described in Chap. 6, are used for many special applications, by far the most common type of industrial motor is the squirrel-cage induction motor. This motor is unusually free from maintenance, many such motors operating for years with no more attention than to see that the bearings are properly lubricated and the windings periodically cleaned.

In an induction motor the rotor conductors have current produced in them by the inductive action of stator currents. No direct electrical connection to the rotor is provided. The rotor of the induction motor acts, therefore, in a manner very similar to the secondary of a transformer in so far as the rotor current is concerned.

The fundamental motor principle of force on conductors carrying current and being located in a strong magnetic field still is the basis of operation. The manner of obtaining this current, however, produces major variations in the operating characteristics of the motor. Since these are important to the use of the motor in industrial situations, it is desirable to study them.

The stator winding of an induction motor is essentially the same as the stator of an alternator described in Chap. 11. The coils are wound in the same way, are fitted into slots in the laminated iron structure, and are connected similarly. The connection may be either Y or Δ and these connections have the same advantages as in the alternator. The windings of the three phases are identical and so produce a symmetrical winding. A balanced three-phase voltage impressed on such a wind-

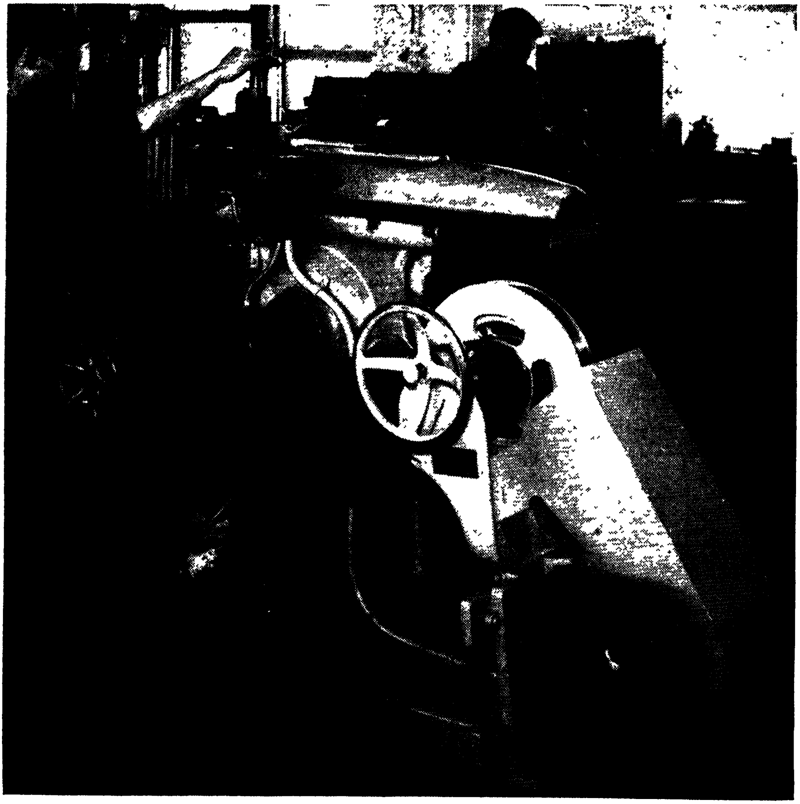

FIG. 12-1. An Induction Motor Driving a Shaper in a Machine Shop.

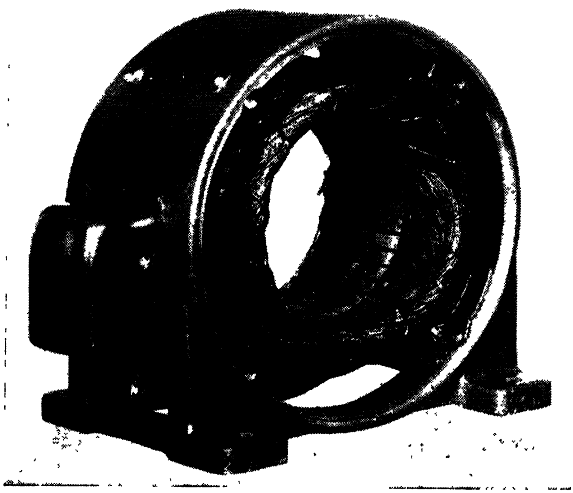

FIG. 12-2. Typical Induction Motor Stator with Balanced Polyphase Winding

ing will give balanced currents in the windings. The currents will therefore be of equal magnitude and will be 120 deg apart in time phase.

Such balanced currents were found by the analysis of armature reaction in Chap. 11 to give a magnetomotive force that

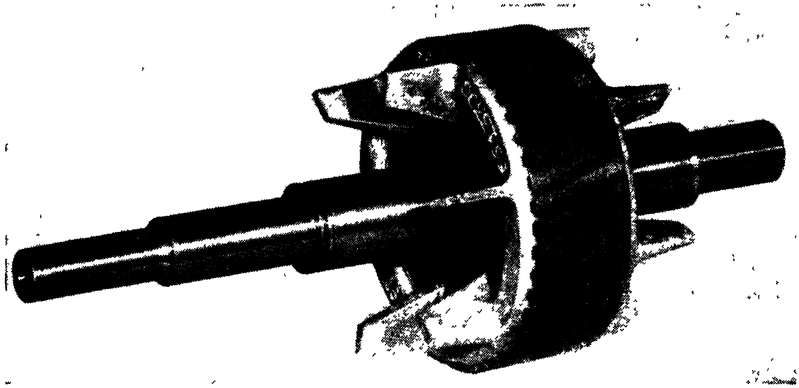

FIG. 12-3a. Squirrel-Cage Rotor for Induction Motor.

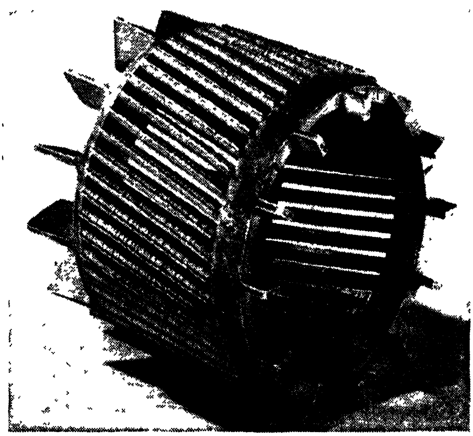

FIG. 12-3b. Squirrel-Cage Winding for Induction Motor. This element was originally cast into a laminated steel core, which was then dissolved away.

rotates around the periphery of the armature at synchronous speed. For instance, for a two-pole winding the speed would be 3600 rpm, for a four-pole winding the speed would be 1800, and for a six-pole winding the speed would be 1200.

Rotor construction of an induction motor. The first requirement of the rotor of the induction motor is that it must provide

a magnetic path of low reluctance for the rotating flux of the stator. This is provided by a stack of sheet-steel punchings, such as are shown in Fig. 12-10, which are riveted together and mounted on the rotor shaft. This magnetic circuit is quite similar, in fact, to the magnetic portion of the rotor of the d-c motor. Bars of copper (or other metal) are inserted into the rotor slots (without insulation), and these bars are brazed to copper or brass rings at both ends.*

A later development in construction of these rotors is the use of aluminum in a die-casting process to produce this conducting structure. Such a rotor structure is shown in Fig. 12-3. Figure 12-3a shows the completed rotor mounted on the motor shaft, while (b) of the diagram shows the cast unit after the iron has been dissolved by acid.

Insulation is not required because of the very low voltages involved. Furthermore, current flowing in the iron will also produce torque, so that it is really not too important that the rotor be laminated.

The air gap is made as small as possible; however, reliable mechanical clearance must be maintained. The magnitude of the air gap varies from about 0.01 in. for motors of a fraction of a horse power to about 0.02 in. for 10 to 25-hp motors. This allows for some bearing wear and a small amount of air flow in the gap.

Qualitative analysis of operation

As has been demonstrated, a polyphase voltage, when impressed on the stator windings, will produce a flux that rotates around the air gap. Since the resistance of these stator windings is relatively low, the current is limited by the reactance. The reactance voltage (for low resistance) was shown in Chap. 7 to be equal and opposite to the impressed voltage. It follows that the rotating flux for the induction motor must have the same magnitude as though the stator were a synchronous generator.†

In the synchronous generator the mmf is supplied by the d-c field. In the induction motor the magnetomotive force is provided by balanced currents flowing in the stator windings.

* This forms a copper structure very similar to some small cages that were formerly popular for exercising captive squirrels, and so the windings are called squirrel-cage windings.

† This neglects leakage reactance since it is not part of the air-gap flux.

These currents lag behind the impressed voltage by almost 90 deg in time phase and are called the exciting currents. The magnitude of these exciting currents depends upon the reluctance of the flux path. Since a major portion of this reluctance is in the air gap, it is kept as small as possible. The lower magnitude of the exciting current improves the power factor of the motor when operating at full load.

The difference between synchronous and actual speed is important in induction-motor performance and has been given a descriptive name. It is called slip and is defined as:

$$\text{slip} = \frac{\text{synchronous speed} - \text{actual speed}}{\text{synchronous speed}}.$$

In a four-pole motor, having a synchronous speed of 1800 rpm and a full-load speed of 1740 rpm, the slip would be

$$\text{slip} = \frac{1800 - 1740}{1800} = \frac{60}{1800} = 0.03 \qquad \text{(or 3 per cent).}$$

When the rotor is stationary, as at starting, the rotating field cuts the rotor conductors at the same rate and frequency as the stator conductors. The rotor frequency at standstill is therefore the same as the power-line frequency. As the rotor gains speed, the relative motion is reduced. As synchronous speed is approached, the frequency of the voltage generated in the rotor conductors approaches zero.

The frequency is therefore directly proportional to slip, so

$$f_r = sf$$

where f_r is the rotor frequency, f is the line frequency, and s is the slip. The rate at which the rotor conductors cut the air-gap flux and the magnitude of the resultant rotor voltage are proportional to the slip. Since the rotor conductors are short-circuited by the end rings, currents will flow in the same direction, and for small values of slip in approximately the same phase, as the voltages that are generated.

Exercise 12-1. A 20-hp 220-v three-phase 60-cycle squirrel-cage induction motor has a rated speed of 875 rpm. (a) How many poles does it have? (b) What is the percentage of full-load slip? (c) How far (in mechanical degrees) will the flux advance in one cycle of applied voltage?

The effect of the rotating air-gap flux on the rotor will be analyzed by first assuming that the rotor is turning at synchro-

nous speed. Under this condition the flux is stationary with respect to the rotor and no voltage is generated in the rotor conductors. When the rotor slows down, a relative motion between the air-gap flux and rotor conductors develops, thus producing a voltage in these conductors. It was shown in Chap. 5 (Fig. 5-3) that the force produced by current flowing in the same direction as the generated voltage is in such a direction as to oppose the relative motion of the conductor and the magnetic field. In the induction motor the relative motion is reduced

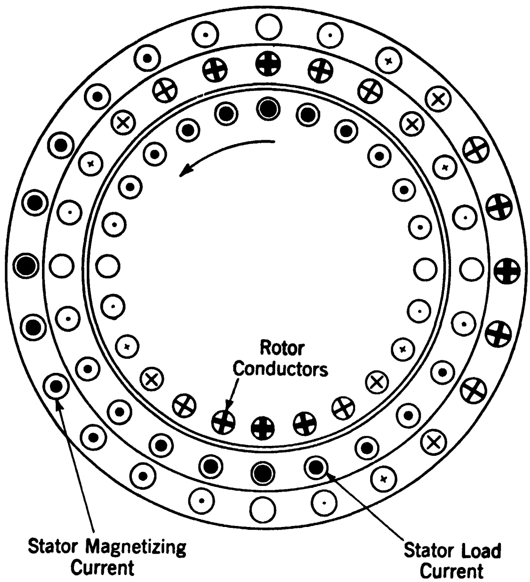

FIG. 12-4. Components of Current in an Induction Motor.

when the rotor approaches more nearly synchronous speed. As the rotor slows down, therefore, it develops additional torque. This torque tends to bring it back to synchronous speed.

The distribution of rotor currents around the periphery of the rotor will follow the distribution of air-gap flux which produces them. Since the flux varies sinusoidally around the air gap, the current distribution will also be so distributed. In Fig. 12-4 this is shown diagrammatically. The flux is shown vertically, produced by a stator magnetizing-current distribution as indicated on the outer ring. The flux, which is sinusoidally distributed, will produce voltages in proportion to the flux. The currents, where limited by resistance only, will be directly proportional to the voltages and so will be distributed as shown

in the inner, or rotor, ring of conductors. To say this in another way, there is a space wave of current density around the periphery of the rotor.* When the maximum of the current space wave is in phase with the flux maximum, the torque distribution around the periphery will be a double-frequency sine wave with an average value equal to one-half of the maximum. The torque under these conditions is proportional to the product of the flux and the current.

$$T = K\phi I$$

When the slip is relatively large, the frequency of the voltage in the rotor bars is sufficient that the reactance cannot be neglected and it is necessary to analyze the motor action on an a-c basis. The rotor current is thus equal to

$$I_r = \frac{E_r}{\sqrt{R_r^2 + X_r^2}}$$

where the values are rotor currents, voltages, resistances, and reactances. *The current lags behind the voltage, and since the voltage is in phase with the flux, there will be a phase angle between the space distribution of the flux and of the current that is equal to the time-phase angle between current and voltage.* The torque then becomes not only proportional to the magnitude of the flux and current but also to the cosine of the angle of phase difference.

$$T = K\phi I_r \cos \theta.$$

Quantitative analysis of current and torque. If the rotor voltage at standstill is designated as E'_r, then the actual rotor voltage at a slip s is

$$E_r = E'_r s.$$

Similarly, if the rotor reactance at standstill is designated as X'_r, then the rotor reactance at any slip s is

$$X_r = X'_r s.$$

Since the magnitude of the current is desired in the torque equa-

* This rotor current reacts on the stator in the same way that the secondary of a transformer reacts on the primary. That is, a primary, or stator, current must flow to neutralize the rotor magnetomotive force, and this is shown in the middle ring of Fig. 12-4. With the rotor mmf neutralized, the air-gap flux remains constant in magnitude.

tion, this is obtained by

$$I_r = \frac{E_r}{\sqrt{R_r^2 + X_r^2}} = \frac{E'_r s}{\sqrt{R_r^2 + (X'_r s)^2}}.$$

The cos θ may be specified as

$$\cos\theta = \frac{R_r}{\sqrt{R_r^2 + (X'_r s)^2}}.$$

The torque may now be written as

$$\begin{aligned} T &= K\phi I_r \cos\theta \\ &= K\phi \frac{E'_r s}{\sqrt{R_r^2 + (X'_r s)^2}} \cdot \frac{R_r}{\sqrt{R_r^2 + (X'_r s)^2}} \\ &= K\phi E'_r R_r \frac{s}{R_r^2 + (X'_r s)^2}. \end{aligned}$$

In order to determine the slip at which maximum torque occurs, the expression for torque must be differentiated and set equal to zero

$$\frac{dT}{ds} = K\phi E'_r R_r \frac{(R_r^2 + X'^2_r s^2) - 2X'^2_r s^2}{(R_r^2 + X'^2_r s^2)^2} = 0.$$

Hence

$$R_r^2 - X'^2_r s^2 = 0,$$

$$R_r = X'_r s.$$

In other words, the maximum torque will occur when the resistance and reactance are equal in magnitude. In most commercial motors this condition exists at a slip of about 0.15, or 15 per cent.

If now the value $X'_r s$ is substituted for its equivalent R_r in the equation for torque, it is found that the torque for maximum conditions is independent of rotor resistance.

Operating characteristics

Induction-motor operating characteristics are usually shown by curves of torque *v.* speed and current *v.* speed with constant impressed voltage.

The condition of maximum torque is a critical point in the analysis; and since it usually comes at about 0.15 slip, the curves naturally fall into two different sections, one of which is at small values of slip (under 0.10) and the other at large values of slip (over $\frac{1}{3}$). At low values of slip the reactance has little effect.

since it is both smaller than, and in quadrature with, the resistance. The rotor current then becomes

$$I_r = \frac{E'_r s}{\sqrt{R_r^2 + (X'_r s)^2}} \approx \frac{E'_r}{R_r} s \qquad \text{where } s < \tfrac{1}{10}.$$

The torque then becomes

$$T = K\phi E'_r R_r \frac{s}{R_r^2 + (X'_r s)^2} \approx \frac{K\phi E'_r}{R_r} s = K's.$$

These approximate equations show that the rotor current and the torque both vary directly with the slip at small values of slip. This is shown by the straight portion of the curves of Fig. 12-5, where the torque and current both rise rapidly as the speed drops below synchronous. This is the normal operating range of the motor.

At high values of slip the resistance is a negligible factor in the impedance, so the current and torque equations become

$$I_r = \frac{E'_r s}{\sqrt{R_r^2 + (X'_r s)^2}} \approx \frac{E'_r}{X'_r} \qquad \text{where } s > \tfrac{1}{3}.$$

$$T = K\phi E'_r R_r \frac{s}{R_r^2 + (X'_r s)^2} \approx \frac{K\phi E'_r R_r}{(X'_r)^2} \cdot \frac{1}{s}$$

$$= K'' \frac{1}{s} \qquad \text{where } s > \tfrac{1}{3}.$$

These approximate equations show that the current tends to become constant for speeds below $\frac{2}{3}$ of synchronous. The torque, on the other hand, varies inversely with slip and so tends to increase as the slip decreases. These relations are shown in the curves of Fig. 12-5.

These motor characteristics may be interpreted physically on the basis that for low values of slip the power factor is relatively high, and so the torque is almost directly proportional to current since the flux is assumed constant. At high values of slip the current is approximately constant, and so the angle of space-phase difference between flux and current becomes the controlling factor. An inspection of the approximate equation for torque at high slip shows that the rotor resistance is included as one of the factors of K''. If, therefore, the rotor resistance is increased by some means, the torque, at low speeds, will also be increased. This increase of torque can also be attributed to

an effective decrease of the phase difference between the flux and the current.

For values of slip between $\frac{1}{10}$ and $\frac{1}{3}$ the above approximations do not hold, and it becomes necessary to use the exact equations. The maximum torque condition occurs in this range, and the two approximate curve sections are joined by a transition curve.

The effect of voltage variation on motor operation is important and can be determined from the above equations.* The

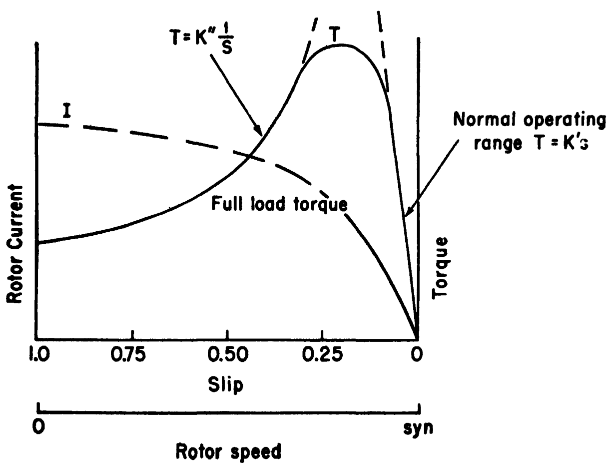

FIG. 12-5. Variation of Induction Motor Torque with Slip.

flux is reduced in direct proportion to the impressed voltage. Since the *secondary current* is dependent upon the flux, this *will also be reduced in direct proportion to the voltage.* The *torque* at any slip, therefore, *will vary as the square of the voltage.*

Rotor reaction on stator. In Fig. 12-4 the flux was assumed to be vertical and caused by an exciting current that was concentrated in the right and left sides of the winding. This current lags 90 deg in time phase behind the voltages impressed upon the windings and leads the internally generated voltage by 90 deg; it is similar to the exciting current of a transformer. The rotor currents produce magnetomotive forces that tend to change the flux. As soon as any change of flux occurs, however,

* These equations are only approximate since the air-gap flux is assumed as proportional to the impressed voltage. There is considerable leakage reactance in the stator windings, and this must be considered for highly accurate results.

the internal generated voltage is no longer equal and opposite to that of the impressed voltage, and so an additional stator current flows to neutralize the rotor current. This neutralization current for normal loads will be approximately in phase with the voltage generated, as explained above.

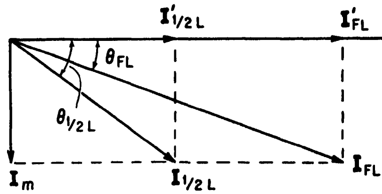

FIG. 12-6. Variation of Power Factor of an Induction Motor with Load.

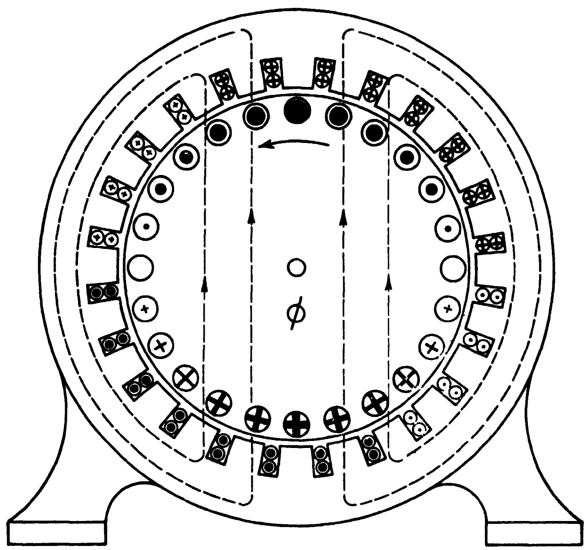

FIG. 12-7. Current Distribution in an Actual Winding of an Induction Motor.

The effect on stator current is shown in Fig. 12-6. The exciting current is from 30 to 50 per cent of full-load current in most motors and is very nearly 90 deg lagging. The load currents, on the other hand, are almost in phase with the voltage, and so the power factor of the motor increases rapidly with load.* It is, therefore, unwise to operate induction motors at

* The manner in which the components of stator current combine to form a single stator current is shown in Fig. 12-7. The current distribution in the stator of this diagram, which is typical of actual construction, is equivalent to the two components in Fig. 12-4.

low loads as it will cause low power factor, which may, in turn, cause high power costs to an industrial establishment.

It was noted above that the induction motor adjusted itself to an increase in load by first slowing down. This increases the rotor voltage, current, and torque. The increase in rotor current causes an mmf that is, in turn, neutralized by the load component of the stator current. This sequence of reaction of rotor on stator is similar to transformer action.

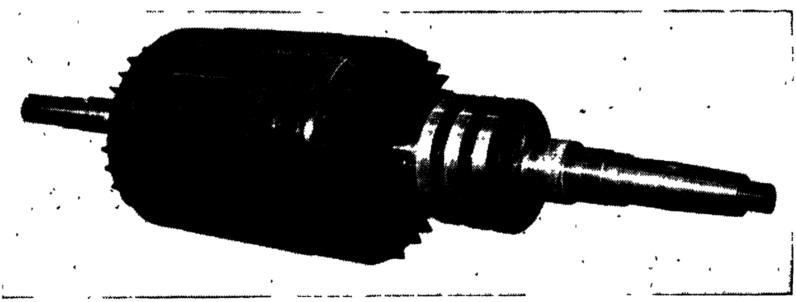

FIG. 12-8. A Wound Rotor for a Variable Speed Induction Motor.

Wound-rotor induction motor

A particular type of induction motor that is used extensively to drive variable-speed centrifugal pumps, fans, hoists, and cranes is known as the wound-rotor induction motor. In this motor the rotor uses a punching similar to that shown in E of Fig. 12-10. A three-phase winding is used with the ends brought out to slip rings as shown in Fig. 12-8. These slip rings are connected to a three-phase resistor, in which the resistance may be gradually reduced. It is thus possible to change the effective rotor resistance over a wide range.

In the previous analysis it was found that the torque of an induction motor is proportional to the rotor current in the operating range. If the rotor resistance is increased to twice its previous value, it will require twice the slip to obtain the same rotor current. This produces a reduction in speed for the same torque. In Fig. 12-9 are shown speed-torque curves for a typical wound-rotor induction motor. The curve that rises most steeply with slip has no external resistance in the circuit and is similar to the torque curve previously studied. The adjacent curve has an added resistance of about one and one-half times that of the rotor, and so increases the slip at rated torque from

4 to 10 per cent. The slope of the torque curves is increased rapidly with increase in rotor resistance.

When starting a load having constant full-load torque, the rotor resistance is gradually decreased so that the operation follows the dotted lines indicated by (a). The stabilized speed for each of these conditions is shown by (b). The stabilized speeds for a load having variable torque (such as a pump) is shown by (c).

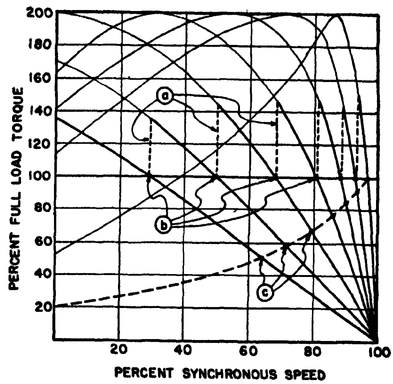

FIG. 12-9. A Seven-Point Control for a Wound-Rotor Induction Motor Showing (a) Starting Steps with Constant Torque Load, (b) Speed Control with Constant Torque Load, and (c) Speed Control with Variable Torque Load.

The rotor efficiency of an induction motor may be shown to be equal to the ratio of actual to synchronous speed. When the speed of the induction motor is reduced by rotor resistance, the efficiency is also reduced. In the case of pumps and blowers the horsepower required drops very rapidly with speed, so a net reduction of power is obtained by increasing the rotor resistance in a wound-rotor motor to reduce the speed, in spite of the lowered efficiency. Wound-rotor motors are started by gradually reducing the rotor resistance from the maximum value. Under these conditions, the starting current of the wound-rotor motor is equivalent (in per cent of full-load values) to the starting torque required by the load. This is a real advantage under some conditions.

The disadvantages of the wound-rotor motor and control are a very considerable increase in cost over the ordinary squirrel-cage motor, the additional maintenance of slip rings, brushes, and controller, and lower efficiency.

Double squirrel-cage induction motors

The chief disadvantages of the low-resistance squirrel-cage rotor are that it draws a high starting current and produces a low starting torque. These objections are overcome by the use of two squirrel-cage windings on the same rotor. One of these is close to the air-gap surface where the reactance is low. In this one the conductor area is small so that the effective resistance

is quite high. The other is embedded deeply in the rotor, so that it has a high reactance. The conductor area, however, is quite large, so that it has low resistance. The rotor lamination for such a double cage is shown in Fig. 12-10C.

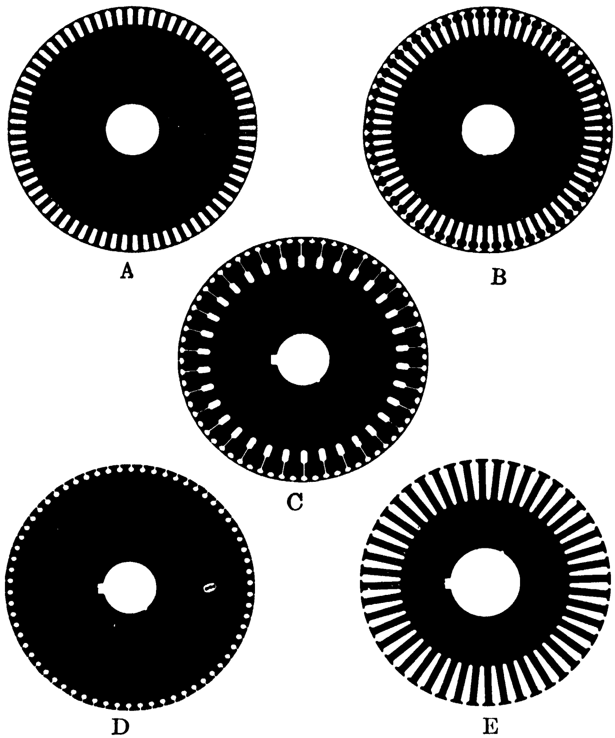

FIG. 12-10. Typical Rotor Lamination for Induction Motors. A—Normal-torque, normal-starting-current squirrel-cage motor. B—Normal-torque, low-starting-current squirrel-cage motor. C—High-torque, low-starting-current squirrel-cage motor. D—High-torque, high-slip squirrel-cage motor. E—Wound-rotor motor.

The action of this double-cage rotor is such that at starting the high reactance of the inner cage prevents a large current flow in it and thus limits the starting current of the motor. The high resistance of the outer cage gives high starting torque. As the rotor gains speed, the frequency of the rotor currents is reduced until the major portion of the flux penetrates the second cage and the equivalent rotor resistance is that of the two cages in parallel. This construction thus gives high starting torque at relatively low starting current and yet provides an efficient, comparatively low-slip motor at normal loads.

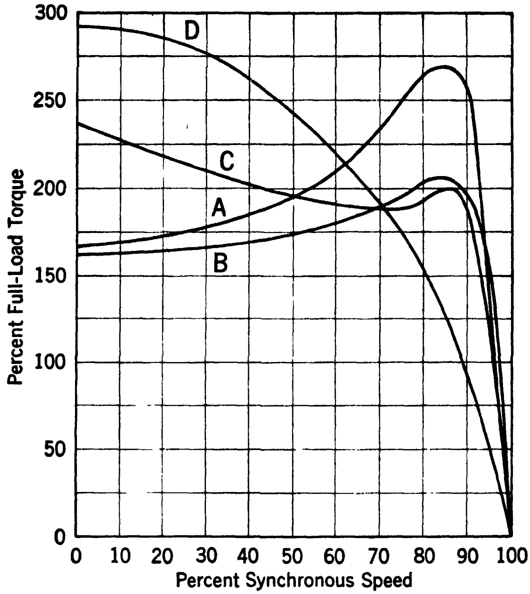

FIG. 12-11. Speed-Torque Curves for Squirrel-Cage Motors. A—Normal torque, normal starting current. B—Normal torque, low starting current. C—High torque, low starting current. D—High torque, high slip.

A similar effect can be obtained by using deep slots in the rotor, as shown in Fig. 12-10B. The effect is not quite so pronounced as in the standard double-cage motor, however.

Standard types of induction motors

It is possible to obtain a wide variety of motor characteristics, using the same stator but with different rotors. Manufacturing costs go up rapidly with the diversity of the product, however, so the electrical manufacturers have agreed to standardize on relatively few types of rotors that will give a satisfactory compromise between the variety of load requirements and cost of production. This compromise limits the rotors to five main types as indicated by the laminations of Fig. 12-10. The characteristic speed-torque and speed-current curves on the first four of these rotors are given in Figs. 12-11 and 12-12.

The speed-torque curve for *A* is typical for a comparatively low-resistance low-reactance rotor. The maximum torque is high and the starting torque is relatively low, while the starting current is high, being 6½ times normal full-load current. Where lower starting current is desired with the same starting torque,

the rotor form shown in *B* is used. This reduces the starting current to less than 5 times normal. The maximum torque is also reduced as well as the power factor.

Where very high starting torque is required with low starting current, the double-cage rotor of *C* is used. It has the disadvantages of slightly higher cost and lower power factor but does have a high operating efficiency.

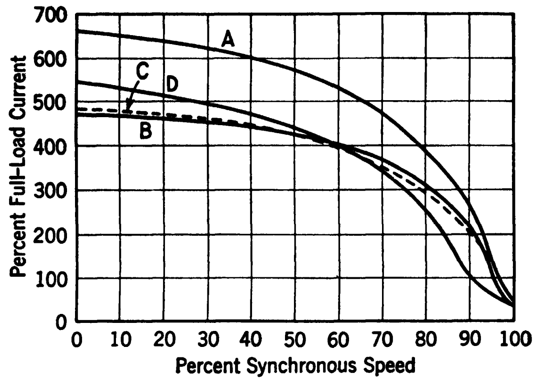

FIG. 12-12. Speed-Current Curves for Squirrel-Cage Motors. A—Normal torque, normal starting current. B—Normal torque, low starting current. C—High torque, low starting current. D—High torque, high slip.

Where very high starting torque is required and operating efficiency is unimportant, or where an appreciable variation in speed with change in load is desired to reduce the strain on the motor, the high-resistance rotor of *D* is used. A typical application for this motor is a punch-press drive with a flywheel to store energy for the punching operation.

The rotor punching marked *E* has large slots to provide room for an insulated three-phase winding. It is used in the wound-rotor motor previously described.

Exercise 12-2. Determine the starting current of a 5-hp 220-volt 1750-rpm normal-torque normal-starting-current three-phase induction motor from the curves of Fig. 12-12. Assume that the motor has a full-load efficiency of 82 per cent and 0.88 power factor.

Exercise 12-3. Determine from Fig. 12-11 and the full-load rated conditions the maximum torque in pound-feet produced by the motor of Exercise 12-2 and the speed at which it occurs.

Exercise 12-4. Repeat Exercises 12-2 and 12-3 for a normal-torque low-starting-current motor.

Exercise 12-5. Repeat Exercises 12-2 and 12-3 for a high-starting-torque low-starting-current motor.

Exercise 12-6. Repeat Exercises 12-2 and 12-3 for a high-starting-torque high-slip motor.

Starting of induction motors

Induction motors are designed so they may be started by closing a switch that connects them directly across the line. This method of starting is the simplest and least expensive, and gets the motor up to speed most rapidly. It is therefore the most common starting procedure for induction motors.

For larger motors, the starting current may place an excessive burden on the power supply line, causing voltage drops that interfere with other equipment. In such instances, some method of limiting the starting current must be used. Two methods are used for squirrel-cage motors. The simplest uses resistors inserted into the stator leads. This reduces the voltage across the stator and thus reduces the stator current. The air-gap flux is reduced in direct proportion to terminal voltage, the rotor current is reduced in direct proportion to the air-gap flux, and since the torque is dependent upon the product of current and flux, it is reduced in proportion to the square of voltage reduction. If the starting current is reduced too much, therefore, the torque will be inadequate to start the load.

The second method of reducing line current has certain advantages over the resistance method but is somewhat more expensive. It also reduces the voltage on the motor, and in this way the limit of voltage reduction is again the torque required to start the load. The voltage reduction is accomplished by the use of an autotransformer. This method has the advantage that the line current is reduced as the square of the voltage reduction rather than as the first power. Where the starting current is limited because of line characteristics, the use of the autotransformer (often called compensator) is usually preferable to resistance.

For "across-the-line starting" the type *B* or *C* motor, shown in Figs. 12-10 to 12, is preferable. The use of this motor, rather than a type *A*, will often make it possible to start across the line where a compensator would be required with a type *A* motor.

Where line current limitations cannot be met by a type *B* or *C* motor, it is *necessary* to use either a starting resistance or a

starting compensator. The amount of starting-current reduction in the case of the compensator depends upon the tap that is used. Normally compensators are supplied with both 65 and 80 per cent taps, which give secondary voltages of that amount. As explained above, the current in the motor is proportional to the voltage, but the current in the line is reduced in proportion to the square of the voltage. Thus the 80 per cent tap gives a line starting current of 64 per cent of full-voltage starting current, and the 65 per cent tap gives 42 per cent of the full-voltage current. The starting torques will also be reduced in proportion to the square of the voltage. The selection of the tap must, therefore, be a compromise between the requirements of starting torque and starting current.

It is well to remember that the entire curve of torque *vs.* speed is reduced in proportion to the square of voltage reduction, whether this be caused by a compensator, a resistor, or low line voltage. Thus, *the available torque of an induction motor at any speed varies as the square of the line voltage.*

In order to demonstrate how the characteristic curves* can be used to select a specific motor and starting compensator and to determine whether they will perform satisfactorily, three illustrative problems will be solved.

Example 1. A load requiring 40 hp at 1750 rpm is to be supplied by a three-phase induction motor from a 220-v three-phase line. The public utility company specifies that the starting current shall not exceed 350 amp. Determine the proper type of motor and starting equipment if the starting torque of the load is 25 per cent of the full-load torque and increases gradually with speed to 100 per cent of full-load torque at rated speed.

Solution: (1) Assume that a 40-hp Class *A* motor can be used. Also assume that the efficiency is 88 per cent and the power factor is 0.90, which are quite probable values for this type of motor.

(2) Compute full-load current.

$$\text{Power input} = \frac{746 \times 40}{0.88} = 34.0 \text{ kw.}$$

$$I_{\text{full load}} = \frac{34.0 \times 1000}{0.90 \times 220 \times \sqrt{3}} = 99.$$

---

* In recent years the increased capacity of public utility power lines usually makes it possible to start Class *B* motors by connecting them directly across the line. This has led to such a general preference for Class *B* motors that they have become the standard or general-purpose induction motor.

(3) Starting current from Fig. 14-12.

I_s @ 100 per cent voltage = 99 × 6.50 = 640 amp,

I_s @ 80 per cent voltage (with transformer starter)
$= 640 \times 0.8^2 = 410$ amp,

I_s @ 65 per cent voltage (with transformer starter)
$= 640 \times 0.65^2 = 275$ amp.

In order for the current to be within the limit specified, it is necessary to use a compensator with the 65 per cent tap.

(4) Determine the starting torque with the 65 per cent tap. From Fig. 12-11, full-voltage starting torque is approximately 160 per cent.

$$T_{(65\text{ per cent})} = 160 \times 0.65^2 = 67 \text{ per cent of rated full-load torque.}$$

This is adequate to meet the starting requirements of the load at zero speed.

(5) Determine the current inrush when the compensator is disconnected from the circuit and the motor is thrown across the line.

In order to make this determination, it is necessary to analyze the speed *vs.* torque curves of the motor and of the load to determine the

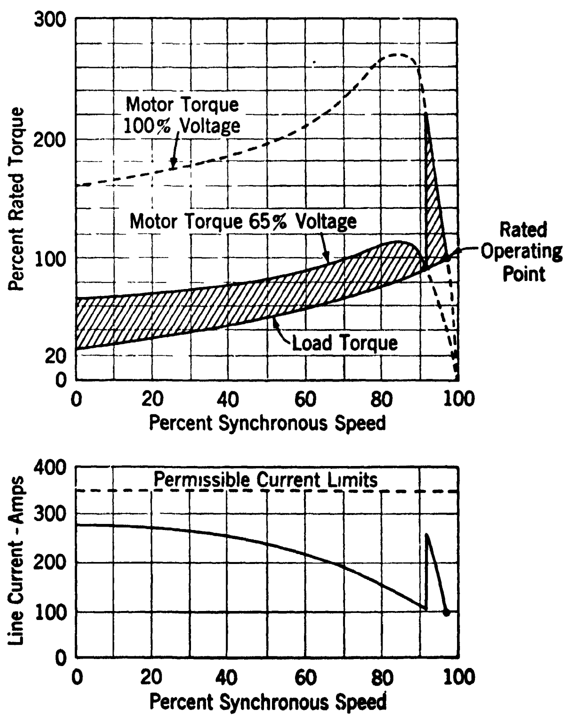

FIG. 12-13. Speed-Torque and Speed-Current Curves for Example 1.

speed that will be obtained with the reduced voltage. These curves and the current *vs.* speed curves are plotted in Fig. 12-13. These are taken from the curves of Figs. 12-11 and 12-12 and are corrected for the effects of reduced voltage of the compensator, as explained in the previous paragraph.

The shaded area indicates the torque available for acceleration. Accelerating torque is available with 65 per cent voltage until 92 per cent of synchronous speed is reached. When the motor is thrown across the line at this speed, the current will increase to 260 amp and then rapidly decrease to rated value. This current inrush does not exceed the limits placed by the power company.

(6) Both Class *B* and *C* motors draw about 490 per cent of rated current at starting, so their full-voltage starting current will be approximately 485 amp. Since neither of these motors can be thrown directly across the line, there would be no advantage to using them. The efficiency, power factor, and pull-out torque (the maximum torque before the motor will stall) are all better for the Class *A* motor than for the others, and therefore it would be preferred. The Class *B* motor would also be satisfactory since its starting current would be lower than and the starting would be equal to the class *A* motor.

Example 2. A load requiring 40 hp at 1700 rpm is to be supplied by a three-phase induction motor from a 220-v line. The public utility company has set a limit of 500 amp for starting current. The starting torque of the load is equal to rated full-load torque and remains uniform with speed. Determine the type of motor and starting equipment to be used.

Solution: It is observed that the data conform with those of Example 1, except that 500-amp starting current is permissible and the load torque is uniform with speed.

(1) From Example 1, the current of the Class *A* motor when starting across the line is excessive, but the current using the 80 per cent compensator is permissible. The starting torque under these conditions is

$$160 \times 0.80^2 = 103 \text{ per cent.}$$

The Class *A* motor and compensator meet the requirements but provide for no reserve of torque.

(2) From the curves of Fig. 12-12 the class *B* motor will have a starting current of $99 \times 4.9 = 485$ amp, which is permissible. The starting torque would be 160 per cent of rated torque.

(3) Since the Class *B* motor would (a) give a better safety factor on starting torque, (b) be lower in first cost because a compensator would not be required, and (c) be simpler in its operation and maintenance, it would be the preferred choice.

Example 3. A load requiring 40 hp at 1700 rpm is to be supplied by a three-phase induction motor from a 220-v line. The starting

current is limited to 350 amp. The starting torque of the load is specified as 125 per cent of rated full-load torque but drops to rated torque at 50 per cent of synchronous speed. Determine the type of motor and starting equipment to be used.

Solution: This is the same as Examples 1 and 2, except for much more severe starting limitations.

(1) None of the motors will meet the current limitations with across-the-line start.

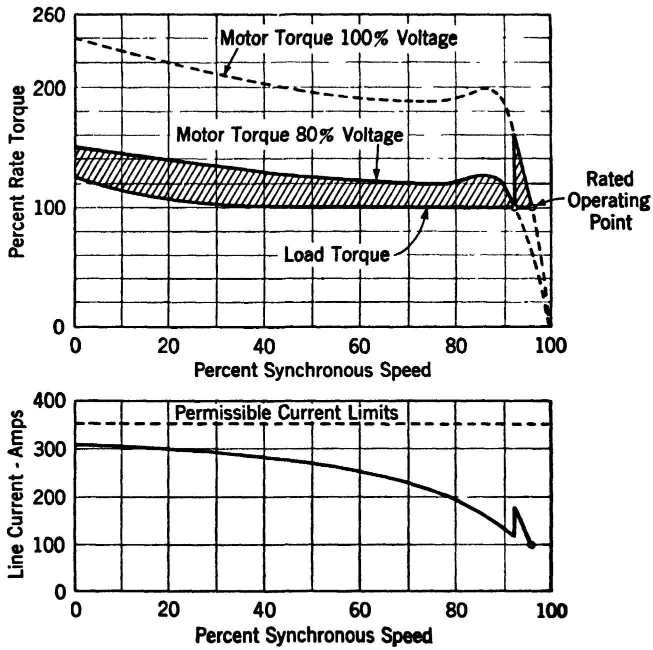

Fig. 12-14. Speed-Torque and Speed-Current Curves for Example 3.

(2) The current and torque with an 80 per cent compensator for the different types of motors, as determined from curves of Figs. 12-11 and 12-12 corrected for compensator ratio, are shown below:

	Class A	*Class B*	*Class C*
Current......................	410	310	305
Torque......................	108	105	144

Since the Class *C* motor alone will meet the starting requirements, it will be necessary to use this motor with an 80 per cent compensator.

(3) Since the second critical current limitation occurs when the motor is thrown across the line, the torque and current *v.* speed diagrams are shown in Fig. 12-14. The torque available for acceleration is low, but it meets the specified requirements. The current when the

motor is thrown across the line is found to be within limits, and the Class *C* motor with 80 per cent compensator is selected.

The above three illustrations not only indicate the method of selecting a motor from the standard types of induction motors, but, in addition, they indicate the reasons for having these various types. The magnitude of starting current that is permissible cannot be arbitrarily specified, since it depends upon many factors. The public utility company will usually specify these limits for any particular situation, and their engineers should be consulted if there is any doubt as to the proper equipment to use. If the power is generated within the industrial plant, the limitations to the starting current must be determined by the best judgment of the plant engineer. Voltage variation permissible on other units on the same power circuit will be one of the important limiting factors.

Exercise 12-7. What is the starting torque (in pound-feet) for a 25-hp normal-torque low-starting-current motor having a rated speed of 1750 rpm?

Exercise 12-8. What is the available starting torque (in pound-feet) for a 15-hp high-torque low-starting-current 1160-rpm motor?

Exercise 12-9. What is the starting torque for the motor of Exercise 12-7 if the line voltage drop (at starting) is 10 per cent of rated voltage?

Exercise 12-10. What is the starting torque and line starting current, in percentage of rated current, for the motor of Exercise 12-7 when an 80 per cent compensator is used for the starting operation?

Exercise 12-11. What is the starting torque and line current, in per cent of rated current, for the motor in Exercise 12-8 when a compensator with 65 per cent tap is used to reduce starting current?

Control equipment for induction motors

Induction motors are usually started and stopped by magnetic contactors, which are controlled by push buttons placed at convenient locations. Figure 12-15 shows such a magnetic contactor for a 5-hp motor. The starting operation is performed by connecting the motor directly across the power line, as illustrated in Example 2 above.

Such contactors are provided with overload protection, so that the motor is disconnected from the line in the case of excessive current values. This overload protection is usually arranged to have a circuit-opening characteristic quite similar

to the heating characteristic of the motor, as shown in Fig. 12-16. This is accomplished in the contactor shown in Fig. 12-15 by conducting the line current through self-supported coils that are wound close to, but insulated from, a bimetallic strip that trips the circuit when it becomes hot. Thus, when an overload occurs on one or more line wires, the coils (which can be seen on the lower right and left sides of the switch) heat the bimetallic strip, which opens the control circuit and

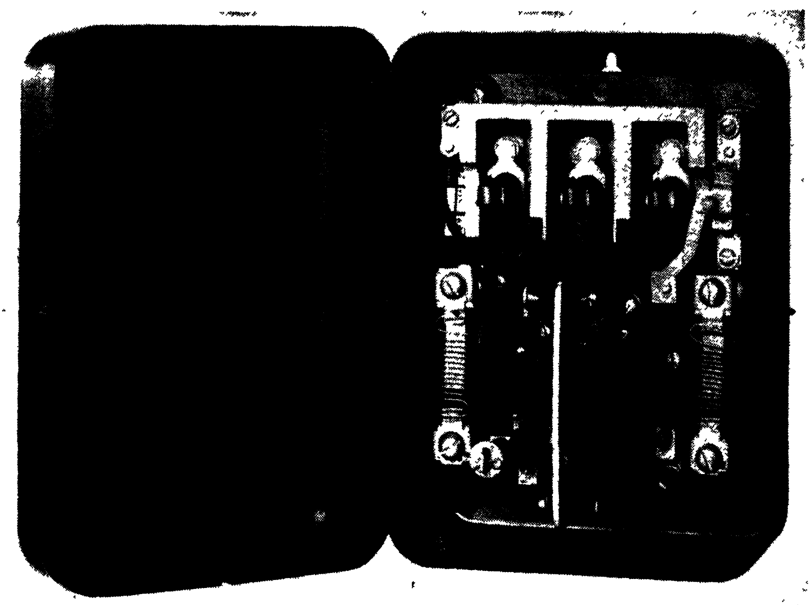

FIG. 12-15. A Three-Pole Magnetic Switch for Starting Induction Motors across the Line.

trips the switch. The coils are adjusted so that the switch will trip before the motor becomes dangerously hot.

Where reduced-voltage starting is necessary to limit the starting current, compensators are generally used, as indicated in Examples 1 and 3 above. Where the starting-current limitations are only slightly exceeded by across-the-line starting, or where economy is more important than minimum starting current, a set of resistances can be placed in the line, as has been previously explained, to limit the starting current. In the case of push-button control, two magnetically operated contactors are required. The switch that short-circuits the resistance

element is delayed by a timer, which permits the motor to come up to speed before operation. Remote-controlled starters using autotransformers also use two magnetically operated contactors with time-delay operation. Thermal overload relays are provided on both the resistance and compensator type of starters. Low-voltage releases are also provided.

Controllers for wound-rotor induction motors are designed to meet special requirements. These should be discussed with

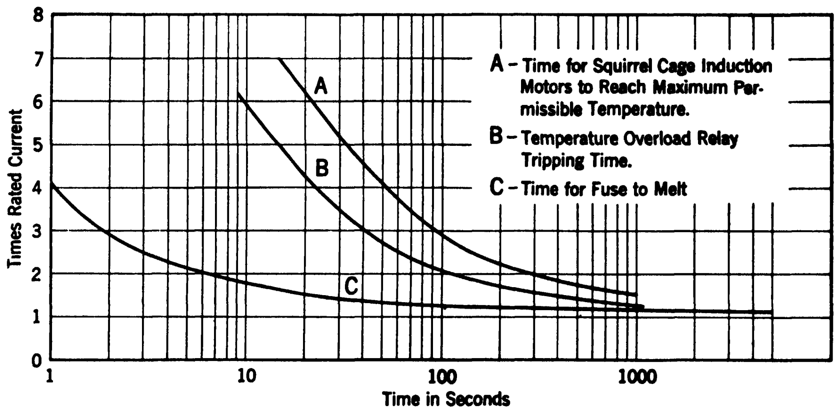

FIG. 12-16. Typical Characteristics of Motors and Protective Devices.

the sales engineer of one or more manufacturing companies before ordering.

Exercise 12-12. A motor is needed to drive a conveyor belt that takes gravel from the washing plant to the concrete mixing plant on a large construction project. The power supply is three-phase and 440 v, and starting currents are limited to 100 amp. The normal load requires a torque of 50 lb-ft at a speed of 1750 rpm. The starting torque may go as high as 100 lb-ft. From manufacturers' catalogues select the correct motor and starting equipment to meet the above requirements.

Exercise 12-13. A centrifugal pump in a chemical plant is required to supply 50,000 gallons per hour of an explosive distillate against a head of 50 lb gage. The pump efficiency is 55 per cent at full load. Select the motor and starting equipment if the power supply is 208 v and three-phase. No arbitrary limitation of starting current is set.

Exercise 12-14. A line shaft is to be driven by a 10-hp 1750-rpm 220-v three-phase induction motor. It is located at the end of a long line having 240 v at the supply end and a line drop of 20 v is

obtained with rated load. It is expected that the starting torque required by the line shaft will be about 125 per cent of rated torque when the bearings and oil are cold. What type of motor should be used? Why?

Exercise 12-15. A belt conveyer requiring a constant torque of 0.9 full-load rated torque and variable speed is driven by a 25-hp 440-v 1750-rpm wound-rotor induction-motor drive, having a full-load efficiency at rated speed of 88 per cent. (a) What is the over-all motor and control efficiency when operating at 1200 rpm? (b) How does the input at 1200 rpm compare with that at 1750 rpm? (c) Rotor resistance is inserted at starting so that just sufficient torque is available to start (at 0.9 full-load torque). What will the starting current be in per cent of full-load current?

Single-phase induction motors

In most residences and other installations using small amounts of power, it is not economical to supply the extra wire for three-phase power. This is particularly true for rural areas where single-phase lines are the rule rather than the exception. There are many places, therefore, where small motors are required and single-phase only is available. For this purpose, single-phase induction motors are used most extensively. The most popular sizes for household use range from $\frac{1}{3}$ down to $\frac{1}{8}$ hp. They operate washing machines and refrigerators in residences, and such tools as grinders, screw drivers, drills, and light hoists in industries. In farm installations the motor size may be as large as $7\frac{1}{2}$ hp, and 3 and 5 hp are common.

The power from a single-phase circuit is pulsating and not continuous. Likewise, the flux from a single-phase winding is fixed in position and alternating in magnitude rather than fixed in magnitude and rotating around the stator. In order to obtain a rotating flux to start the motor, a second winding is placed on the motor in space quadrature—that is, with the axis of the winding displaced a quarter-phase from the axis of the main winding. In this winding it is necessary for the time phase of the current to be different from that of the current flowing in the main winding. This condition will produce a flux that becomes a maximum in successive positions around the periphery of the motor, and thus rotating flux is obtained, although it is not usually uniform in magnitude.

If the current in the second winding, usually called the *starting winding*, could be made to lead that in the main winding by 90 deg in time phase (by connecting a capacitor in series with

it) and could be of such magnitude that it would produce the same ampere-turns as the main winding, a uniform rotating field would result. If, however, this ideal situation does not exist, the field may be considered as being composed of a rotating field plus a stationary field of alternating magnitude. The starting torque is proportional only to the rotating component of this field. If there is no quadrature magnetomotive force, no starting torque is possible and the motor will not start.

The resistance, or split-phase, type of starting winding uses fine wire in the quadrature field to obtain a higher resistance

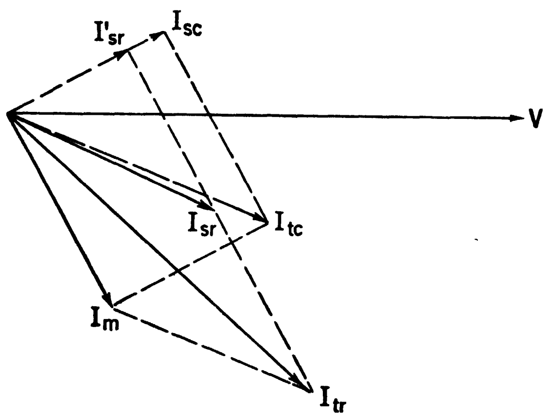

FIG. 12-17. Current Diagram for Single-Phase Induction Motors when Starting.

and therefore has a current that does not lag as much as the current in the main winding. This is shown in Fig. 12-17, where I_m is the current in the main winding and I_{sr} is the current in a starting winding, using resistance to obtain the power-factor difference. The component of I_{sr} which is effective in producing starting torque is that which is in quadrature time phase with I_m. This is shown on the diagram as I'_{sr}. The total current is the vector sum of the two currents and is almost twice the blocked rotor current of the main phase alone. It is shown in the diagram as I_{tr}.

In most modern motors it has been found advantageous to place a small condenser or capacitor in series with the starting winding. This causes the current in this winding to lead the voltage and to be approximately in time-phase quadrature with the current in the main winding. This quadrature current is shown in Fig. 12-17 as I_{sc} and is observed to have a quadrature component about 50 per cent greater than the ordinary split-

phase type; therefore, the starting torque is approximately 50 per cent greater. The starting current is considerably less than with the resistance or split-phase winding. Motors with a capacitor in the starting winding are known as *capacitor-start motors*. The very satisfactory starting performance of these motors has caused them to become the most popular type of single-phase motors. Since the starting winding usually does not have sufficient current-carrying capacity to operate continuously, it is cut out by a centrifugal switch when the motor has reached about two-thirds of rated speed.

Running operation on main winding only. The performance of a single-phase motor, with the main winding only connected,

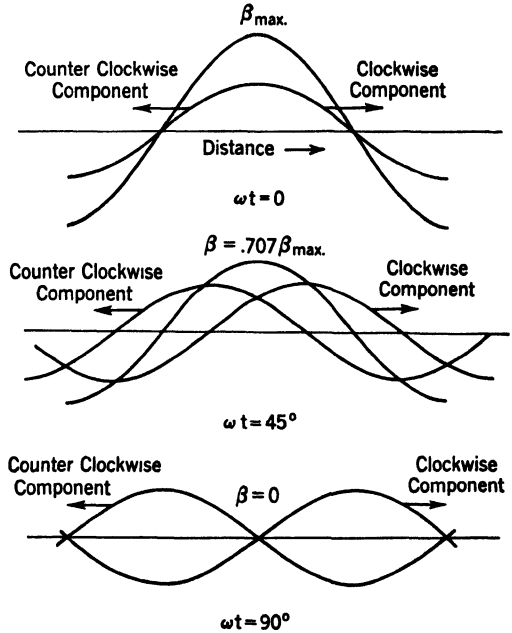

FIG. 12-18. A Stationary Flux Wave Whose Magnitude Varies Sinusoidally with Time Is Replaced by Components Fixed in Magnitude but Rotating around the Periphery of the Motor.

can be most simply visualized by dividing the magnetomotive force of the single-phase winding, which is varying in magnitude from positive to negative maximum values, into two components. The sum of these components must, at all times, be equal to the original magnetomotive force. This requirement is met by two sinusoidal waves of magnetomotive force, the

magnitude of each being equal to one-half the maximum value of the original wave and constant in magnitude, but moving in opposite directions around the periphery of the armature at synchronous speed. Figure 12-18 shows the original pulsating wave and the two components for three successive instants which are 45 deg apart in time phase. The first represents the condition when the flux density is a maximum. The next shows one of the components 45 deg to the right, while the other has moved 45 deg to the left. Their sum produces the stationary wave, but with a magnitude reduced to 0.707 of its maximum value. The next position shows the components after they have moved 90 deg in their respective directions. Their resultant is zero, as it should be, since the original magnetomotive force is zero at 90 deg after the maximum value.

Each of these component rotating flux waves may be considered to act on the rotor independently of the other. In Fig. 12-19 the clockwise torque is plotted from synchronous speed in the clockwise direction to zero speed and then on to synchro-

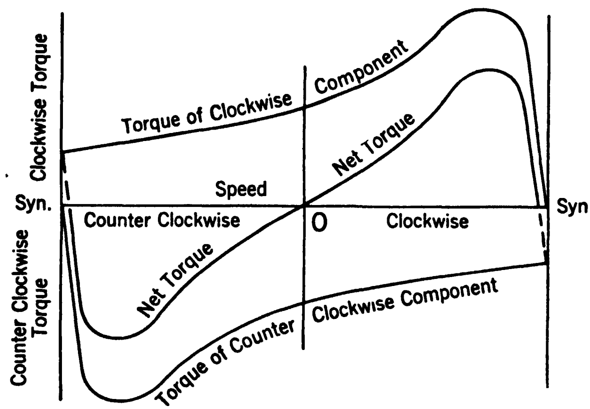

FIG. 12-19. Speed-Torque Curves of a Single-Phase Induction Motor.

nous speed in the negative direction. The torque with rotation in the direction opposite to the direction of rotation of the flux is less because of the increasing value of rotor reactance and reduced power factor as the rotor frequency is increased beyond synchronous frequency.

The torque resulting from the counterclockwise component of flux is also plotted. The net, or resultant, torque is the sum of these two torques in opposite directions and is likewise plot-

ted in Fig. 12-19. It is seen that at zero speed the net torque is zero, so that there is no starting torque, which confirms the previous conclusions. If, however, a rotational velocity is given to the rotor in either direction, a net torque will be developed that will tend to continue the rotation in that direction. Thus, when the starting winding is disconnected at about two-thirds speed, there is sufficient torque available to continue the acceleration to the normal operating speed.

In some of the larger single-phase motors where increased power factor and smoother, quieter operation is desired, the capacitor winding and capacitor are built with sufficient current-carrying capacity to operate continuously. Such motors are somewhat more expensive than capacitor-start motors, but are used for fan service and other applications where a low starting torque is adequate.

Another type of single-phase motor will be mentioned only. This is the *series motor*. When d-c motors were being discussed, the series motor was considered. It will be noted that, if both the field and the armature current are reversed, the torque continues in the same direction. Thus, a series motor may operate on alternating current if it is built so that the losses and inductive effects are reduced to a minimum. Such a motor built in small sizes may be operated on either alternating current or direct current and is called a universal motor. When built in large sizes, it is often used for electric railways.

A form of induction motor used primarily for starting has a commutator and is essentially a series motor. It is called a repulsion motor, and since it is now being replaced by the capacitor motor, it will not be discussed in detail.

Synchronous motors

General. If a squirrel-cage winding composed of conducting bars connected to end rings is placed in the pole faces of the rotor of a synchronous generator as shown in Fig. 12-20, a common type of synchronous motor will be obtained. It is started as an induction motor, the poleface windings acting as the squirrel-cage rotor. When it reaches normal operating speed as an induction motor, the slip is small. The d-c circuit providing the magnetomotive force for the rotor poles is then closed and the north and south poles of the rotor lock in with the stator poles of opposite polarity so that the rotor revolves at synchronous speed.

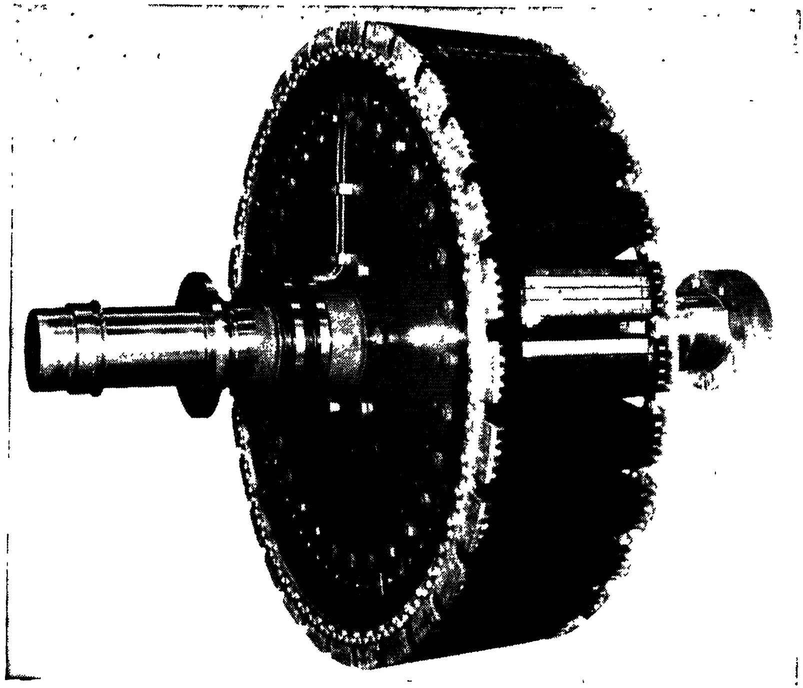

FIG. 12-20. Rotor for a Large Slow-Speed Synchronous Motor.

Magnetizing current in the armature. The generation of voltage in the stator conductors of a synchronous motor is similar to that of an alternator of the same number of poles and the same speed. Let it first be assumed that the motor is supplying no load and that the d-c exciting current does not provide enough mmf to give a flux that will generate (in the stator conductors) a voltage that is equal and opposite to the impressed voltage. From circuit studies and from the studies of the induction motor it is known that the internal generated voltage must be equal to the impressed voltage except for the resistance drop, which is small. Sufficient stator current must therefore flow so that the combined magnetomotive force of the d-c field winding and the armature winding will produce an air-gap flux that will generate in the armature or stator conductors a voltage that is equal and opposite to the impressed voltage. In order for this to be true, it is necessary that the armature mmf of the stator (in Fig. 12-21) aid the d-c magnetomotive force. It is thus necessary that the currents in the conductor groups B''', C'', B', and C be flowing into the paper and the currents in C''', B, C', and

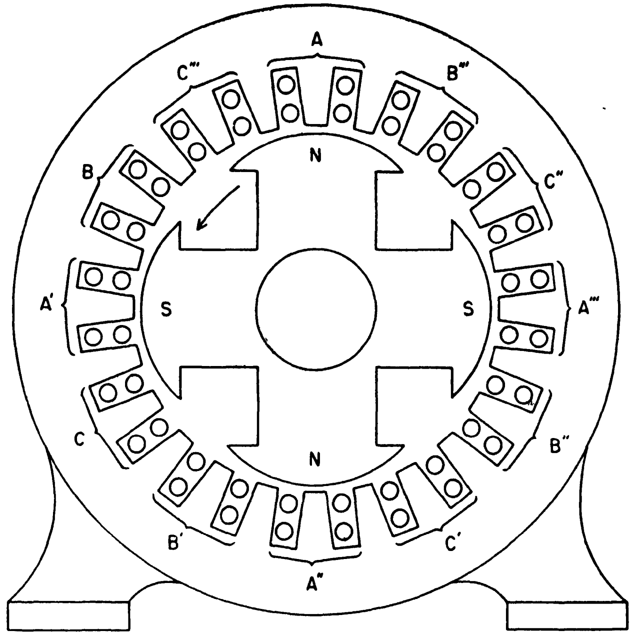

FIG. 12-21. Elementary Diagram for Analysis of Winding Currents and Magnetomotive Forces.

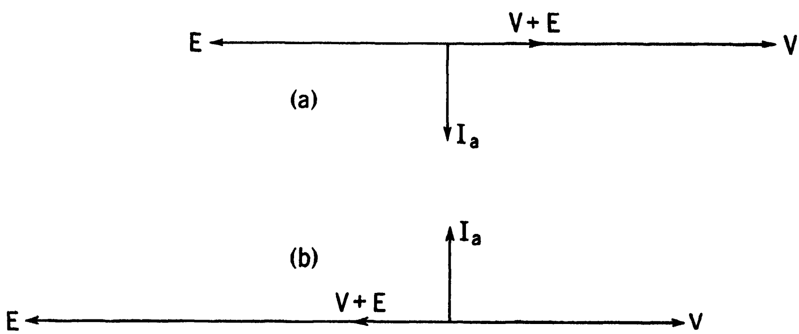

FIG. 12-22. Vector Diagrams for Synchronous Motor—No Load.

In Chap. 11 it was demonstrated that the voltage generated in phase *A* would be out of the paper and the impressed voltage would therefore be opposite to this direction. Analysis will show that the currents in the stator conductors will lag 90 deg behind the impressed voltage in time phase, which is correct for an inductive circuit. A phasor diagram of this is shown in Fig. 12-22a. It is seen that the voltage *E*, which would be pro-

neutralize the impressed voltage V and there is thus a resultant voltage $\boldsymbol{V} + \boldsymbol{E}$ that causes an armature current to flow that lags 90 deg behind it in time phase.

If now the d-c exciting current is increased, the magnitude of $\boldsymbol{E}$ will be increased until it is greater than $\boldsymbol{V}$ so that $\boldsymbol{V} + \boldsymbol{E}$ is reversed in direction, as shown in Fig. 12-22b. The armature current will again lag the net voltage by 90 deg in time phase, which means that it is reversed from the current in Fig. 12-22a. Referring again to Fig. 12-21, it is found that this reversal of current means that the armature magnetomotive force now opposes the d-c exciting mmf. In other words, the air-gap flux remains essentially constant regardless of the d-c exciting current. The armature current either aids or opposes the d-c excitation in order to bring this about.*

Load current in the armature. The retarding torque of the load has been assumed as zero in the above discussion. If now, with the excitation voltage equal to the impressed voltage, a load is placed on the motor, it starts to slow down and the

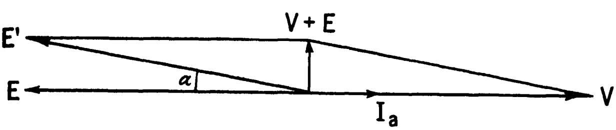

FIG. 12-23. Phasor Diagram for Synchronous Motor—Loaded.

countervoltage will drop behind the original position by an angle shown in Fig. 12-23. The resultant voltage will be equal to the vector sum of the impressed and countervoltages, which gives a resultant voltage leading the impressed voltage by 90 deg. This causes a current to flow, which is in phase with the impressed voltage. The magnitude of this current is dependent upon the angle by which the rotor drops behind its position of phase opposition. As soon as it has dropped behind sufficiently so that the current flowing in the stator produces enough torque to overcome the load, the tendency to slow down is eliminated and the rotor will continue to turn at synchronous speed. The power taken from the line is only that required to overcome the retarding torque of the load and supply the losses in the motor.

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* It should be remembered that the stator windings are embedded in slots surrounded by iron, and so a large amount of flux is produced by the stator windings that does not cross the air gap. Because of this the air-gap flux does vary, but the effect on the motor characteristics is not greatly altered by this approximation.

Thus, the synchronous motor adjusts itself to varying load torque by a variation in the angular position of the rotor.

In Fig. 12-23 the excitation voltage was assumed to be equal to the impressed voltage. This gives an armature current that is essentially in phase with the impressed voltage and gives a minimum current in the armature for a specified power output. When the motor is underexcited, as in Fig. 12-24a, the armature current must not only provide for load torque, but must also

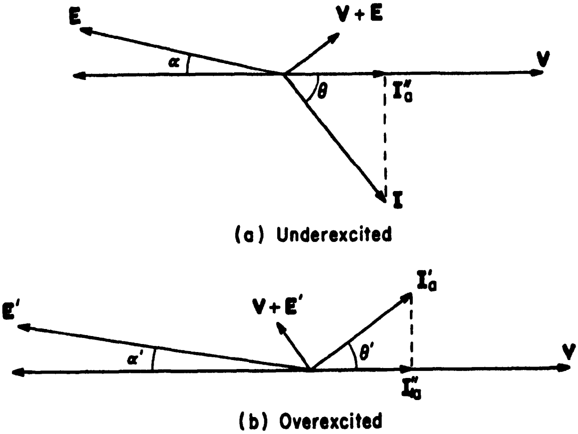

FIG. 12-24. Phasor Diagrams for Loaded Synchronous Motor.

supply the additional magnetomotive force to obtain the proper air-gap flux. It is seen that the current lags behind the impressed voltage by the power-factor angle θ.

When the motor is overexcited as in Fig. 12-24b, the power component of armature current I''_a remains essentially constant, but the phasor $V + E'$ swings forward so that the current I'_a, which lags about 90 deg behind it, eventually leads the impressed voltage by a power-factor angle θ'. There is little commercial importance attached to the operation of a synchronous motor with lagging power factor, as it holds no advantages. With a leading power factor, however, it is possible to correct for the lagging power factor of induction motors in industrial plants.

The action of the synchronous motor is somewhat analogous to the action of a coupling that is connected by coiled springs in tension, as shown in Fig. 12-25. When there is no load on the coupling, the spring tensions will be balanced. When a load is placed on the driven shaft, however, it is displaced back-

ward and the driving torque is approximately proportional to the angular displacement for small angles. This corresponds to the displacement between the poles of the stator and the rotor.

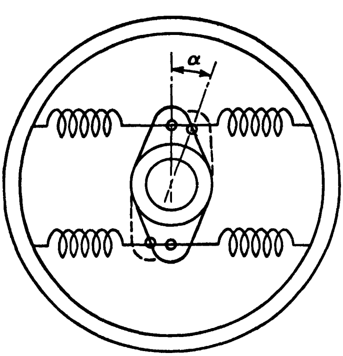

Fig. 12-25. A Spring Coupling.

A common method of testing the performance of a synchronous motor is to record the variation of armature current with field current for zero, one-fourth, one-half, and full rated load. Such a set of curves are known as *V* curves because of their general form and are shown for a typical motor in Fig. 12-26.

It is noted from these curves that if the synchronous motor is to provide power-factor correction, it must operate in the region of overexcitation. This requires that for an 0.8 leading power factor the armature must carry 25 per cent more current

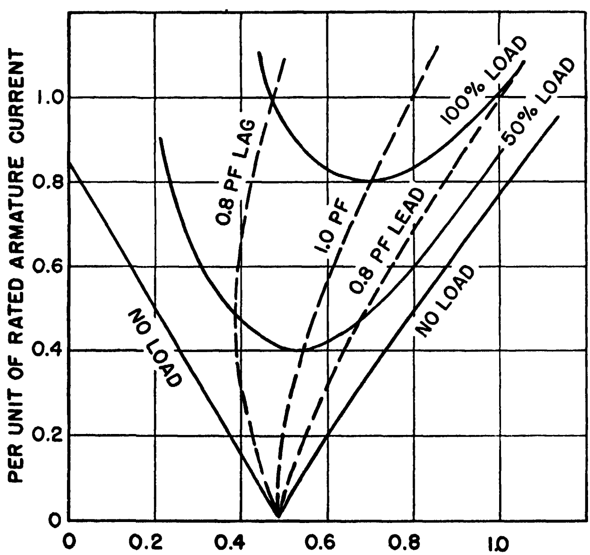

Fig. 12-26. Typical "V" Curves and Compounding Curves for an 80 Per Cent Power Factor Synchronous Motor.

than is necessary at unity power factor, and that the field winding must carry from 50 to 90 per cent more current than at unity power factor. If a motor is to operate continuously under

these conditions, it will require considerably more copper in the windings than is necessary for unity-power-factor operation. When it is desired to operate a synchronous motor for power-factor correction, it is normal to purchase a motor that is designed to operate at 0.8 leading power factor. When no power-factor correction is necessary, the unity-power-factor motor may be purchased at a considerably reduced cost if its starting torque is adequate.

Advantages and use of synchronous motors. Reference has been made to the low power factor of induction motors. The additional current-carrying capacity of generating, transmission, and distribution equipment of the power companies with low and lagging power factor often causes them to make an extra charge when the power factor is low and lagging or to give a bonus when the power factor is high or leading. Under these circumstances it is possible to use a synchronous motor to drive some of the equipment and to overexcite the field, so that it will take leading current, which neutralizes some of the lagging current drawn by the induction motors.

In small sizes, the synchronous motor and starter are considerably more expensive than an induction motor of the same horsepower. It is seldom economical, therefore, to use synchronous motors for power-factor correction, except in ratings of 100 hp or over.

Synchronous motors of any size may be used where exactly constant speed is desired. They have come into extensive use as elements in control devices. Small synchronous motors to drive clocks operate without a d-c field and depend only on the hysteresis of the rotor steel or on the bunching of the flux at the rotor poles to produce torque. Modern synchronous electric clocks are accurate because the frequency is closely controlled at the power station, and any deviation is compared with a standard clock. Corrections are made so that the clocks normally do not vary more than a few seconds from the correct time.

CHAPTER 13

Electric Motor Applications

Characteristics of industrial machinery

The problem of selecting motors and control equipment for driving industrial machinery involves the matching of motor and control characteristics to torque, speed, and control requirements of the industrial equipment. Since this subject is as broad as the whole of industrial development, it is necessary to study certain typical applications, the drives and control devices used, and to learn from them certain generalized procedures for selection of motor and control equipment.

Students often do not have specific information as to the characteristics of typical industrial machinery, and so this chapter will first present some of this information, which can then be used as a basis for typical problems.

Pumps. One of the most common industrial machines is the pump. Most industrial pumps are centrifugal and are composed of impellers that are mounted on shafts and rotate within a casing designed to control the flow of the liquid. Centrifugal pumps ordinarily operate at quite high speed, so the motor is directly connected by means of a flexible coupling. The torque required at starting is friction torque only. As the speed builds up, the torque increases as the square of the speed. The power, however, builds up as the cube of the speed.

The motor drive most commonly used is the general-purpose squirrel-cage induction motor. Such a motor is almost ideal for constant-speed operation. Direct-current shunt motors are used where only d-c power is available. For larger pumps synchronous motors may be used if power-factor correction is desired.

When it is desired to control the flow by pump pressure, a variable-speed motor may be required. Multiple-speed induction motors may be used with a throttling valve for intermediate control. If a continuously variable speed is desired, the wound-rotor induction motor may be used. Although this motor is

fundamentally inefficient at low speeds, the pump power required is so low that it makes little difference. Thus if the speed is reduced to $\frac{1}{2}$, the power required is reduced to $\frac{1}{8}$ and the 50 per cent (or lower) efficiency of the motor is unimportant. Adjustable-speed d-c motors would be used for this service if d-c power is available.

For pumps operating at high pressures and small flow, reciprocating displacement pumps are often used. Rotary pumps, using two rotating elements to effect a displacement pumping effect, are also extensively used. The rotary pumps are most often used for heavy oils, syrups, and other similar liquids. Both types of pumps run at low speed and motors are ordinarily connected to them by belts, gears, or chains. Motors will usually have to start these pumps against normal pressure, so high-torque induction motors are most commonly used.

Nearly all types of pumps will have an efficiency of from 65 to 80 per cent, with the most common value about 75 per cent. Motor horsepower to drive a pump may be computed from the pump output, using this efficiency with reasonable assurance that the error will not be great.

Exercise 13-1. A centrifugal pump operates continuously at its rated speed of 1750 rpm and requires 20 lb-ft of torque. Power of 220 v three-phase 60 cps is available. Specify the most economical motor and starting equipment.

Fans, blowers, and compressors. Fans and blowers are quite similar to centrifugal pumps in their characteristics, since they are essentially pumps applied to a gaseous fluid rather than to a liquid. As in the case of pumps with no change in outlet orifice, the volume, or flow, will vary directly with change of speed. The pressure will vary as the square of the speed, whereas the horsepower requirement varies approximately as the cube of the speed.

Centrifugal fans and propeller fans perform differently when the discharge area is restricted to control the flow. The centrifugal fan, in which the inlet is in the center of the fan and the discharge is radial to a spiral casing, depends upon the centrifugal force of the gas to produce the pressure. When the discharge opening is reduced in size, the gas flows through the blades less rapidly, and the horsepower required is reduced. For the propeller type of fan, which moves the air in an axial direction, the closing of the discharge opening increases the back

pressure and thus increases the horsepower. This effect may be so severe as to require several times the original power. It is important, therefore, that air-flow control for process purposes not be accomplished by throttling if the blower is of the propeller type, as it may cause a motor to overheat.

Compressors may be of either the centrifugal or the reciprocating displacement type. In general, the centrifugal type is preferred for larger volumes and limited pressure. So far as motor application is concerned, they are similar to centrifugal pumps except that the compressors are of higher speed and will usually require gears to step up the motor speed. General-purpose squirrel-cage, synchronous, or wound-rotor motors are the most common drives, and the choice will depend upon control requirements, the necessity of power-factor correction, and other conditions.

Reciprocating compressors are essentially slow-speed high-pressure units. They usually operate at constant speed and use either squirrel-cage or synchronous motors. In the smaller sizes the motors are high-speed and may use short center belts or self-contained gears as in gear-motors. In the larger sizes the motors are often direct-connected slow-speed synchronous motors. Engine-type motors, in which the rotor is supplied for mounting on the compressor shaft and the stator requires no end bells, are frequently used for this service.

Exercise 13-2. An induced-draft fan requires 500 lb-ft of torque at its maximum speed of 585 rpm. The friction is approximately constant at 25 lb-ft. The remainder of the torque varies as the square of the speed. It is to be operated at speeds that are continuously variable down to 300 rpm. The power supply is 440 v three-phase 60 cps. Specify the motor size, type, and control.

Machine tools. Rapid progress is being made in the machine tool industry. More and more automatic machines are being built with corresponding variations in speed and control characteristics of the motor drive. For these specialized applications it is usually best to work with a well-qualified representative of one of the large electrical manufacturing companies to determine the motor specifications. There are many common types of machines, however, the motors for which can be specified by the operating engineer.

Saws are usually constant-speed and are driven by general-purpose squirrel-cage motors if alternating current is available. If direct current only is available, the motor will be a shunt or cumulative compound motor, depending upon the type of duty.

Shears, punch presses, and forging machines are often provided with flywheels to store energy and assist in equalizing the load. In such machines high-slip squirrel-cage motors are used so that the motor will slow down somewhat when the load increases and permit the flywheel to deliver its stored energy to the operation. If direct current only is available, a cumulative compound motor should be used.

Cutting and turning machines usually require speed adjustment. In many small machines this is obtained by belts, gears, or other mechanical means. In larger machines, however, it is economical to provide much of this speed adjustment through the motor. The adjustable-speed shunt motor is most extensively used for this duty. If there are a number of machines, it is usually desirable to provide a motor-generator set, or rectifier unit, to supply constant-voltage direct current for the group of motors. If only one or two machines are involved, it may be more economical to use grid-controlled rectifiers for each motor, since this gives an extremely wide range of control possibilities. The determination of the size of the motor may be made from the cubic inches of metal to be removed per minute. This is dependent upon the type of tool and upon the metal. Constants for the more common metals are given in the following table for lathes and drills.

TABLE 13-1

HORSEPOWER REQUIRED TO REMOVE 1 CU IN./MINUTE OF DIFFERENT METALS BY MEANS OF LATHES AND DRILLS

	Lathes	Drills
Brass (and similar alloys)	0.2 to 0.3	0.4 to 0.6
Cast iron	0.3 to 0.5	0.6 to 1.0
Wrought iron and mild steel (0.3 to 0.4 per cent carbon)	0.6	1.2
Hard steel (0.5 per cent carbon)	1.0 to 1.3	2.0 to 2.5

These constants can be used for milling machines, planers, shapers, and slotters also. In most of these the horsepower required will follow more nearly the character of lathes than that of drills.

Exercise 13-3. An engine lathe requires a 20-hp motor with speeds continuously adjustable from rated to half-rated value. The power supply is 440 v, three-phase alternating current. It is expected that the motor will operate most of the time at about half-speed and will require full speed only about 10 per cent of the time. Give three possible solutions with a comparison of advantages.

Cranes and elevators. Cranes and elevators may use either d-c or a-c power. The d-c power is preferable because of better control possibilities. When the crane or elevator is used only occasionally, satisfactory control can be obtained by using wound-rotor induction motors. If accurate control is not necessary, high-slip squirrel-cage motors may be used with primary-resistance control. When the cranes or elevators are in continuous operation on production work, the added speed of operation and accuracy of control will usually justify d-c motors with a conversion unit to change from a-c to d-c power.

Other industrial machinery. No specific discussion is included here of punches and dies, rolling operations, drawing of wire, manufacture of paper, and a thousand and one other industrial operations that require electric motors. Once the requirements of the individual industrial situation are known, the following principles may usually be applied to determine suitable electrical equipment.

Exercise 13-4. The belt conveyer in a gravel-washing plant must deliver 50 tons of gravel per hour to a bunker that is 100 ft above the supply. The friction load is $2\frac{1}{2}$ hp. The starting torque may run as high as 200 per cent of the full-load value. The power supply is 230 v three-phase 60 cps. Specify the motor and control.

Electric motor characteristics

Most of the characteristics of d-c and a-c motors have been discussed when the theory of these machines and the reasons for those characteristics were being studied. It may be desirable to summarize these characteristics briefly and to make certain comparisons between d-c and a-c motors.

Alternating-current induction motors operate normally at constant speed. The starting current of squirrel-cage motors is large, from four to seven times normal full-load current. The starting torque for these large currents is usually only from 100 to 200 per cent of normal full-load torque.

Adjustable speed may be obtained (within limits) by the use of double windings to give two or even four speeds. Such motors are, however, more expensive than the ordinary motors and not as efficient. Variable speed and high starting torque per ampere may be obtained by the use of wound-rotor motors with external resistance inserted in the rotor circuit. These motors vary in speed with changes in load and are inefficient so far as power utilization is concerned.

Alternating-current synchronous motors run at exactly constant speed, but have the advantage of power-factor control. At slow speeds and in larger sizes they are also less expensive and more efficient than induction motors.

Direct-current motors are versatile, having an almost infinite variety of characteristics that can be obtained by suitable control equipment. Shunt motors operate at almost constant speed in spite of load changes. This speed may be easily adjusted, however, by means of a small field rheostat. An adjustable-speed d-c motor will normally have a speed range up to four times the normal base speed. It is, however, essentially a d-c shunt motor.

When series windings are added to the fields of d-c motors, the control possibilities become much broader. If in addition some method of controlling the current flow is established, they can be made to produce constant torque, constant tension, or other specialized characteristics.

With the development of more complex industrial machines and the simultaneous development of simple rectifiers, the use of d-c motors in industrial plants is rapidly increasing.

Types of motor housings

Service conditions often require special types of motor housing. Providing such types of housing might be considered as *packaging the motor* to meet the conditions under which it will operate and, in general, does not involve the speed, torque, and control characteristics previously discussed. It is important, however, that the proper type of housing be specified if the motor is to give continued satisfactory service.

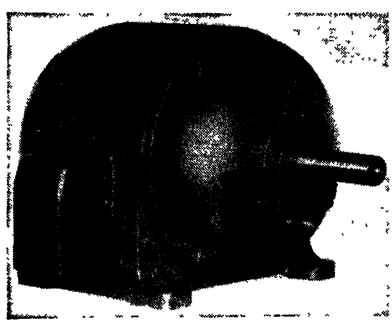

Fig. 13-1. Sleeve-Bearing Squirrel-Cage Polyphase Induction Motor.

Open motors are motors that have no restriction to the flow of ventilating air, except such as is necessitated by mechanical construction. Motors of this type are most common, are cheaper than other types, and may be used wherever service conditions are sufficiently satisfactory that no special protection is required (see Fig. 13-1).

Protected motors have all ventilating openings covered with

wire screen, expanded metal, or perforated metal covers in order to prevent accidental contact with live or rotating parts (see Fig. 13-2).

Drip-proof motors have their ventilating openings so constructed that drops of liquid falling on the motor at an angle not greater than 15 deg from the vertical will not enter the machine.

Splash-proof motors are so constructed that liquid coming at the motor in a straight line, not more than 100 deg from the vertical, cannot enter the ventilating openings Motors of this type could be used for outdoor applications.

Totally enclosed motors are, as their name implies, totally enclosed to prevent interchange of air between the inside and outside of the motor housing. In case the motor shaft is provided with a fan for exterior cooling, as shown in Fig. 13-3, this motor is called a *totally enclosed fan-cooled motor.*

A totally enclosed motor that will exclude water applied in a form of a hose stream is called a *water-proof motor.* Motors of this type are used for slaughter houses and similar applications in which the area is washed out with a hose at regular intervals. They are also used where a considerable amount of dust or dirt exists in the air surrounding the motor.

An *explosion-proof motor* is provided with an enclosed case designed and constructed to withstand an explosion of a specified gas or dust that may occur within it, and to prevent the ignition by the internal explosion of the same gas or dust that surrounds the motor.

There are other special types of housing, but the above are the more important ones. These special housings are available for either d-c or a-c motors in nearly all of the various operating characteristics. A motor with a special housing costs more than the open-type motor. The totally enclosed motor, in fact, costs considerably more because the difficulty of cooling necessitates a larger motor for the same rating.

Bearings. A mechanical feature of the motor that is sometimes important is the type of bearings. Horizontal motors have as standard equipment an oil-lubricated sleeve bearing, and as long as reasonable attention is given to the oil supply, these bearings are very satisfactory. In some applications the motor is difficult to reach, and in these cases it may be advisable to specify ball bearings, which will operate for long periods without attention. Ball bearings are also used where the motors

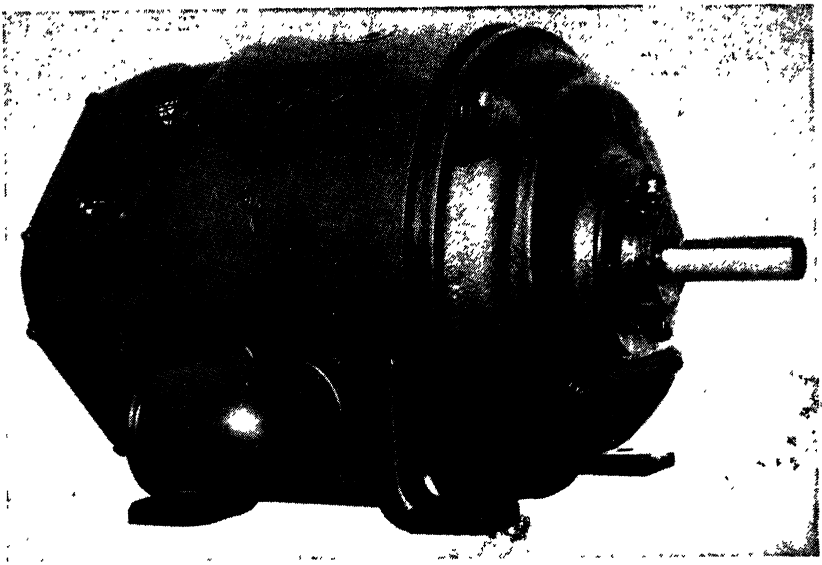

FIG. 13-2. Standard Horizontal Ball-Bearing D-C Motor, Protected with Screen Covers.

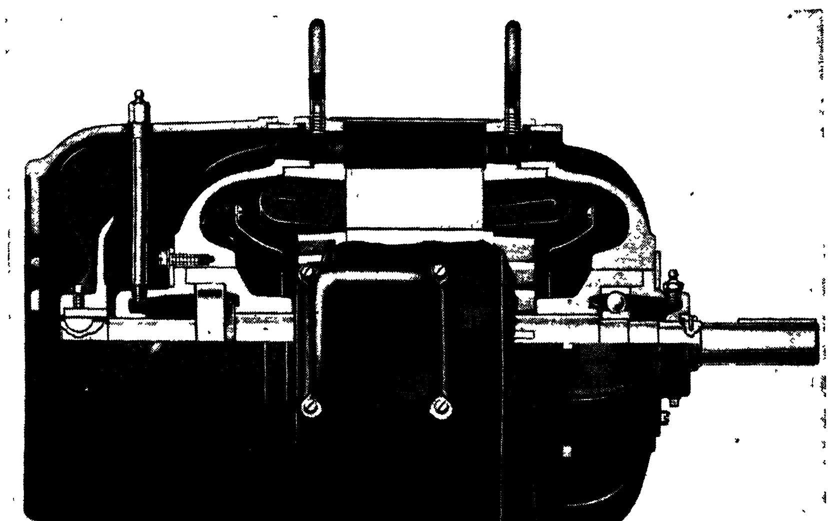

FIG. 13-3. Totally Enclosed Fan-Cooled Squirrel-Cage Induction Motor with Parts Cut Away to Show Construction.

are mounted at an angle with the horizontal and where the motor is subject to any type of axial thrust.

Gear motors. Most electrical motors are essentially high-speed machines. It is not economical to build them in small and medium sizes for speeds under 900 rpm. For the lower driving speeds gear motors using 1750-rpm motors are cheaper and more efficient. In most cases the motors are mounted on the gear case. These gears reduce the speed of the output shaft to the desired value, and so speeds from 4 to 1400 rpm are avail-

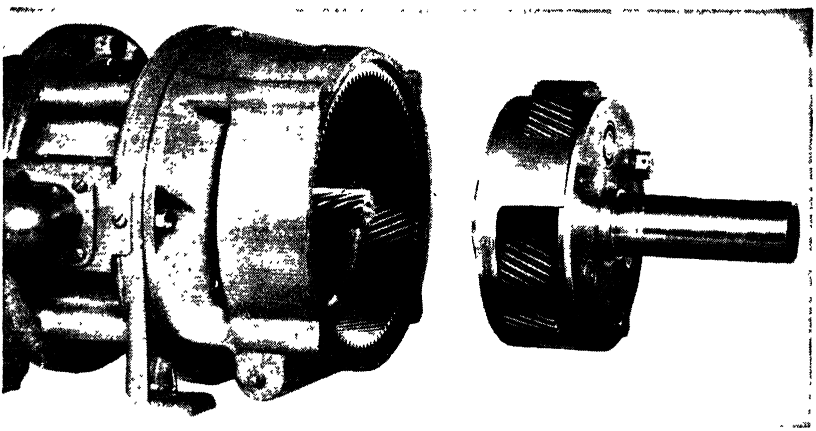

FIG. 13-4. Gear Motor, Ball-Bearing, Squirrel-Cage Polyphase Induction Design Output Shaft Speeds 600 to 154 RPM, with Gear Cage Removed.

able from stock. They are applied to the load just as any low-speed motor would be applied.

Motor ratings

All electrical machines are rated on the basis of output. Thus a 5-hp motor is guaranteed to deliver 5 hp at its rated speed when supplied with rated voltage of the type specified on the name plate. The limiting factor in motor loading is the heating, and general-purpose motors are guaranteed to have no more than a 40° C rise in temperature when carrying continuous full load. It is assumed that the temperature of the surrounding air does not exceed 20° C. If the air is cooler, the motor will carry some additional load without exceeding a safe temperature.

Some types of motors are designed to have a 50° C rise, and these are so specified on the name plate. Totally enclosed motors are rated for a 55° C rise in temperature. These tem-

peratures are based on the use of Class A insulation which consists of cotton, silk, paper, and similar organic materials filled with various impregnating compounds. Class B insulation consists of asbestos, glass, mica, or similar inorganic materials bound with special high-temperature compounds. Motors using Class B insulation are permitted a much higher operating temperature and so are used where it is necessary to reduce the weight of the motor, or where especially severe conditions of operation are expected.

Intermittent ratings. When the motor load is not continuous but intermittent, a special motor rating may be used to advantage. Typical applications for motors with *intermittent rating* are cranes, shovels, elevators, drag lines, and certain types of machine tools. The most common intermittent time ratings are for 1 hour, $\frac{1}{2}$ hour, 15 minutes, and 5 minutes. The temperature rise allowed depends upon the class of the insulation, being usually 55° C for Class A and 75° C for Class B insulation.

Motors having rating times of 15 minutes or more are given a horsepower rating. Those rated on a 5-minute basis are given a rating on the basis of foot-pounds of starting torque. Such motors are used for valves, presses, and other devices where the motor makes only a few revolutions. They must develop sufficient torque to start and carry the load through its operating cycle.

In giving a motor a $\frac{1}{2}$-hour rating the load will be applied at full rated value for $\frac{1}{2}$ hour continuously and then be shut off for several hours until the motor cools off. The actual loading situation may deviate from the ideal; however, it should give approximately the same heating effect as the above to take advantage properly of the $\frac{1}{2}$-hour rating.

Many industrial machines will have continuous operation with a wide cyclic variation of power drawn from the motor. The question of determining the proper size of motor for such an application is one of determining the equivalent heating effects of the cyclic load current, since motor ratings are based on temperature limits.

Much of the motor heating is caused by the copper losses of load current. These losses are proportional to the square of the current; so to determine the average heating effect it is necessary to take an average of the square of the currents and extract the square root of this average to obtain the equivalent load

current. Such an average is called the root-mean-square, or rms, load.

To obtain the average it is necessary to divide the integrated sum of the squared current values by the time of the cycle. This can be done usually without too much error. However, the motor does not cool as rapidly when it is stopped as when it is running. It is therefore usual to reduce the cycle time by applying a factor of $\frac{1}{3}$ to the time at rest. The technique of determining an equivalent rms load can be demonstrated by the following illustration.

Example. A drag line handling coal in a storage pile is to be driven by a 220-v three-phase wound-rotor induction motor. It is estimated that the duty cycle will approximate the following:

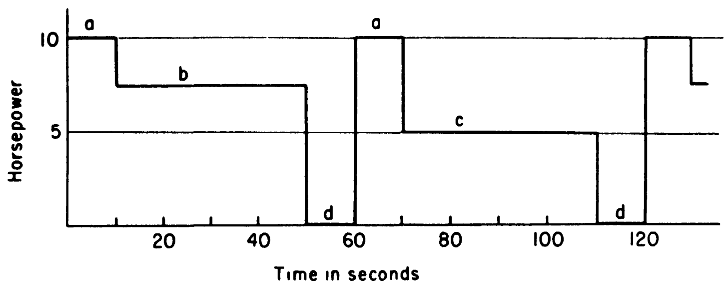

FIG. 13-5. Motor Duty Cycle for Computation of RMS Load.

Solution: (1) The equivalent cycle time is found from the sum of the times for each portion of the cycle with rest periods divided by 3.

$$20 + 40 + 40 + \tfrac{20}{3} = 106.7.$$

(2) The equivalent rms load is determined from the square root of the average squared values of load.

$$\text{rms hp} = \sqrt{\frac{10^2 \times 20 + 7.5^2 \times 40 + 5^2 \times 40}{20 + 40 + 40 + \frac{20}{3}}}$$

$$= \sqrt{\frac{5250}{106.7}}$$

$$= \sqrt{49.3} = 7.01 \text{ hp} \qquad (\textit{Ans.})$$

•Motor control for industrial loads

Control devices for motors achieve the following objectives.

1. To start and to stop the motor.

2. To disconnect the motor from the power source in case of excessive or continued overload.

3. To disconnect the motor from the power source when the voltage drops below a safe operating value.

4. To cause the motor to operate at a predetermined speed or torque.

5. To give the motor certain speed-torque characteristics that are desired for the industrial machine being driven.

These objectives are achieved in many different ways by different manufacturers. They vary widely also with the type of motors used.

Starting and stopping is usually accomplished by magnetic contactors. These contactors close the circuit to the motor when the control magnet is energized by a push button at a remote location. In the case of induction motors of relatively small size a single multipole contactor is sufficient when provided with satisfactory overload and undervoltage protection as described in Chap. 12.

On d-c motors where resistance is inserted in the armature circuit during starting, the closure of several switches in sequence is necessary. This may be accomplished by the operation of several contactors in time sequence. In small motors it is usually accomplished by a single contactor with several poles that make contact in sequence caused by a mechanical time-delay device. Such a controller is shown in Fig. 13-6. The solenoid on the right energizes the device and places a tension on the horizontal rod supporting the movable contacts. The rotation of this rod is controlled by a ratchet device at the lower left of the switch. This allows the contacts to close in a predetermined time sequence, so that the motor speeds up as the armature resistance is shorted out. Overload and undervoltage protection are similar to that in the a-c switch.

In many industrial applications this simple on-or-off control is inadequate. The speed must be controlled to a fraction of one per cent, as in the case of a paper mill, or the motor must be continually starting, stopping, and reversing, as in the case of an electric shovel. The problems involved in these widely varying motor applications are so extensive that books have been written regarding them. Only a brief comment may be made on them to call attention to the fact that it is possible to do almost anything electrically if one is willing to pay the money.

In complicated reversing or variable-speed controls it is usually assumed that d-c motors are used. If a-c power only is available (as is usually the case), the d-c is obtained by motor-

generator sets or by thyratron rectifiers. Therefore, the problems of speed control, dynamic braking, reversing, and plugging usually refer to d-c motors.

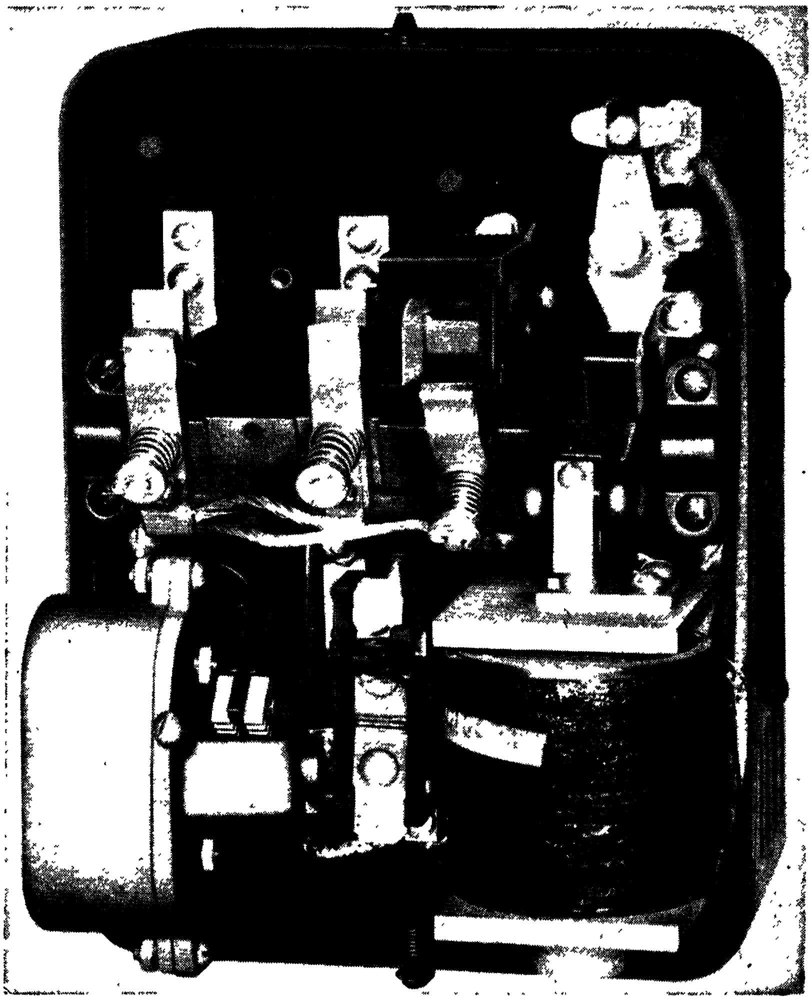

FIG. 13-6. Magnetic D-C Controller without Cover.

Speed control in d-c motors, when they are supplied from a constant-voltage d-c source, is usually accomplished by adjusting the rheostat in the shunt field. Speed variations of as much as 4:1 are obtained in this way. To obtain greater variation or to obtain speeds below the normal base speed it is possible to

insert armature resistance. Speed control by armature resistance gives very poor speed regulation with varying load and is inefficient.

Although field control on shunt motors is common, many d-c applications are for special machines that require a flexibility of control beyond that available with simple shunt motors. It is often necessary to provide a motor-generator set to supply the direct current, and since a special power source is provided, it is usual to use the d-c generator as part of the control system. In this case full shunt-field excitation is maintained on the motor. The generator voltage is then varied to obtain speed control and reversal of the motor.

Speed control may be obtained on induction motors by varying the frequency, and this is done in some high-speed applications where a frequency changer is used. This frequency changer is in the form of a wound-rotor induction machine driven in the reverse direction to obtain a frequency from the rotor circuit higher than that of the supply.

The entire problem of motor application is essentially that of matching a power supply from a group of electric wires to a driven shaft on a machine. The number of steps necessary in the transfer is dependent upon the form of electric power available and upon the variability and sensitivity of power requirements at the shaft. To properly select a motor and its control it is therefore necessary to follow through a number of steps that will be described in subsequent paragraphs.

Steps in selecting motor and control

(1) **Specify electric power available.** Industrial plants are ordinarily supplied with either d-c or a-c power but seldom with both. The power system used in the plant will therefore control to a great extent the selection of the motor for a particular industrial application. When selecting a motor, therefore, specify first the voltage, phase, and frequency of the power supply if a-c; if d-c specify the voltage and the fact that it is d-c.

(2) **Determine the horsepower needed to drive the load.** The horsepower needed to drive the conveyer, pump, or other machine may be determined in several ways. If it is a machine that is being purchased, the power required to operate it may usually be obtained from the manufacturer. If it is a machine that is designed locally, it will usually be best to run a test to determine the power for various operating conditions. To do

this a motor of a size larger than is estimated to be required may be temporarily connected or belted to the machine and the input measured with a wattmeter. The horsepower required is

$$\text{Hp} = \frac{\text{watts measured} \times \text{motor efficiency}}{746}.$$

The motor efficiency can be estimated with sufficiently accurate results to determine motor size.

If the machine to be driven is not available, as in the case of a machine that has been locally designed but not yet built, the horsepower must be computed from basic mechanics, or the motor size must be determined from handbooks. This procedure usually leads to the selection of a motor that is too large, since the tendency is always to allow safety factors in estimating. (Considerable economies may often be accomplished by a survey of the loading of motors in industrial plants and by replacing motors with those of smaller sizes where possible. This reduces power losses and improves power factor.)

(3) **Determine speed of motor.** The speed of the driven shaft must be known and this speed must be matched to standard motor speeds. If the shaft has a high speed that corresponds to one of the standard motor speeds, then direct connection is indicated. Since high-speed motors are much cheaper and more economical than low-speed motors, low-speed shafts on machines must be matched to motor speeds by belts and pulleys, by gears, or by the use of gear motors.

Where a variation in speed is required, it must be determined whether it will be obtained by some form of variable-speed mechanical drive or by variation in motor speed. If the speed variation is to be obtained by an adjustment of motor speed, then the various types of adjustable-speed motors must be considered that will fit the power available. The characteristics of these different motors have been briefly given in Table 13-2 and have been described in greater detail in the chapters on d-c and on a-c motors.

(4) **Determine starting torque required.** The manufacturer of the driven machine can often give the torque required to start. If the machine is in the plant, a spring balance on the end of a pipe wrench will give the pound-feet required to start the machine. The ratio of this torque to rated full-load torque of the motor chosen is then computed; and if it is less than about 1.75, a normal-torque or general-purpose induction motor may

be used with across-the-line starter. If this ratio is greater than 1.75, it is advisable to use a high-starting-torque induction motor. If d-c power is available and if the control is properly adjusted, the d-c motor will give adequate starting torque.

(5) **Determine the proper type of motor enclosure.** If ordinary industrial conditions exist, it is best to use a standard open motor because it is least expensive and most easily ventilated. Splash-proof motors, as indicated in the name, are used in packing houses and other places where splashing liquids exist. These motors will be satisfactory for outdoor installation in mild climates, but regular checks of insulation should be made. For outdoor installation the best motor is the totally enclosed fan-cooled type. In extreme cold these may require heaters for the lubricant. These totally enclosed motors will also be used when excessive moisture and fumes are prevalent. The use of special insulation is often desirable. For explosive gas atmospheres one of the several types of explosion-proof motors will be used. Special consideration must be given to such installations, and qualified representatives of the motor manufacturer will assist in the motor specification.

(6) **Decide on type of bearing.** Sleeve bearings are entirely satisfactory for general utility where the motor is installed with the axis horizontal. Where there is a possibility of the motor receiving a very considerable end thrust or where it is mounted on a tilt or in a vertical position, ball-bearing motors are required. Ball-bearing motors may also be preferred where maintenance is likely to be inadequate.

(7) **Determine power line limitations.** If the motor is large, permission should be obtained from the power company before plans are made to start it across the line (full voltage). Check also on probable voltage variation at the point of installation to determine whether it will interfere with other operations in the plant. If reduced voltage starting is required, determine motor and control as indicated in Chap. 12.

Determine what advantage, if any, will be obtained with use of a synchronous motor with power-factor correction. If the advantages of power-factor correction justify the additional expense, and if the synchronous motor is a suitable driving motor, it should be specified.

(8) **Specify the motor.** With the above information and manufacturers' catalogues it should be possible to specify the motor. Even if there are some special features that require

expert advice, it is still important to assemble the above information as a guide to your counselor. Manufacturers' representatives can be very helpful, although the engineer in charge must always be responsible for the decision where conflicting recommendations are made.

The material in this chapter has been summarized in Table 13-2. The motor groups are divided on basis of speed characteristics. This table should be helpful, but the latest manufacturers' catalogue material should always be consulted as the final authority.

CHAPTER 14

Electron Tubes and Circuits (Diodes)

The place of vacuum tubes and circuits in engineering

As the extent of scientific knowledge becomes greater, new tools become available to the engineer. One of the most versatile of these tools has been the electron tube and its associated wealth of circuit possibilities.

Some of the desirable characteristics of the electron tube are listed briefly below. (The reasons for these characteristics will be developed in the study of the tube theory, and some of the possible applications then will be discussed.) First, the electron tube, due to the low inertia of electrons, can respond almost instantly to a control stimulation. In fact, for most commercial applications, the response time is so rapid that it can be entirely neglected and the tube can be considered as giving instantaneous response. Second, the tube requires an almost negligible quantity of activating or control energy which provides a sensitivity greatly needed by many commercial applications. Third, it is extremely versatile because, with special circuits to take advantage of the characteristics of these tubes, almost any desired response can be obtained. Currents can be controlled from microamperes up to thousands of amperes. Voltages can be obtained and controlled from microvolts to several hundred thousand volts.

Historical development of electron tubes

During Thomas Edison's early experiments with the electric lamp, he observed that if a metallic plate were sealed in the glass bulb of the lamp, a current would flow from the plate to the filament when the plate was connected to the positive terminal of the filament, but that no flow would occur when the plate was connected to the negative terminal. This discovery was made and recorded in 1883, but the phenomenon was not understood. J. J. Thompson explained the phenomenon when he discovered and identified the electron at the turn of the

century. Following the appearance of the Fleming valve in 1904, and the invention of the audion in 1906 by Dr. Lee De Forest, the commercial development of vacuum tubes progressed rapidly. Tubes of a wide variety of sizes, types, and characteristics are now available for use.

The already extensive use of vacuum tubes for measurement and control in industrial operations is rapidly expanding. The opportunity for engineers to make contributions to the field of their particular specialization by the intelligent application of this tool is great.

Movement of molecules and electrons in a vacuum

The science of physics teaches that the atmosphere is composed of a tremendous number of molecules moving about in random fashion, bumping into each other and striking the solid surfaces in contact with the atmosphere. The rapidity of movement is dependent upon temperature. The average distance that the molecule will travel before colliding with another molecule is called the *mean free path* of the molecule under the stated conditions. This mean free path at ordinary atmospheric pressure is only about four millionths of an inch. That is, the average molecule only goes about four millionths of an inch in its random movement before it strikes another molecule, and the two bounce like a couple of billiard balls to assume new directions and velocities.

In a vacuum tube most of the molecules are pumped out of the tube so that only relatively few are left. In modern vacuum tubes only about one ten-billionth of the original molecules are left in the tube. This still leaves fifty billion or more molecules per cubic inch, but, since they are so small, the gas molecules or ions have a greatly increased average distance between collisions. Under the high vacuum obtained in most electron tubes, this path will be on the order of two or three inches. The electrons, being much smaller than the gas molecules, will have a mean free path or average distance between collisions of four or five times the gas molecules. Their mean free path is, therefore, of the order of magnitude of ten inches. Since in most vacuum tubes the distances between various parts of the tube are usually only a fraction of an inch, most electrons may be expected to go from one part of the tube to the other without striking gas molecules. Thus, even though there are actually a considerable number of gas

molecules in each vacuum tube, the electronic behavior is similar to that in a true vacuum. This ability to control the motions of electrons without the complications of molecular collisions is usually desirable, and so, high-vacuum tubes are evacuated as much as possible and in all cases sufficiently to assure that the mean free path is considerably greater than the distance between the tube elements.

Thermionic emission—the source of electrons

The movement of molecules and electrons in solid bodies such as metals is greatly restricted as compared to their movement in gases. The molecules do, however, vibrate within their crystal structure and electrons go bouncing to and fro with a general drift in the direction of the electric field, and thus constitute the electric current discussed in earlier chapters. The agitation or velocity of movement is dependent upon the temperature of the conductor. The surface of the conductor provides a sufficient barrier, however, so that at ordinary temperatures no electron gets beyond the surface.

At high temperatures, however, the velocity of some of the electrons becomes so great that their kinetic energy carries them through the surface. This results in a cloud of electrons surrounding the hot metal in very much the same manner that the earth is surrounded by its atmosphere. Eventually this atmosphere of negatively charged electrons will drive just as many electrons back into the metal (because of the repelling action of the electric forces) as escape because of high kinetic energy, and equilibrium will be obtained. This atmosphere of electrons surrounding a hot body is called a *space charge.* A tungsten filament or other body, when placed in a vacuum tube with a positively charged plate or anode and heated until it is a source of electrons, is called a *cathode.*

Since the molecular attractive forces at the surface are different with different materials, varying temperatures are required to give sufficient kinetic energy to the electrons to permit them to get beyond the effects of the surface barrier.

Types of commercial cathodes

Extensive experimentation has developed three commercial types of cathodes. The first is a pure tungsten filament. This operates at a high temperature and thus requires considerable energy to maintain the temperature. It has the advantage,

however, of being extremely rugged and will not be harmed by the bombardment of the few positively charged ions that are created in the tube operation. It is used, therefore, for high-voltage tubes where this bombardment would be destructive to the other types of cathode.

The second type of cathode is quite similar except that the tungsten filament is impregnated with thorium oxide. The thorium gradually migrates to the surface of the filament and forms a molecular layer of thorium which permits a larger supply of electrons at a much lower temperature. Although quite stable, the surface may be damaged by the high-velocity positive ions that are found in high-voltage tubes. This type of cathode is therefore used in tubes of medium voltage.

The third type of cathode, and that used for low-voltage tubes, is composed of a metal surface coated with the oxides of barium or strontium. This cathode provides a large supply of free electrons at a comparatively low temperature and so requires only a small loss in the cathode heating circuit. This type of cathode, however, will not withstand the bombardment of high-velocity positive ions, and so is limited in its use to tubes of comparatively low voltage. Oxide-coated cathodes are used in nearly all of the small radio receiving tubes and in many of the intermediate sizes as well.

The tungsten and the thoriated tungsten are usually in filament form, and the tube circuit is connected directly to the filament. In the case of the oxide-coated cathodes it is possible to form the oxide coating on the filament, but it is more common to insulate the filament or heating circuit and place it in a nickel or nickel-alloy sleeve. The oxide coating is then formed on this sleeve which is connected to the external circuit by a separate wire. Since with this design the heater is insulated from the cathode, the heaters in various tubes may be connected in series or parallel as desired, thus giving greater flexibility in circuit design. It likewise permits alternating current to be used as the heating source without causing hum, or interference with the signals.

Conventional symbols for representing vacuum tubes

A vacuum tube is made up of a cathode or electron source, an anode or plate to which the electrons are attracted, and often one or more grids or wire meshes which are used to control the electron flow. These are all sealed in a glass or metal

envelope which has been evacuated to a very low pressure. The indication of connections in a circuit requires that certain conventional symbols be used. These are shown in Fig. 14-1.

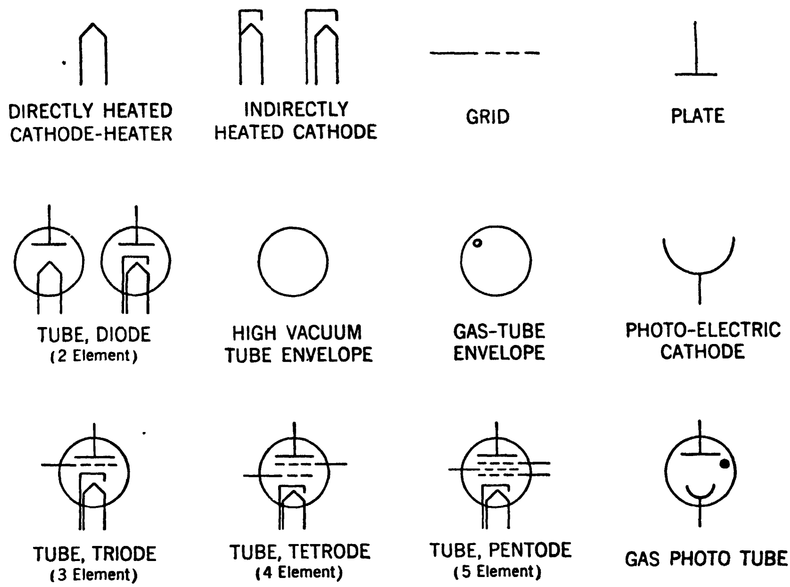

FIG. 14-1. Conventional Circuit Symbols.

High-vacuum diode tubes

The simplest type of electron tube is one composed of a cathode, or source of electrons, and a plate or conducting surface which is used to attract and collect these electrons by virtue of its positive polarity. The cathode may be of the filament type, using tungsten or thoriated tungsten, or of the indirectly heated type with an oxide coating. In order to determine the performance of the diode in an electric circuit and so use it to best advantage, it is necessary to study its characteristics. An understanding of the behavior of the diode is likewise necessary for an intelligent interpretation of the performance of the triode and other more complicated tubes.

The formation of an electron cloud or atmosphere around the cathode has already been described and is illustrated in Fig. 14-2(a). A cathode of the oxide-film type, using a cylindrical sleeve heated by an insulated filament, is located in the center of a metal cylinder which acts as the plate or anode. The cloud

of electrons is indicated as dense near the cathode and becoming quite thin as the plate is approached.

When an electron is emitted from the hot cathode, its velocity carries it into this cloud of electrons. It is repelled by the negative charges of the other electrons and so loses velocity. A few of the electrons having the highest velocity will carry past the center of the cloud, but most of them will be turned

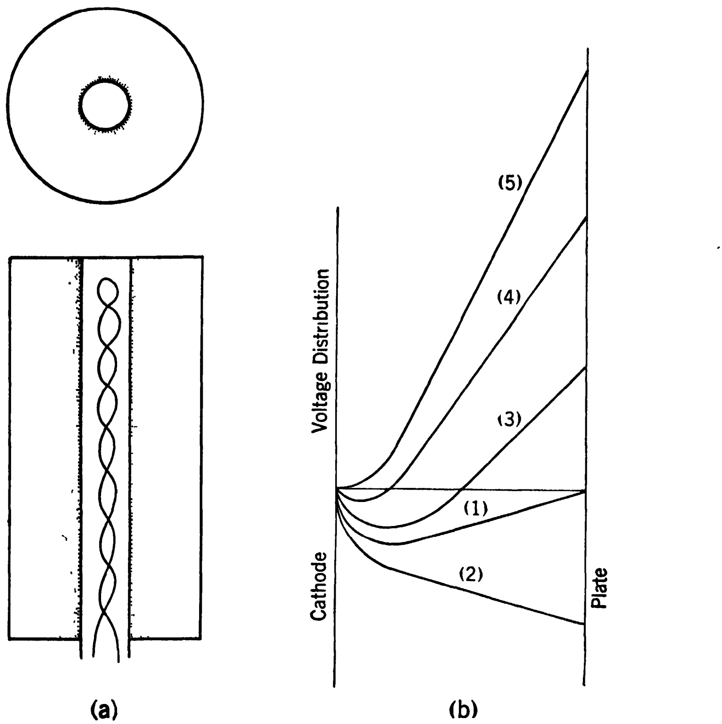

FIG. 14-2. Space Charge and Voltage Distribution in a Diode.

back and will re-enter the cathode. If the plate has a positive potential, the electrons on the outer edge of the electron cloud or *space charge* are attracted to the plate. Because of the inertia of the electrons, their migration to the plate requires a small but finite time and these outer electrons are quickly replaced by the electrons emitted from the cathode.

The acceleration of the electron is dependent upon the field strength or rate of potential variation with distance. If a study is made of the potential variation from cathode to plate for various plate potentials, much can be learned regarding the

performance of the vacuum tube. In Fig. 14-2(b) a number of curves are drawn showing the variation in potential between cathode and plate. The curve marked (1) represents the condition with no voltage on the plate. Under this situation the electrons that are emitted are practically all driven back into the cathode by the space charge, and no electron drift or current is obtained.

When a negative voltage is put on the plate, as indicated in curve number (2), any electrons that get beyond the center of the space charge are repelled by the plate and electron drift is thus definitely stopped.

When a positive voltage is applied to the plate or anode, however, as in curve (3), the electrons on the outer edge of the space charge are attracted to the plate and a drift of electrons results. This drift constitutes an electric current from the plate to the cathode.* As the voltage of the plate is raised to the successively higher values, as in curves (4) and (5), the acceleration of the electrons toward the plate is increased and the space charge is decreased. In fact, when the voltage of the plate becomes as high as that of curve (5), the space charge has practically been eliminated and the electrons are attracted to the plate as fast as they are emitted. Thus, when the voltage is raised still further, no further current is obtained because all of the electrons emitted are already being drawn to the plate.

The results of these phenomena are observed experimentally when, with a constant value of filament current, the voltage of the plate is gradually raised and the resultant current is measured. With a filament current of I_{f_1}, the current is found to vary as indicated in Fig. 14-3. The current is zero as long as the plate is negative. It increases continuously with an increase of positive voltage until a critical value is reached, after which it remains essentially constant. These results confirm the conclusions reached by the analysis of the field and space charge of Fig. 14-2. When the current is limited by the number of electrons emitted, the tube is said to have reached temperature saturation.

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* The choice of positive and negative polarity in electricity was made long before the existence of electrons, which have a negative charge, was known. This arbitrary selection has resulted in the positive direction of current flow being opposite to the direction of electron drift, and has thus caused some confusion. In this text the standard terminology of current flow will be used, and, when electron drift or electron flow is used, it will be so designated.

If the experiment is repeated with a higher value of filament current, thus giving a higher temperature cathode, a new curve of current vs. plate voltage is obtained and is labeled as I_{f_2}. It is observed that over much of the range there is very little difference between the two curves and that they do not tend to deviate appreciably until the region marked (a) is reached. With plate voltages below this critical magnitude, the current is controlled primarily by space charge. For

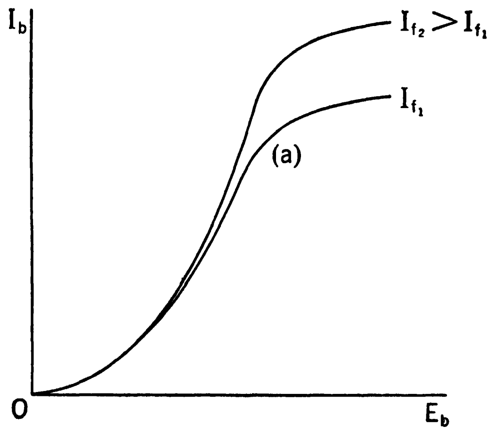

FIG. 14-3. Limitation of Diode Current by Temperature.

voltages greater than this, the current is primarily controlled by the number of electrons emitted; and since the emission is greater with the higher temperature cathode, the second curve becomes constant at a higher current value.

Tube rating

When the electrons reach the plate, they have received considerable acceleration and thus contain an appreciable amount of kinetic energy. The kinetic energy per electron is directly proportional to the voltage between plate and cathode. The bombardment of the plate by these electrons produces heating; and since the tube operation requires that the plate be relatively cool, this usually limits the current flow that may be permitted. Very seldom in commercial applications, therefore, does the current in a vacuum tube reach a value even approaching temperature saturation.

Thus the diode normally operates with a considerable space charge, and the electron drift or current flow is determined by a

balance between the plate voltage and space charge. The voltage of the plate and the current flow through the tube are usually dependent upon the circuit in which the tube is used.

Gas tubes

Although many electron tubes are highly evacuated, another type of tube is becoming increasingly important in the field of industrial application. In these tubes a small amount of gas is allowed to remain, and so they are known as *gas tubes*.

The movement of molecules and electrons in a gas was discussed in an earlier paragraph. Electrons in an electric field are accelerated, and the velocity attained is dependent upon the voltage through which they drop. Thus, an electron that has fallen through a potential of 10 v will have a velocity somewhat greater than 1000 miles per second; and when it has fallen through 20 v, it will have a velocity greater than 1500 miles per second. When an electron obtains this high velocity, it has sufficient energy to tear an electron from some of the gas molecules. The potential through which an electron must drop before it has sufficient kinetic energy to disrupt a normal atom of a gas, and thus produce an electron and a positively charged ion, is called the *ionizing potential* of the gas. For the gases used in electron tubes, this ionizing potential is between 10 and 20 v.

The operation of the tube is influenced greatly by the amount of gas in the tube. When a large amount of gas is present, the length of the mean free path is so short that the free electrons never have a chance to attain an ionizing velocity before they strike a molecule and lose much of their velocity. As the amount of gas is reduced, the length of the mean free path is increased, and a few electrons achieve ionizing velocities. When this occurs, the current for the same potential drop across the tube is increased. As the gas pressure is still further reduced, the number of electrons attaining ionizing potential is increased and the current flow increases. This process continues until the length of the mean free path begins to approach the distance between the cathode and anode or until the number of gas molecules becomes so few that ionizing collisions are reduced instead of increased when the pressure is reduced. Gas tubes are manufactured to operate at a pressure close to the optimum ionizing condition.

When the gas in the electron tube is ionized, it not only

supplies additional electrons for current flow but also provides a large number of positive ions which, because of their large mass in comparison with the electron, drift relatively slowly toward the cathode. These positive ions tend to neutralize the space charge and thus facilitate the flow of electrons from the cathode to the plate.

The presence of the gas produces a tube which has a much different current-voltage characteristic than that of the high-vacuum diode. This is shown very strikingly in Fig. 14-4, where

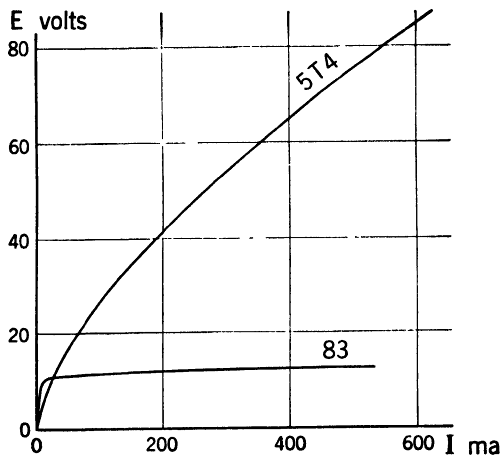

FIG. 14-4. Plate Characteristics of a High-Vacuum (5T4) and Gas (83) Rectifier Tube.

the current-voltage characteristics of a high-vacuum diode and a mercury-vapor diode are plotted on the same chart. It is noted that when a potential difference of 12 v is reached, the current increases almost indefinitely with no increase in voltage.

Since diodes are used most extensively for rectifier service, and since the voltage drop across the tube represents a power loss, the gas diode is much more efficient than the high-vacuum diode. The current is limited by the resistance of the load, with only sufficient voltage appearing across the tube as is needed to supply the ionizing potential.

Rectifier circuits

Electrochemical processes, variable-speed motors operated from a-c power lines, and many types of electrical instrumentation require the use of electronic rectifiers, which are, therefore, one of the most common electronic devices. These rectifiers

vary in size from a single tube a fraction of an inch in diameter to units of six or twelve tubes, each more than a foot in diameter and supplying several thousand amperes of direct current. In fact, some large rectifier installations are composed of as many as twelve units of twelve tubes each, all operating in parallel so that they might be said to use a total of 144 of these large diodes.* Rectifier voltages vary from a few volts needed for small instruments to several hundreds of thousands of volts for testing cables and for operating Cottrell-type precipitators to recover waste material in chemical processes.

The single-phase rectifier

When a single diode is used in a circuit similar to the one shown in Fig. 14-5, it is said to be a half-wave rectifier because

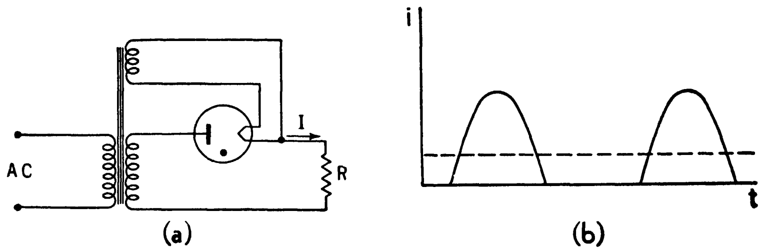

Fig. 14-5. Half-Wave Rectifier.

it passes current during only the half wave in which the plate is positive with respect to the cathode. The output current is shown in Fig. 14-5(b) as occurring in pulses of sinusoidal form. This assumes that the voltage drop in the tube is negligible. The average value of current is shown dotted and is $(1/\pi)$, or 32 per cent of the maximum value. The half-wave rectifier is seldom used because the periodic or pulsing character of the current is usually undesirable.

When two diodes, or the equivalent *double diode*, which is a single tube with two anodes and a common cathode, are used in a circuit such as shown in Fig. 14-6, a full-wave rectifier is obtained. This gives a current such as is shown in Fig. 14-6(a), and, although it is not continuous, it is much more nearly so than the current from the half-wave rectifier.

In this circuit the transformer secondary is provided with a center tap to which the load is connected. The current, which

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* See Chapter 16, page 325.

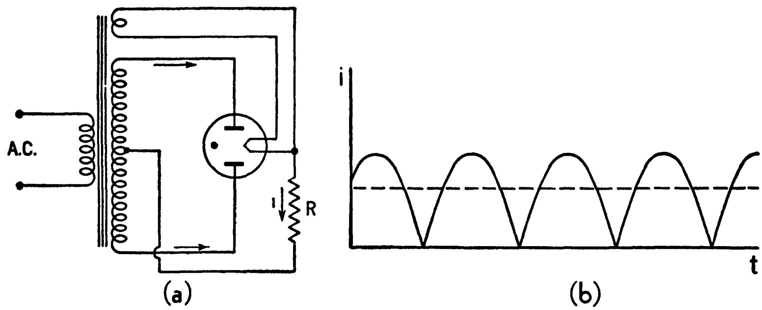


FIG. 14-6. Full-Wave Rectifier.

is unidirectional in the load, flows first in one direction and then in the other in the transformer, and so normal a-c flow is obtained in the transformer. The average current flow in the load is twice that of the half-wave rectifier of the same voltage. This gives a value of $2/\pi$, or 64 per cent of the maximum value.

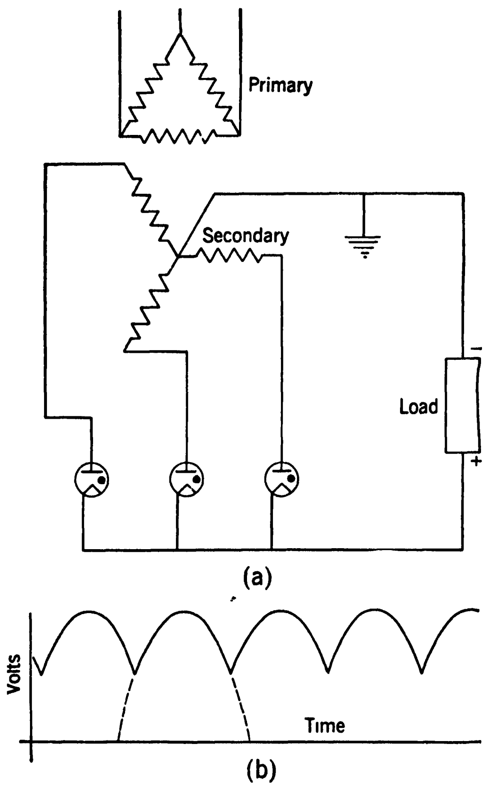


FIG. 14-7. Elementary Three-Phase Diode Rectifier.

Polyphase rectifiers

When polyphase voltages are available, it is possible to use more diodes and obtain both additional current-carrying capacity and greater uniformity of voltage. Thus, using a three-phase power source with the secondary of the transformers connected in Y, the connection diagram of Fig. 14-7(a) is obtained. Since the voltage across a resistance is directly proportional to the current flow, the voltage of cathodes above ground is proportional to the current flow. This is shown

in Fig. 14-7(b). Current will flow in any diode only when the anode is more positive than the cathode. This occurs, in a three-phase circuit, only during that one third of the cycle in which the voltage of the anode is higher than that of the anodes of either of the other tubes. Each tube, therefore, carries current only one third of the time, and since the heating of the anode is proportional to the average current, three times as much current can be carried as with a single-phase half-wave rectifier using the same type of tube. The average voltage and current have risen to 83 per cent of the maximum value.

When a transformer connection similar to that shown in Fig. 14-8 is used with six diodes, a six-phase rectifier results. Here, since each tube carries current but one sixth of the cycle, the capacity of the rectifier is still the sum of the average current capacity of all the tubes. The average current and voltage have now reached 95 per cent of the maximum value and so are quite constant.

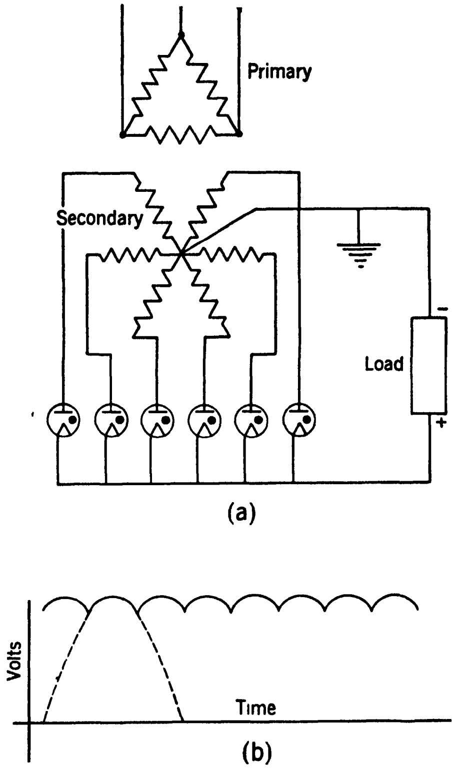


FIG. 14-8. Elementary Six-Phase Diode Rectifier.

This reduction in the fluctuation of current and voltage is one of the important advantages of the polyphase rectifier. In fact, in some rectifiers the transformer connections are arranged to obtain as many as twelve phases and thus develop even more constant voltage.

The above conclusions have been based upon the assumption that the voltage drop in the tubes was small in comparison to the total voltage. Where the tube drop of ten to fifteen volts is an appreciable portion of the total voltage, some corrections in the analysis are necessary for accurate results.

Mercury-arc rectifiers

The oldest form of electronic rectifier is the mercury-arc type. In this type, a pool of mercury is used as the cathode. It becomes a source of electrons when an arc is drawn from an auxiliary contact. After the initiation of the arc, it is maintained by current flow from one of several electrodes which draw current continuously through the tube. The mercury surface is bombarded by the positive ions in the arc stream, which produce a high temperature at the surface of the mercury, which in turn provides the source of electrons. The chief advantage of this type of tube is the ability of the mercury pool to supply an almost limitless number of electrons, thus enabling the tube to carry a large current.

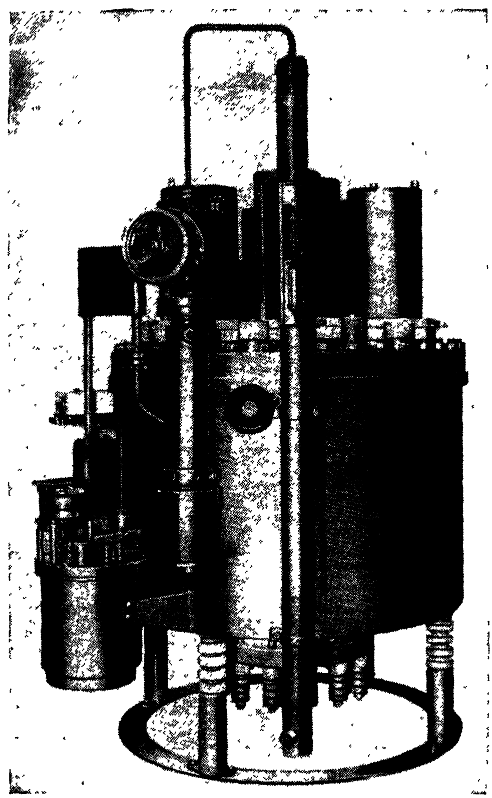


FIG. 14-9. Metallic-Tank High-Voltage Mercury-Arc Rectifier.

Mercury-arc rectifiers are built in large sizes and usually have three, six, or more anodes for a single mercury pool. Fig. 14-9 shows a photograph of a large steel tank rectifier, and Fig. 14-10 shows a schematic wiring diagram for a six-anode rectifier.*

Operation of gas-tube rectifiers

Although hot-cathode gas tubes are more efficient than vacuum tubes for rectification, they will not withstand such

* A further discussion of polyphase rectifiers is included in Chapter 16, where the use of the ignitron for rectifiers is explained. The ignitron itself is discussed in Chapter 15.

high inverse voltages. That is, the high voltage from plate to cathode may cause some ionization, and when this occurs, the tube may carry current in the reverse direction, which (in polyphase rectifiers) creates a short circuit. For very high voltages, it is usual, therefore, to use vacuum-tube diodes as rectifiers.

When gas tubes are used, the cathode must be permitted to reach normal temperature before the plate voltage is applied. If the plate voltage is applied when the cathode is cold, an insufficient number of electrons will be available to supply normal current, and an excess of potential will develop across the tube. This will give the positive ions so much acceleration that they will damage the oxide coating of the cathode.

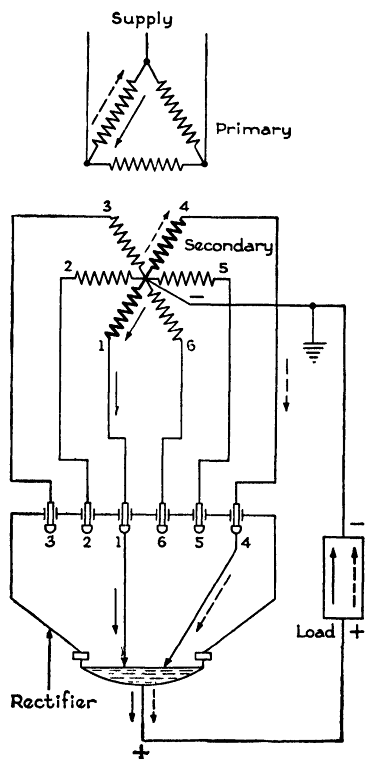


FIG. 14-10. Schematic Diagram for Six-Anode Mercury-Arc Rectifier.

Filter circuits

Although the voltage obtained with even a single-phase rectifier is satisfactory for many industrial applications, certain instruments as well as radio transmitters and receivers require much more constant voltage. This constant voltage is obtained from rectifiers by the use of filter circuits composed of inductors and capacitors, or of resistors and capacitors.

The most common filters are composed of inductors and capacitors as shown in Fig. 14-11. The inductance, being of low resistance, has a very small voltage drop as long as the current is constant. When, however, the current changes, voltages are set up in the inductance tending to oppose the change. This tends to compensate for voltage variations and thus produce a more uniform current in the load.

A small voltage variation will nevertheless appear across the load, and since the condenser is in parallel with the load, any change in voltage will cause a current to flow in the condenser circuit. These currents in the condenser tend to stabilize

the voltage across the load still further and will usually give satisfactory load voltage.

In cases in which very accurate voltage stabilization is required, a second filter section is added as shown in Fig. 14-11(b). Each section will permit only a small percentage of the voltage ripple to pass through it. For instance, if one section permitted 5 per cent of the voltage ripple to pass, then two sections would pass only one fourth of 1 per cent of the

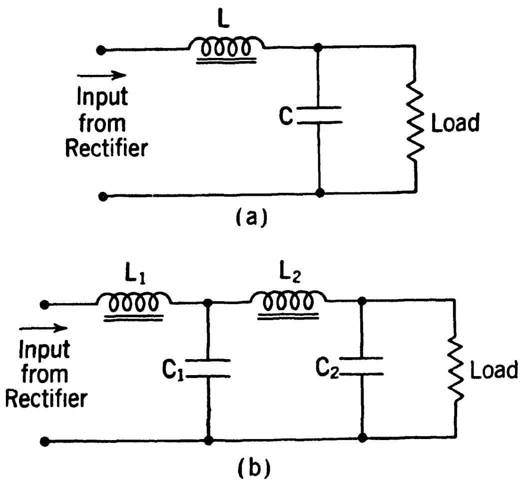


FIG. 14-11. Choke-Input Filter Circuits. (a) Single-Section. (b) Two-Section.

voltage ripple. Since the ripple voltage is much smaller than the d-c voltage, a single filter section will usually reduce the ripple voltage to about 1 percent of the d-c value even in a single phase rectifier.

The phototube

One of the very useful two-element tubes is the phototube, which is conducting when light strikes it. Some substances emit electrons when light shines on them. This property is used in the manufacture of the phototube. A large surface of light-sensitive material forms the cathode, while a rod of metal that is not light-sensitive acts as the anode. These are placed in an evacuated glass envelope and the anode is given a positive potential. The circuit is shown in Fig. 14-12, and the current flow is proportional to the intensity of illumination on the tube. Variations of intensity of light appear as variations in voltage

across the load resistance which, when amplified, may be used to trigger thyratron tubes, which will be studied in Chapter 16.

Phototubes are made both with high vacuum and as gas tubes. When they are highly evacuated, they are more stable and respond more quickly. The advantage of the gas tube is in its ability to develop a larger current for the same electron emission due to ionization, and therefore it is somewhat more sensitive.

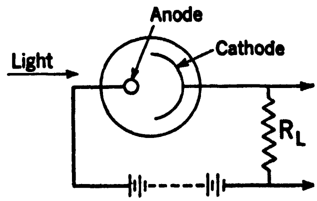


FIG. 14-12. Circuit for Phototube.

These tubes are used to perform many useful and ingenious control functions, such as opening doors, counting, sorting, grading, maintaining precise register in printing, actuating safety devices, and a host of similar jobs.

The glow tube

The last of the diodes to be discussed is the glow tube. In this tube, which is gas filled, the cathode is cold. Electrons are pulled from the cold cathode surface by a high-potential gradient or voltage differential at the cathode.

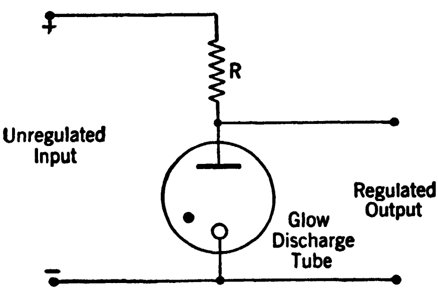


FIG. 14-13. Regulated Voltage Supply.

Since the emission of electrons requires a critical value of voltage, the glow tube is useful as a voltage control tube. In other words, it will maintain a constant voltage with a considerable current variation. This is shown in the circuit of Fig. 14-13, where the voltage across the tube will remain constant even though there is a wide variation of input voltage and current. These variations of voltage are absorbed in the voltage drop across the resistance R as the current varies. Such a constant voltage is often used as a reference voltage in electronic control devices to maintain a constant voltage supply, in spite of considerable variation of the source voltage. It may also be used as a constant reference for control of speed or for other industrial control operations.

CHAPTER 15

Electron Tubes and Circuits (Triodes and Other Multi-Element Tubes)

Construction and operation of a vacuum-tube triode

In the diode it was found that the current flow was considerably affected by the space charge. If a third tube element composed of fine wires spaced at relatively large intervals is

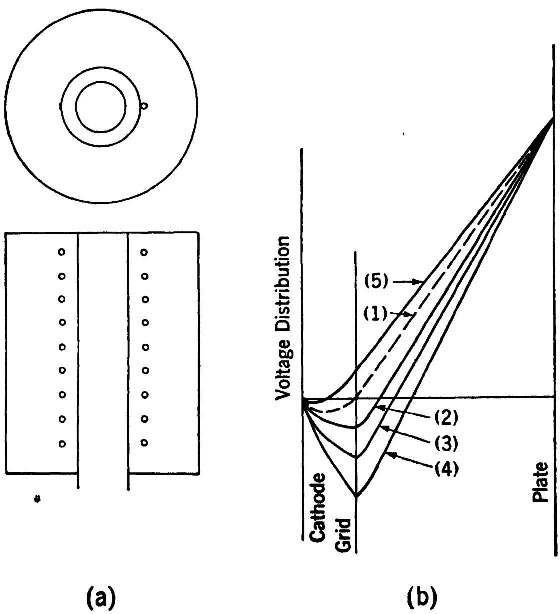


FIG. 15-1. Voltage Distribution in a Triode.

placed between the cathode and the plate, it is possible to control the density of the space charge and the current flow in the tube by a small variation in the voltage of this element. In early tubes, this third element was similar in its construction to a gridiron, and so it was called a grid. The name has continued even though the structural form of the element has changed.

In Fig. 15-1, a tube similar to the diode of Fig. 14-2 is shown. It has a heated nickel cylinder coated with barium or strontium oxide for a cathode and a cylindrical plate which has a positive polarity. It has, in addition, a helical wire grid which surrounds the cathode. The dash line indicating potential distribution in Fig. 15-1(b) corresponds to curve (4) in Fig. 14-2. If the grid is at cathode potential as indicated by curve (1), it will have no effect on the current flow. If, however, its potential is made more negative, it tends to repel the electrons of the space charge and will thus reduce the electron flow.

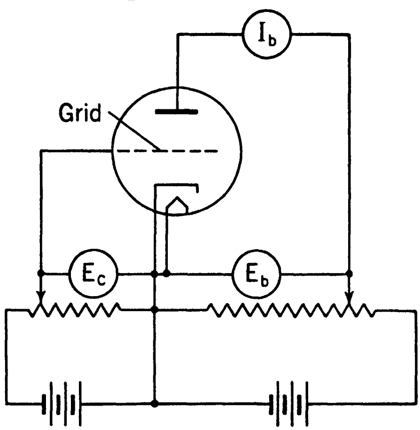


FIG. 15-2. Circuit for Obtaining Triode Characteristics.

The grid wires are widely spaced, and so, for small negative potentials, the electrons close to the grid will be repelled, but the electrons between the grid wires will behave very much as they did before. The space charge may be considered to bulge out between the grid wires similar to the tufts of a mattress. A somewhat more accurate description is that the higher-speed electrons which are directed toward the space between the grid wires may be deflected somewhat by the grid, but they will not be turned back. The slower electrons will be deflected so much that they will be turned back into the space charge. As the grid becomes more and more negative, many of the higher-speed electrons are deflected sufficiently so that they are turned back, and the electron flow becomes less and less. Eventually the grid becomes so highly negative that all electrons are turned back, and the flow ceases entirely. When the grid is given a potential other than that shown in (1), the potential distribution is radically changed as shown in curves (2), (3), (4), and (5). Although these curves do not indicate the leaky character or progressive action of the grid potential, they do show its average effect on the space charge.

Since the grid is located close to the cathode, a change in its potentials will have a much greater effect on current flow than the same change in potential when applied to the plate.

These relationships are demonstrated in a study of the characteristic curves of triodes.

Characteristic curves of triodes

The variation of plate or anode current with changes in grid and plate voltages may be studied by the use of the circuit in Fig. 15-2.

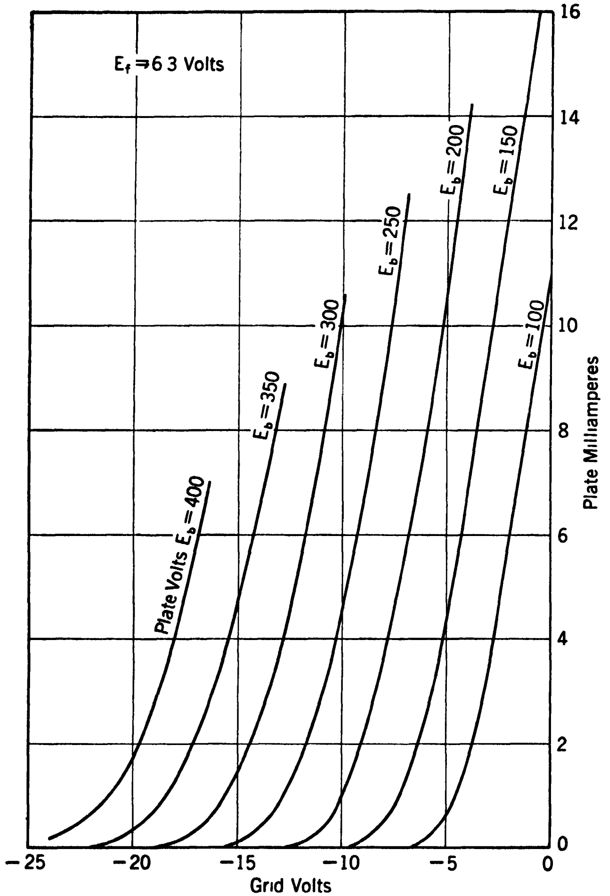


Fig. 15-3. Mutual Characteristics of 6J5 Triode.

In order to determine the effect of grid-voltage change, the plate voltage E_b is held constant and the grid voltage E_c is varied. The relationship between the grid voltage and plate current, which is called the *mutual characteristic*, can be plotted on a graph. Fig. 15-3 shows a family of such mutual charac-

teristics, each with a different constant plate voltage. By interpolation between curves, it is possible to determine the plate current for any combination of plate and grid voltages.

These curves are nearly straight lines for most of the operating range, so that it may be said that the variation in plate current is approximately proportional to the variation in grid voltage. This characteristic of a triode makes it useful as an amplifier, as will be explained later.

The same information given in Fig. 15-3 may be shown by plotting the variation in plate current with change in plate

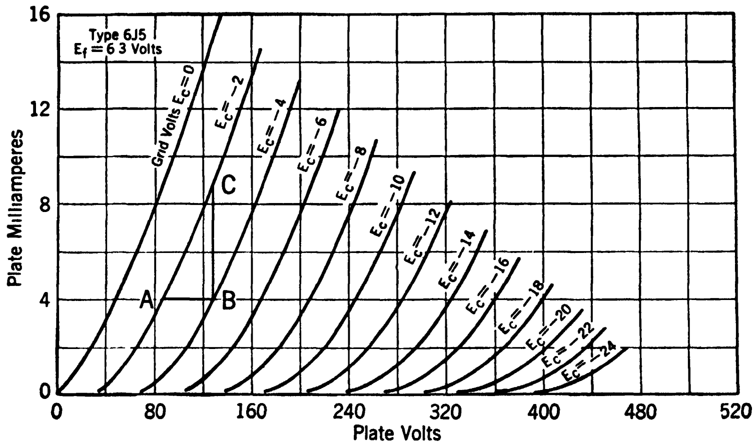


FIG. 15-4. Plate Characteristics of 6J5 Triode.

voltage when the grid voltage is held constant. Such a family of curves is shown in Fig. 15-4, and these are known as *plate characteristics*. Under some circumstances, the mutual characteristic curves are the more convenient, but for most circuit computations involving vacuum-tube triodes, the plate characteristics are preferred.

Tube characteristics

Although the families of curves discussed above are often used in the analysis of tube performance, it is sometimes desirable to use certain parameters called *tube characteristics*. These are known as the *amplification factor*, the *mutual conductance*, or *transconductance*, and the *internal*, or *plate resistance*.

The amplification factor, usually designated by the Greek letter μ (mu), is the ratio of the change in plate voltage to the

change in grid voltage necessary to maintain a constant magnitude of plate current. Thus

$$\mu = \frac{\Delta E_b}{\Delta E_c}$$

for constant plate current.

Referring to Fig. 15-4, the distance from A to B represents a plate-voltage variation of 40 v necessary to neutralize a grid-voltage change of 2 v. The amplification factor of this particular tube would, therefore, be equal to 20. In most triodes the amplification factor ranges from 10 to 40, although special tubes may have factors considerably beyond this range.

Mutual conductance, or transconductance, is designated by the symbol g_m, defined as the ratio of a small change in the plate current to the small change in grid voltage which produced it. Thus,

$$g_m = \frac{\Delta I_p}{\Delta E_c}$$

for constant plate voltage.

In Fig. 15-4, a change of 5 ma in plate current is represented by B-C and is caused by a grid voltage change of 2 v.

$$g_m = \frac{0.0050}{2} = 0.0025 \text{ mho.}$$

In order to get away from the decimal values, most tube transconductances are listed in terms of micromhos. Thus the transconductance of the tube shown in Fig. 15-4 would be 2500 micromhos. This constant is a measure of the effectiveness of the grid in controlling plate current.

The internal or plate resistance is designated by r_p and is defined as the ratio of a small change in plate voltage to the change in plate current which results. Thus,

$$r_p = \frac{\Delta E_b}{\Delta I_p}$$

for constant grid voltage. In Fig. 15-4, a change of 40 v on the plate A-B produces a change of 5 ma in plate current.

$$r_p = \frac{40}{0.005} = 8000 \text{ ohms.}$$

From the above definitions, it is evident that

$$\mu = r_p g_m.$$

It is thus possible to obtain any one of the tube characteristics if the other two are known.

To the experienced engineer, these characteristics give much information as to the merit of a tube for use in any particular circuit.

The triode as a relay or valve

A simple application of the triode is observed in its use as a very sensitive relay. In Fig. 15-5 it is used to control the temperature of a water bath. In this application, a sensitive mercury thermometer has electrodes mounted in the stem so that the mercury completes the connection to the grid circuit of a 6J5 tube, the characteristics of which are shown in Fig. 8-3. A 6-volt negative grid bias is used with a 1-megohm resistor in series. The plate potential is 100 v. When the thermometer does not complete the connection, the grid blocks nearly all the plate current so that the relay does not operate, and the power supply to the water heater is connected. When the temperature rises sufficiently for the mercury to make contact with the upper electrode, the grid is brought to zero potential, and the plate current jumps to 8 or 10 ma (depending upon the resistance of the relay), which operates the relay and disconnects the water heater. The mercury column of the thermometer is required to make and break a circuit carrying less than one one-hundred thousandth of an ampere at 6 volts, so little maintenance difficulty is experienced. This illustrates the previous reference to the very small amount of energy required to operate the grid, which is one of the chief advantages of the vacuum tube for industrial applications.

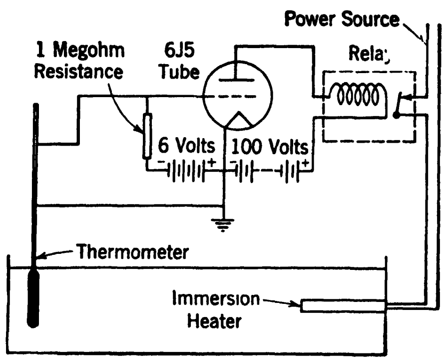


FIG. 15-5. Temperature Control on Constant-Temperature Bath.

The triode as an amplifier

Many of the uses of the vacuum-tube triode involve its ability to amplify a very small variation in voltage, which is impressed on the grid, into a much larger current or voltage

variation in the output circuit. This output may then be used to accomplish many different objectives. One instance of its use is to amplify the voltage variation across fine resistance wires cemented to the surface of machinery or structural elements, such as the wings of an airplane, so that stresses on power dives may be determined. These resistance wires are stretched or compressed as the surface of the structural element undergoes these same deformations. When the variations of resistance are amplified and recorded on oscillographs (ammeters which will respond to high frequency and record instantaneous variation of currents), the record can be used to study the problems of structural design. In most of these applications it is necessary to use several amplifier tubes in tandem, and the manner of doing this will be taken up in a later paragraph. The fundamental operation of the triode as an amplifier and methods of circuit computation must first be mastered.

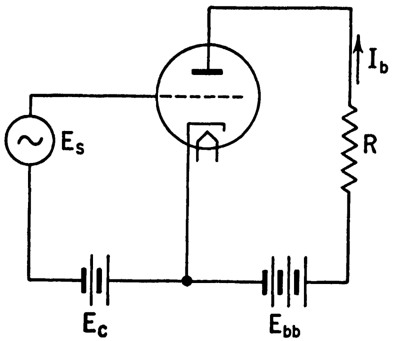


FIG. 15-6. Triode with Resistance Load.

The basic connection diagram is given in Fig. 15-6, which is observed to be similar to the connection used in Fig. 15-5 but without the complications involving the practical application. The vacuum tube has a mutual characteristic (including the load resistance) for the constant battery voltage E_{bb} as shown in curve A of Fig. 15-7. Grid voltage is plotted along the horizontal axis and plate current along the vertical axis as in Fig. 15-3. The constant grid-bias voltage E_c is shown at n and the corresponding plate current at o. Two additional diagrams are superposed on the mutual characteristic. One is a time-grid voltage variation in the form of a typical radio signal voltage shown in the lower left portion of the figure. The time starts at s and continues to g. The grid voltage is composed of the addition of the constant grid bias E_c and the signal voltage e_s.

The variation of the grid voltage produces a similar variation of plate current with time, and this is shown in the upper

right portion of the figure. Here time starts from s_1 and continues to g_1. When the grid voltage is at E_c, the plate current is at o. It then decreases to about one half of E_c and the plate current rises to b. When the signal voltage reverses, the grid bias increases to about one and one-half E_c and the plate current drops to a. This continues, with the plate current having a

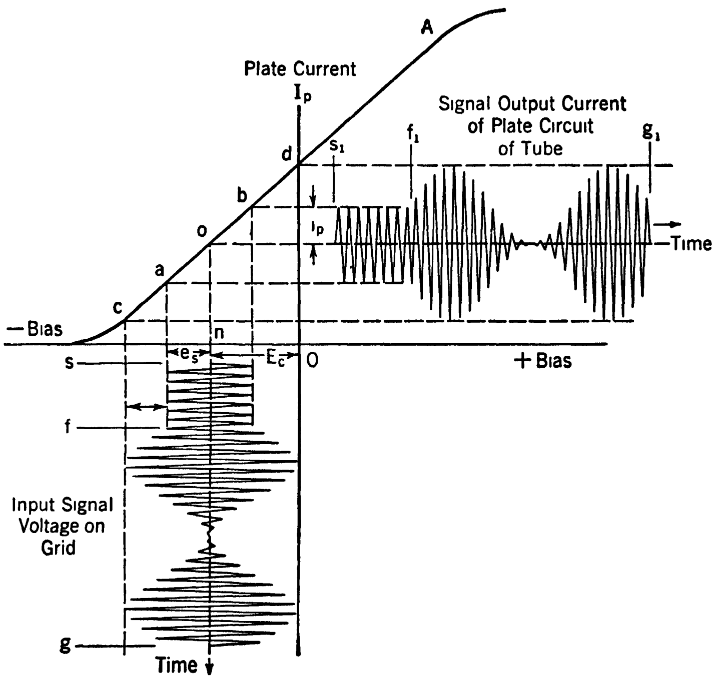


FIG. 15-7. Grid Voltage-Plate Current Relations in a Vacuum-Tube Amplifier. (The waves are sine waves. They are shown as straight lines for convenience only.)

variation with time that is exactly similar to the input signal voltage, as long as the portion of the mutual characteristic involved is a straight line.

The conversion of grid-voltage variation into a plate-current variation permits amplification of the signal, because this variation of current can in turn be converted to a variation in voltage through the use of the voltage drop across the load resistance R. In general, this variation of voltage across the load will be much greater than the input-signal voltage, which is demonstrated in the following paragraph.

The equivalent circuit of the triode

The plate resistance of the tube was defined as the ratio of plate-voltage change to plate-current change. This is shown in the form of an equivalent circuit in Fig. 15-8(a), where a variation of plate voltage e_p is introduced into the circuit. This will produce a resultant variation in plate current i_p.

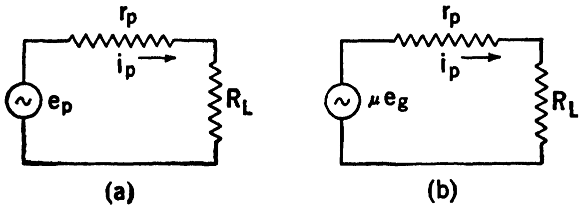


FIG. 15-8. Equivalent Circuit of the Vacuum-Tube Triode.

The form of this current variation is shown in Fig. 15-9. In the equivalent circuit the constant flow of I_b or average plate current is not considered, since it contributes nothing to the amplification of the input signal, and attention therefore is concentrated on the effect of the current variation. This

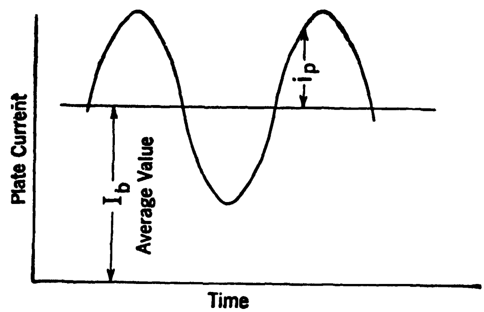


FIG. 15-9. Components of Plate Current in a Vacuum-Tube Triode.

equivalent circuit is of little use except to show how a similar equivalent circuit, using grid-voltage input, may be arranged.

Since a variation in grid voltage will produce μ times as much change in plate current as the same variation in plate voltage, the input voltage in Fig. 15-8(b) is considered to be μe_g. This equivalent circuit makes it possible to compute the voltage variation across the load resistance when the input grid voltage is specified. If the tube under consideration is the 6J5, whose characteristics are shown in Figs. 15-3 and 15-4, the μ is 20 and the plate resistance is 8000 ohms. As a particular case, the load

resistance will be assumed to be 20,000 ohms. If the variation of resistance of a wire strain gage cemented to the surface of an airfoil produces a voltage variation across it of $\frac{1}{10}$ mv, then the voltage inserted in the equivalent circuit would be

$$\mu e_g = 20 \times 0.0001 = 0.002 \text{ v.}$$

The change of current would be

$$i_p = \frac{\mu e_g}{r_p + R_L} = \frac{0.002}{8000 + 20{,}000} = 7.2 \times 10^{-8} \text{ amp.}$$

The change of voltage across the load resistance would be

$$e_{R_L} = i_p R_L = 20{,}000 \times 7.2 \times 10^{-8} = 1.44 \times 10^{-3}.$$

The ratio of the change in voltage across the load resistance to the grid voltage (the voltage amplification of the amplifier) is then

$$\frac{e_{R_L}}{e_g} = \frac{1.44 \times 10^{-3}}{10^{-4}} = 14.4.$$

If the voltage variation across the load resistance is now applied to the grid of another tube, the same process may be repeated.

Exercise 15-1. A triode having a μ of 12 and an r_p of 8000 ohms has a resistance load of 30,000 ohms. What will be the voltage variation available across the load resistance when a grid-voltage variation of 20 mv is impressed?

Exercise 15-2. What is the actual voltage amplification for an amplifier stage having $\mu = 18$, $r_p = 25{,}000$ ohms, $R_1 = 50{,}000$ ohms, and $e_g = 300\ \mu$v?

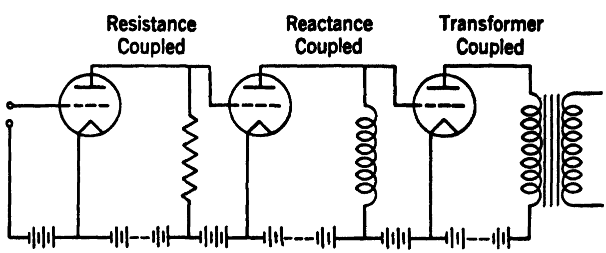


FIG. 15-10. Methods of Loading and Coupling Vacuum-Tube Triodes.

Amplifier stages in series

When several vacuum-tube triodes are used in series to amplify small variations in voltage, it is necessary to apply the voltage variation of the load of one tube to the grid of the next

tube. If each tube is supplied with its own plate voltage and the grid-bias voltage, then the various types of direct connection illustrated in Fig. 15-10 are possible. The impedance of the load (for instance, an inductive reactance) would appear in the equivalent circuit for each type of coupling used.

Since it is not economical to provide separate voltage supplies for each tube, it is common practice to insulate the grid of the

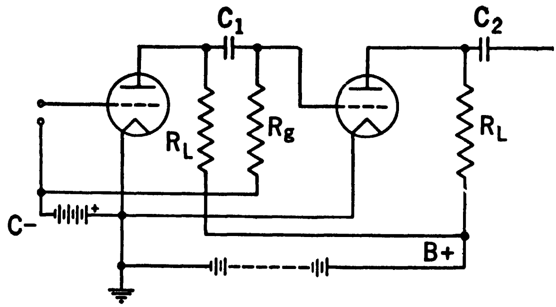


FIG. 15-11. Coupling Condensers (or Transformers) Permit a Common Set of Batteries to Be Used for a Number of Tubes.

following tube from the d-c component of the plate voltage by means of a condenser. Such a circuit, using a resistance as the plate load, is shown in Fig. 15-11. Here a common plate-voltage supply and a common grid bias are used. The voltage variation is transmitted to the grid of the second tube through the condenser C_1. The average grid voltage is maintained at the proper value by the high resistance (R_g) connection to the grid-bias voltage supply.

Example. Assume that the vacuum tube of Fig. 15-11 has a μ of 20 and r_p of 8000 ohms. The circuit values are as follows:

$$R_L = 20{,}000 \text{ ohms}$$
$$R_g = 1 \text{ megohm}$$
$$C_1 = 0.004\ \mu\text{f.}$$

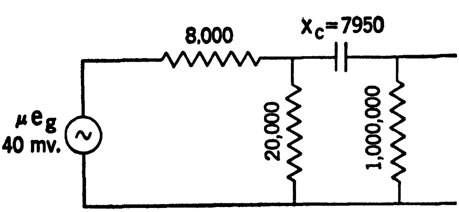


FIG. 15-12.

Determine the amplification of the first stage when the grid voltage is 2 mv at a frequency of 5000 cycles.

Solution: Since the resistance of the batteries to the high-frequency current is negligible, the equivalent circuit will be as shown in Fig. 15-12.

(1) The reactance of the condenser is

$$X_c = \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 5000 \times 0.004}$$
$$= 7950 \text{ ohms.}$$

(2) The impedance of the 20,000-ohm resistance with the 1 megohm and 7950 capacitive reactance in parallel can be computed as indicated in Chapter 7, or an approximation may be used. Since the 1-megohm resistance is 50 times the load resistance, an error of less than 2 per cent will result if the impedance of the parallel circuit is assumed to be 20,000 ohms. Therefore, this assumption will be made.

(3) The plate current flow is

$$i_p = \frac{\mu e_g}{r_p + R_L} = \frac{0.04}{8000 + 20{,}000}$$
$$= 1.43 \times 10^{-6} \text{ amp.}$$

(4) The voltage across R_L is

$$\begin{aligned} e_{R_L} &= 20{,}000 \times 1.43 \times 10^{-6} \\ &= 28.6 \times 10^{-3} \text{ v} \\ &= 28.6 \text{ mv.} \end{aligned}$$

(5) The voltage across R_g is equal to the voltage across the load resistance multiplied by the ratio of the resistance R_g to the impedance of R_g and X_c in series.

$$\begin{aligned} Z_g &= \sqrt{R_g^2 + X_c^2} \\ &= \sqrt{1{,}000{,}000^2 + 7950^2} \\ &= 1{,}000{,}000 \text{ approx.} \end{aligned}$$

The ratio of R_g to Z_g is therefore approximately equal to unity. The voltage across the grid of the next tube is then equal to the voltage across the load resistance, which is 28.6 mv.

(6) The voltage amplification for the stage is then

$$\frac{28.6}{2.0} = 14.3.$$

The above example indicates several general conclusions with respect to the capacitor coupling to the grid of the following circuit. First, if the grid-circuit resistance is large in proportion to the load resistance, the parallel R_g circuit will have very little effect on the voltage across the load resistance. Second, as long as the frequency of the signal is sufficiently large that X_c is small with respect to R_g, then the voltage impressed on the grid of the next tube will be approximately the same as the

voltage across the load resistance, because the voltages across X_c and R_g are added in quadrature. The condenser coupling of the grid, as the above type of circuit is called, greatly simplifies the problem of providing the various voltages needed for the operation of a vacuum-tube amplifier, because a single battery or rectifier can be used for many tubes in parallel.

As shown in Fig. 15-10, it is possible to use an inductive reactance load on the tube in place of the resistance R_L. This has the advantage that it lowers the over-all impedance of the equivalent circuit because the resistance of the tube and load reactance are in quadrature. The lower impedance makes possible a larger effective current variation and a correspondingly larger voltage variation across the load. The disadvantage of the inductive reactance load is that it is sensitive to frequency variation, and, where a wide range of frequencies must be amplified, it is not satisfactory. This voltage variation can be transferred to the grid of the next tube by a condenser coupling just as with the resistance load of Fig. 15-11.

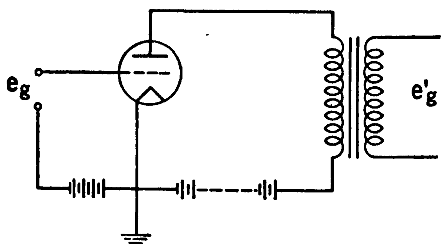


FIG. 15-13. A Transformer-Coupled Amplifier.

When a transformer is used as the coupling device, it insulates the grid circuit of the following tube from the d-c plate voltages, and so no coupling condenser is required. It has the further advantage that some additional voltage multiplication may be obtained by using a larger number of turns on the secondary winding of the transformer. The following example shows how this circuit may be calculated.

Example. The transformer-coupled circuit shown in Fig. 15-13 uses a transformer having three times as many turns on the secondary as on the primary. The primary has an inductance (with the secondary open) of 4 h and a resistance of 2000 ohms. Determine the voltage amplification when a voltage of 5 mv at 500 cycles is impressed on the grid of the tube. The tube has a μ of 20 and a plate resistance of 6700 ohms.

Solution: (1) The impedance of the plate circuit is

$$\begin{aligned} Z_p &= \sqrt{(r_p + r_t)^2 + X_t^2} \\ &= \sqrt{(6700 + 2000)^2 + (2\pi 500 \times 4)^2} \\ &= \sqrt{8700^2 + 12{,}600^2} = 15{,}400. \end{aligned}$$

(2) The signal plate current is

$$i_p = \frac{\mu e_g}{Z_p} = \frac{20 \times 0.005}{15{,}400} = 6.5 \times 10^{-6} \text{ amp.}$$

(3) The voltage across the primary of the transformer is (since the grid circuit of the following tube is essentially an open circuit)

$$\begin{aligned} e_t &= i_p Z_t = 6.5 \times 10^{-6} \sqrt{2000^2 + 12{,}600^2} \\ &= 6.5 \times 12{,}800 \times 10^{-6} \\ &= 83.2 \text{ mv.} \end{aligned}$$

(4) The grid voltage on the next tube will be three times the voltage of the transformer primary, since the turn ratio is 3 to 1, so

$$e_g' = 3 \times 83.2 = 250 \text{ mv.}$$

(5) The amplification ratio is

$$\tfrac{250}{5} = 50.$$

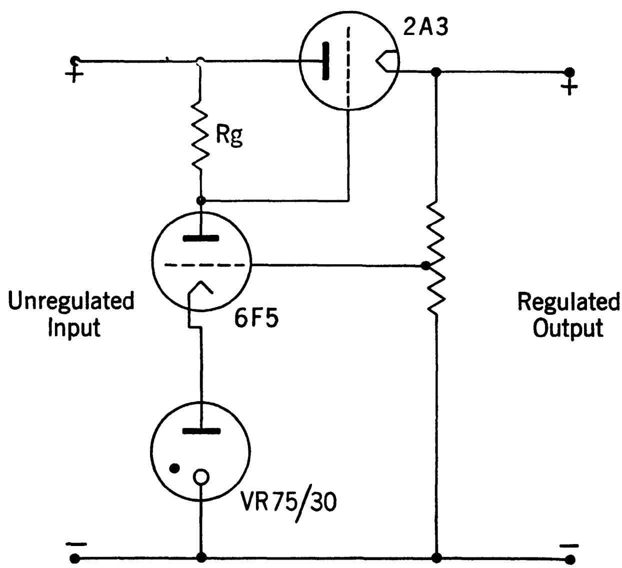


FIG. 15-14. Electronic Voltage Regulator.

The selection of the proper condenser and resistance values and other design features of amplifier circuits is beyond the scope of this text. The above explanation indicates, however, the manner of making performance calculations on amplifier circuits.

Exercise 15-3. An amplifier stage has, as a plate load, an inductor with a resistance of 1000 ohms and an inductance of 2 h. The μ of the tube is 25 and the plate resistance is 20,000 ohms. The grid of the following tube is coupled through a capacitor of 0.004 μf and has a grid resistor of 1 megohm. Compute the amplification ratio of the amplifier when a 1-mv signal is impressed on the grid at a frequency of (a) 10,000 cycles and (b) 100 cycles.

Exercise 15-4. Determine the amplification ratio for the following transformer-coupled amplifier at a frequency of 1000 cycles.
Tube constants:

$$\mu = 40 \qquad r_p = 25{,}000 \text{ ohms.}$$

Transformer constants, primary impedance:

$$r_t = 2500 \text{ ohms.} \qquad L_t = 3 \text{ h.}$$

Turn ratio, 4 secondary to 1 primary.

Exercise 15-5. The VR-75/30 tube in Fig. 15-14 will start to draw current at about 73 volts and this current will increase to about 30 milliamperes at 76 volts. The other two vacuum tubes are normal triodes. Analyze the operation of the circuit in maintaining a constant output voltage.

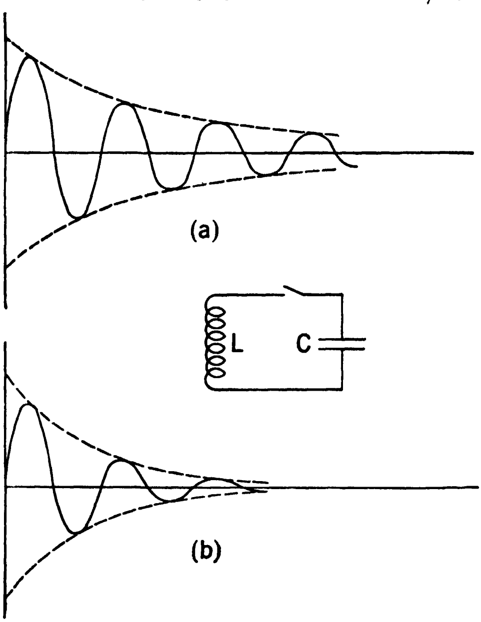


Fig. 15-15. Current Oscillations in an L-C Circuit.

High-frequency oscillators

In the study of a-c resonant circuits in Chapter 7, it was learned that energy is stored alternately in the magnetic field of a coil and in the dielectric field of a capacitor. When a coil with very small resistance is connected in parallel with a capacitor, very little external power is required to maintain a large oscillation of power between the coil and the condenser.

If an isolated coil and capacitor are connected after the condenser has been charged, this same oscillation of power will occur at the resonant frequency of the coil and capacitor.

It cannot be maintained indefinitely, however, as the energy stored in the capacitor will be dissipated in the resistance of the wires of the coil and connections. This is shown in Fig. 15-15, where the oscillations of current are shown for a coil and a capacitor. The magnitude of the oscillating current gradually decreases, just as a swing will gradually stop when no one pushes it. If the resistance of the coil is fairly high, the current will die down rapidly, as in (b), but if the resistance is very low the oscillations may continue for some time as in (a) of the figure.

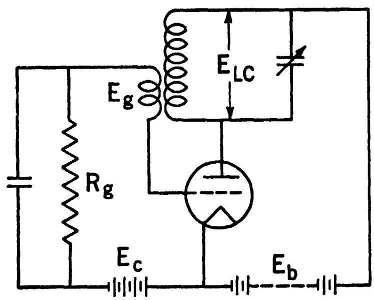


FIG. 15-16. A Vacuum-Tube Oscillator.

In a high-frequency oscillator, such a natural oscillatory circuit is connected in the plate circuit of a triode, as indicated in Fig. 15-16. A second coil having a mutual coupling with the main coil is connected to the grid of the tube. The voltage variation developed in this secondary coil, when applied to the grid, causes a high-frequency current to flow in the plate circuit. *The high-frequency current in the plate circuit is sufficient to supply the losses of the coil, and so the oscillation continues.*

The energy to maintain the oscillations is drawn from the source of d-c plate voltage. This is analogous to the supply of energy to the balance wheel of a watch from the mainspring. In the watch, the escapement mechanism converts the continuous source of spring pressure to impulses which maintain the oscillation. In power oscillators used for high-frequency heating of gears, for surface hardening, and for dielectric heating of plastics, this same principle of impulses is used. In this type of oscillator, the grid bias is sufficiently large that no current will flow except on the positive peak of the grid-voltage wave. Thus, the impulse that comes with the positive peak of the grid-voltage wave of each cycle is used to maintain the oscillation.

The chief advantage of this method of operation is that considerably more power can be developed with the same tube. One of the main limitations on tube operation is the temperature of the plate, which must not get so hot that it melts or becomes distorted. The heating of the plate is caused by the bombardment of the electrons and so is almost directly propor-

tional to the product of the average current flow and the plate voltage. The continuous current portion of the plate current contributes nothing to the stimulation of the high-frequency oscillations, but it does produce heating of the plate. If this direct-current component is reduced, then the stimulation may be considerably greater without overheating the tube.

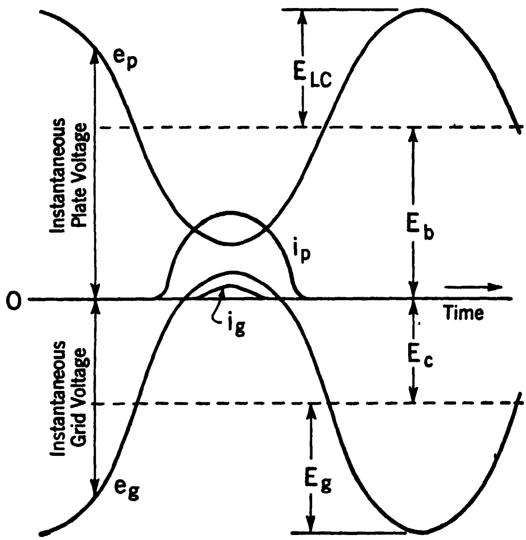


Fig. 15-7. Instantaneous Currents and Voltages in the Triode of a Power Oscillator.

Such a method of tube operation is shown in Fig. 15-17, where the variation in voltages on the plate and grid are plotted for a circuit similar to Fig. 15-16. In this case, the grid is biased considerably beyond cut-off so that current does not begin to flow in the plate circuit until E_g nears its maximum positive value. Since the grid voltage does swing positive, the plate current for this short time is quite high. When the grid is positive, there is a small electron flow to the grid which increases the negative grid bias of the tube, since it takes an appreciable length of time for the charge to leak off the grid through the high resistance R_g. This action limits the magnitude of the positive swing of the grid and in turn the excitation of the oscillatory, or tank, circuit.*

* The biasing effect of grid current during the positive swing of the grid is adequate to provide all of the negative grid voltage. The battery E_c of Fig. 15-16 is not needed and so is not normally used.

The magnitude of the oscillations will depend upon the grid bias, the ratio of E_g to E_{LC}, the tube characteristics, and the resistance of the oscillatory circuit. The ratio of E_g to E_{LC} must be high enough to maintain oscillations. Also, the grid bias must not be so large that adequate plate current cannot be obtained. The balance between these various factors and the natural losses in the circuit is automatically controlled by the self-biasing action of the grid. This action is obtained when the positive grid-voltage peaks become so large that the electron flow to the grid is increased and consequently the negative grid bias is also increased. The oscillations that cause the grid voltage are therefore reduced.

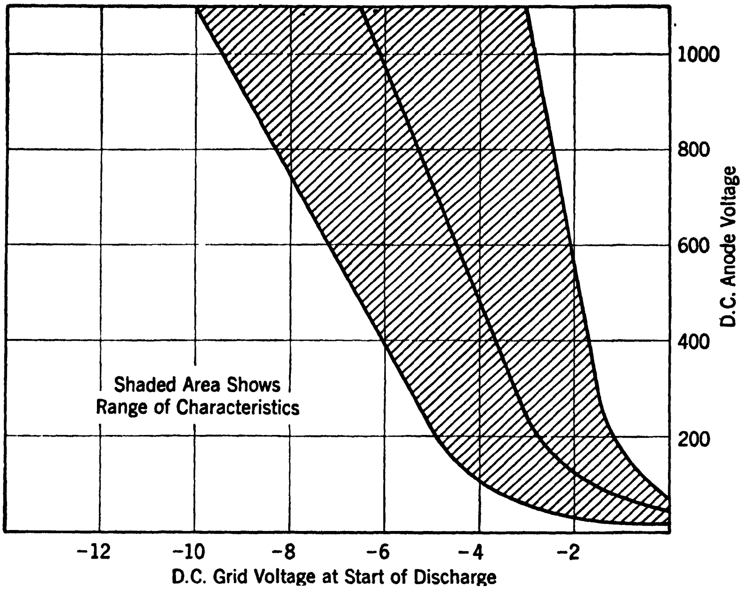
Only one of the more elementary types of oscillator has been discussed. In some oscillators the mutual coupling to the grid is through a portion of the condenser in the main oscillatory circuit. In others, the oscillatory circuit is located in the grid circuit rather than in the plate circuit. One example of a commercial oscillator for high-frequency heating is given in Chapter 16.

Theory of the gas triode or thyratron

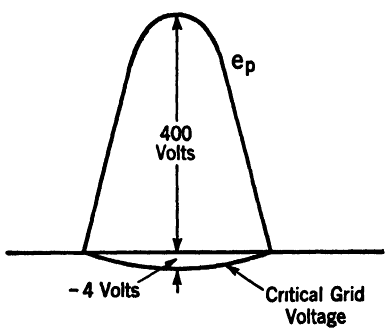
The operation of the gas diode was discussed in Chapter 14. The addition of a grid to this type of tube produces characteristics which make it very useful in a wide variety of industrial applications.

When the grid is highly negative with respect to the cathode, all of the electrons that are evaporated from the cathode are repelled, and none of them attain sufficient velocity to ionize the gas atoms. There is, therefore, no appreciable current flow in the tube, regardless of the plate voltage. As the negative grid potential is reduced, eventually a few electrons will escape past the grid, and these are accelerated to the ionizing velocities. As soon as ionization occurs, the tube becomes highly conducting and the voltage difference between the cathode and anode drops to about 20 v, the minimum voltage necessary to produce ionization. When ionization occurs, the tube is said to *fire*. It acts as an open or closed switch with a constant voltage drop of about 20 v. The grid voltage at which ionization will occur is somewhat dependent upon the magnitude of the anode voltage, and this relationship is shown in Fig. 15-18(a). Here the graph shows the negative values of grid voltage at which the tube will

fire with different anode voltages. These values are called the *critical grid voltages* because with grid voltages more negative, no current flows, and with grid voltages less negative, complete



(a)



(b)

FIG. 15-18. (a) Typical Thyratron Control Characteristics. (b) Critical Grid-Voltage Characteristics of a Thyratron Tube with an A-C Plate Voltage.

ionization is obtained. In other words, the tube characteristic is discontinuous at this point.

When the tube is once ionized, it is impossible to block the flow of current with the grid because the positive ions form a neutralizing cloud around the grid when it becomes negative

and ionization continues. In order to *turn the switch off* or to stop the current flow in a thyratron, it is necessary to remove the positive voltage on the anode long enough for the tube to de-ionize. Since this involves the combination of a large number of electrons with positive gas ions, the time required is in the order of one ten-thousandth of a second. Because it is necessary to remove the anode voltage in order for the grid to regain control of the tube, it is common practice to use an a-c voltage on the anode. With 60 cycles, it is then possible for the grid to gain control 60 times per second.

It is often convenient to plot the critical negative grid voltage against time with an a-c voltage on the anode. This is done in Fig. 15-18(b) and indicates clearly the grid voltage necessary for the tube to fire at any time during the half cycle that the anode is positive. In a typical thyratron, a negative grid potential of four volts will prevent firing with an anode voltage of four hundred volts, while if the anode voltage is raised to five hundred volts, the negative grid voltage must be increased to five in order to continue to prevent ionization.

Grid control of the thyratron. The gas triode makes an excellent relay since it operates at a fixed predetermined grid voltage. When it fires, full-load current is immediately obtained. It has high current-carrying ability with low-voltage drop between plate and cathode. The grid may be controlled by a photoelectric tube, a vacuum-tube triode, a sensitive contacting device, or any other method that will give a grid voltage less negative than the critical when it is desired to have the tube operate.

The thyratron may also be used to supply a unidirectional current of variable average magnitude for various control purposes. This is usually done by controlling the phase of the grid potential so that the tube will fire at different portions of the half cycle during which the plate is positive. This is shown diagrammatically in Fig. 15-19. The method of obtaining the phase shift is not indicated, but the effect of such a phase shift on the average current flow is shown for several phase positions of the grid voltage. In Fig. 15-19(b) the grid voltage is in phase with the plate voltage and the maximum current flow is obtained.

The average current is proportional to the area under the current curve. In part (c) of the diagram, the grid potential has been given a lag of 90°, so that the critical negative grid

voltage is not reached until the anode voltage has reached almost maximum value. The area under the current curve is thus reduced to almost half of the previous value. When the phase of the grid voltage lags still further, the time of firing of the tube is delayed still more and the area under the curve (the average value of current) is reduced to a small value indeed.

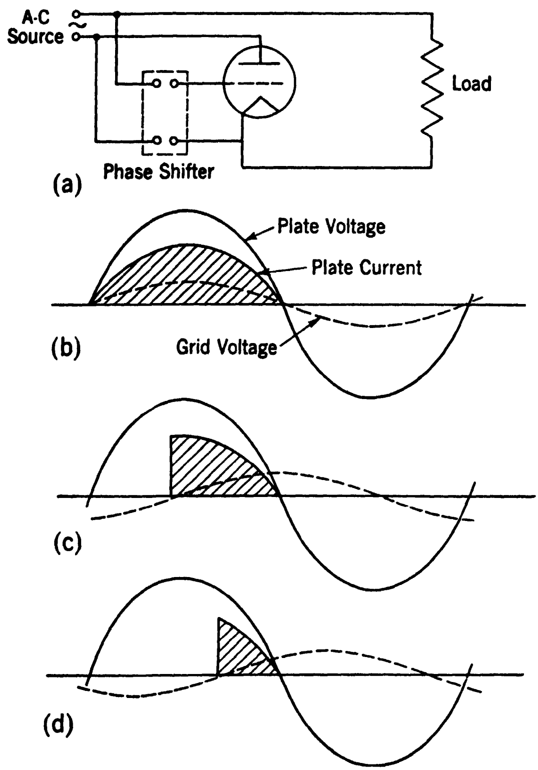


FIG. 15-19. Phase-Shift Grid Control in a Thyratron.

It is thus possible to vary the average value of current to a motor or other device over a wide range by the variation of the phase of the grid voltage of a thyratron. This can be done with small loss and often with comparatively inexpensive equipment.

A phase-shifting circuit

The use of phase-shifting circuits is so general that one type of such a circuit will be discussed. In Fig. 15-20 a voltage

is supplied by a transformer which is in phase with the main voltage source. This is shown as V_{ab} in (b) and (c) of the diagram. The output of this transformer is impressed on a circuit composed of a constant inductive reactance and a variable high resistance. When the resistance is large as compared to the inductive reactance, the current flow will be small and lagging by only a small angle, as indicated by the diagram at (b).

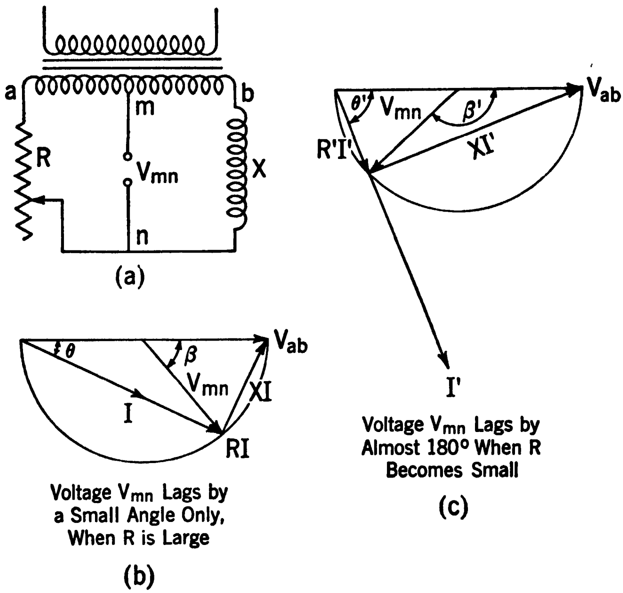


FIG. 15-20. Phase-Control Circuit.

The voltage V_{mn}, measured from the midpoint of the transformer secondary to the connection between X_s and R, will lag by an angle β which is twice as large as the angle of current lag θ. When the resistance has been greatly reduced, the current flow in the circuit will be much larger and will lag by a large angle θ'. The voltage V_{mn} will continue to have the same magnitude but will lag by an angle β' which is still equal to twice θ'. Thus the voltage V_{mn} is constant in magnitude but varies in phase position as the resistance value is changed. A similar result can be obtained by using a fixed capacitor and variable resistor. The more usual variation is the use of a fixed resistor and a variable reactor of the saturable core type.

The use of such circuits to provide the phase control for thyratrons (or ignitions) in various types of industrial equipment is quite common.

Thyratron control of d-c motors

In Chapter 6 it was shown that the speed of d-c motors could be controlled by a variation of either the field current or the armature voltage. In order to obtain the benefits of the control characteristics of d-c motors when a-c power only is available, it is necessary to supply the motor through some form of rectifier.

Thyratrons (or ignitions) are a common form of rectifier used for this purpose. Furthermore, by applying phase control to the grids of the thyratrons, it is possible to establish a highly efficient, extremely sensitive, and unusually versatile control of these motors. This has been found to be of particular value in the drives for many modern machine tools.

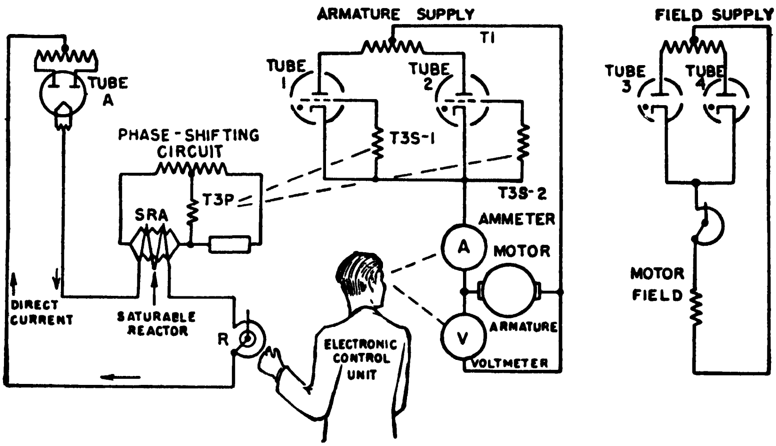


FIG. 15-21. Simplified Diagram of Thyratron Motor Control.

A simplified diagram of this type of control is shown in Fig. 15-21. At the right of the diagram, the field circuit of the motor is supplied by a rectifier using two gas diodes. In the center portion of the diagram, the motor armature is supplied by another rectifier using two thyratron tubes with phase control on the grids.

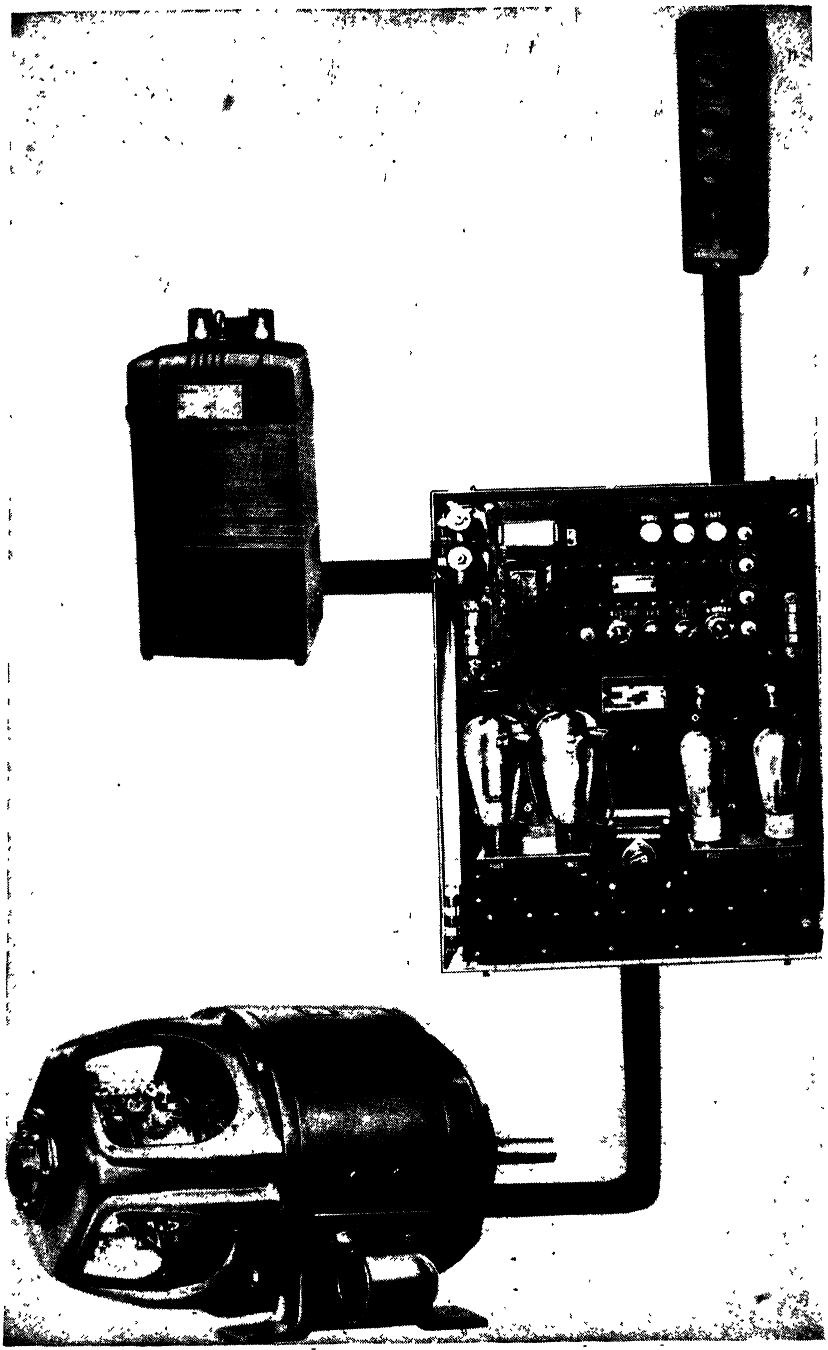


Fig. 15-22. Four Basic Units of Which "Thy-Mo-Trol" Drive Is Made Up: D-C motor, "Thy-Mo-Trol" panel, anode transformer, and control station.

This phase control is shown in the left section of the diagram and is similar to the control described in Chapter 7, except that in this case the resistance is constant and the magnitude of the inductance is varied by means of a saturable reactor. The voltage from the midtap of the supply transformer to the phase-shifting point between the resistor and the inductor is impressed upon the primary of the transformer marked *T3P*. The two secondaries (T3S-1 and T3S-2) of this transformer are independently connected to the grids of the thyratrons. They control the firing time of these tubes and thus the magnitude of the average armature current.

The action of the saturable reactor is described more fully in Chapter 19. It is used here to control phase shift because of the ease with which vacuum tubes can vary the inductance by control of the d-c current from a third rectifier shown in the upper left-hand corner. Although, for simplicity, the d-c current controlling the saturable reactor is shown in Fig. 15-21 to be varied manually by a rheostat, this current is electronically controlled in the actual equipment. Thus, the electronic control unit is activated by changes in armature voltage (which is proportional to speed) or in armature current (which is proportional to torque) and in turn varies, through its vacuum-tube circuits, the average current in the saturable reactor and therefore in the d-c motor.

By adding grids with phase control to the rectifier tubes supplying the field, it is possible to obtain field control also. With this versatile control and with the electron and sequence timers available, it is possible to produce complicated motor operations automatically.

Many refinements are added to nearly all commercial forms of this control. These can be worked out from the manufacturers' literature when necessary. Sales representatives are also glad to assist in explaining the operation of their equipment.

The ignitron tube

The ignitron tube is a three-element tube having characteristics similar to the thyratron except that it can be manufactured to carry currents up to several thousand amperes.

The diagram of Fig. 15-23 shows the construction of the ignitron tube. The cathode is a mercury pool located in the bottom of the tube. The anode is usually a graphite cylinder supported by an insulator in the top of the tube. The main current connections are shown by the heavy black conductors at the top and bottom of the tube. The ignitor is the silicon-carbide (Carborundum) pencil which dips into the mercury.

In operation a current pulse is applied from the ignitor to the mercury when it is desired to fire the tube, or to cause it

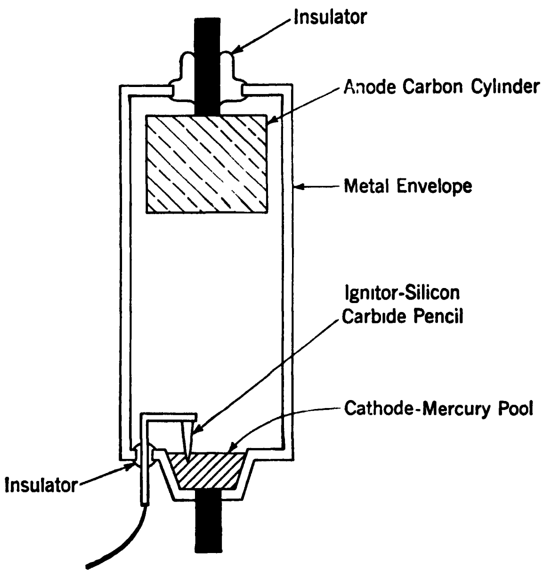


FIG. 15-23. Construction of an Ignitron.

to conduct. The action of this pulse is such that the current flow causes one or more minute arcs at the contact surface of the carborundum pencil and the mercury, thus providing an initial supply of electrons which starts the ionization process. This pulse is usually supplied by a thyratron, as it is necessary to fire the tube each cycle when it operates on alternating current. Various methods are used for timing the pulses, depending upon the application of the tube.

The ignitron has two main industrial uses. The first is in the precision control of current in spot welding and seam welding many different types of metal. The second is in the rectification of alternating current for industrial use. An important advantage of the ignitron as an electrical relay is that there

are no moving parts, and therefore low maintenance results. Also, special foundations are not required and fire hazards are reduced.

Multigrid high-vacuum tubes

So many special tubes have been constructed with the vacuum-tube triode as the base that it is impossible to discuss most of them. The screen-grid tube (the tetrode) and the pentode are used so extensively, however, that they must be mentioned.

In the tetrode, or four-element tube, a second grid is placed between the control grid and the plate. When the tube is used as an amplifier, this grid is given a positive potential which will remain constant regardless of load. This potential is often equal to the maximum plate voltage. It performs two important functions. First, it prevents any electrostatic coupling from the plate to the grid, and thus eliminates the possibility of unwanted voltages being fed back into the grid circuit, which might cause distortion or even oscillation. It is called a screen grid because it *screens* the control grid from the effects of variable plate voltage. The second function of the positive potential is to maintain uniform acceleration of electrons in the tube in spite of variation of plate voltage. Since the triode plate voltage normally goes down, because of the drop in the load as the plate current increases, this tends to operate in opposition to the effect of the grid. When the screen grid with constant potential is inserted, it increases the amplification of the tube considerably. When used as a straight amplifier, the operation is much the same as that of the triode, but with the above-mentioned improved characteristics.

When the plate becomes considerably less positive than the screen grid, some flow of electrons from the plate to the screen grid is obtained due to the emission of electrons at the plate caused by the bombardment of the plate by the electrons from the cathode. This emission is called *secondary emission.* A number of circuits have been devised that use the peculiar screen-grid current characteristics of this tube, but they are beyond the scope of this text.

In the pentode the current flow in the screen-grid circuit due to secondary emission was recognized as undesirable for most amplifier applications, so another grid at cathode potential was inserted between the screen grid and the plate of the tube.

This low-potential grid prevents the electrons of secondary emission from being attracted to the screen grid.

Cathode-ray tubes

The present use of cathode-ray tubes in meters and instruments and the probable future extended application of them to the instrumentation problems of industry require that they be described and their operation explained. A diagram of a typical cathode-ray tube is shown in Fig. 15-24.

The hot cathode K supplies the electrons. The control grid G permits those electrons which are in the center of the tube to pass through to the accelerating electrodes F and A. Because of the special geometric form of these electrodes,

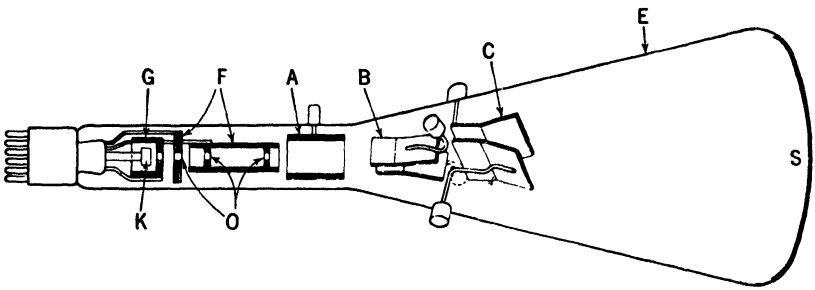


FIG. 15-24. Schematic Arrangement of Electrodes in a Cathode-Ray Tube of the Electrostatic-Deflection Type.

they not only accelerate the electrons but also focus them into a narrow stream or beam, which is directed to the screen S. This screen, which is circular, is covered with a luminescent material which glows when bombarded by the high-velocity electrons.

The stream of electrons passes between the parallel plates at B. If a difference of potential is applied to these plates, it will cause the electron stream to be deflected toward the plate that is more positive. By causing the plate potential to vary with time, the spot may be made to sweep across the screen. The voltage on this sweep circuit usually increases linearly with time for a short period of time and then returns suddenly to zero again, after which the voltage again starts to rise. When a signal voltage is placed across the control plates at C (which are perpendicular to those at B), and when the periodicity of the sweep circuit is synchronized with the signal frequency, the signal appears on the screen. The wave

form shows its variation with time. Since the electron beam has almost no inertia, it can respond to very high frequencies, and so is useful beyond the range of most meters.

One of the most promising uses for this type of tube is in the field of television, where it acts as a television receiver. It has, however, found many applications in the field of military and industrial measurements.

CHAPTER 16

Heating, Welding, and Electrochemical Processes

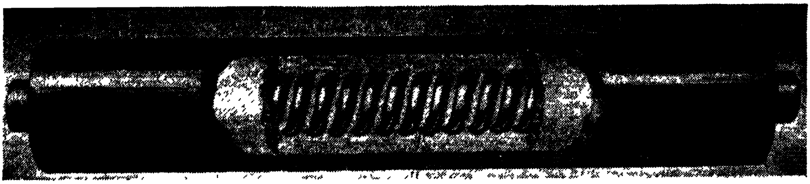
Heating

The advantages of electric heating for industrial purposes include the ease with which it can be supplied locally, the accuracy with which the temperature can be controlled, the ability to control the atmosphere within a furnace, the extremely high temperatures which can be obtained, and in some types of heating the ability to localize the heat in a particular portion of the article to be heated.

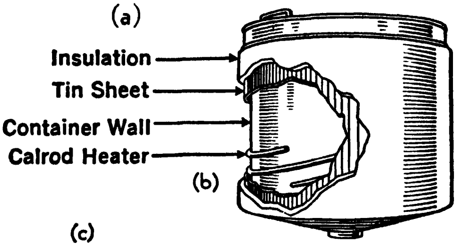
In kettles and ovens (below 400° C) the heating is usually accomplished by forcing current through resistor heater elements, which, by conduction, convection, and radiation, transmit the heat to the industrial product. In electric furnaces (above 400° C) the heating may be accomplished by resistor units, by having the current flow through the material to be heated, by inducing current in the material to be heated electromagnetically, or by concentrating the power in an electric arc which by radiation and reflection from the walls of the furnace heats the product. Since it is impossible to cover all of the various methods of heating in detail, representative examples only will be given. More detailed discussions will be found in the collateral reading.

Resistor-type heating. In most cases the resistor is entirely separate from the material being heated. Thus, in a kettle for cooking purposes, the resistance units are usually placed outside of the kettle but are so arranged that the transfer of heat from the unit to the kettle is facilitated. This is shown in the photograph and diagram of Fig. 16-1.

Exercise 16-1. A 50-gal kettle is to be heated by electric power. (a) How much power will be required if the kettle is filled at 70° F and must be brought to a boil at 220° F within one-half hr? (It may be assumed that 2.5 watt-hours are required to raise 1 gal of the charge 1° F.) (b) What current would be required to supply this kettle at 220 v d-c or single-phase a-c? (c) What current would be required at 220-v three-phase a-c? (d) What power would be

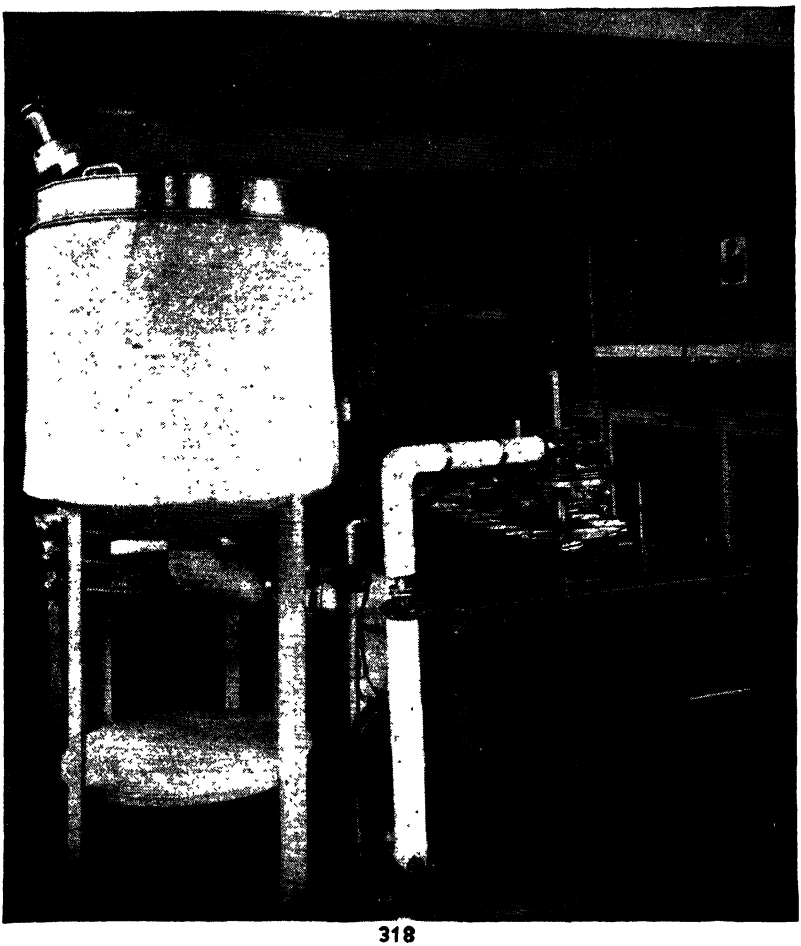


(a)



(c)

FIG. 16-1. (a) Calrod Heating Unit with Cut-Away Section to Show Helical Conductor Imbedded in Magnesium Oxide Powder, and the Whole Enclosed in a Steel Tube. (b) Typical Construction of Heating Tanks for Industrial Use. (c) Installation of a Tank for Heating Ointment Prior to Filling Containers.



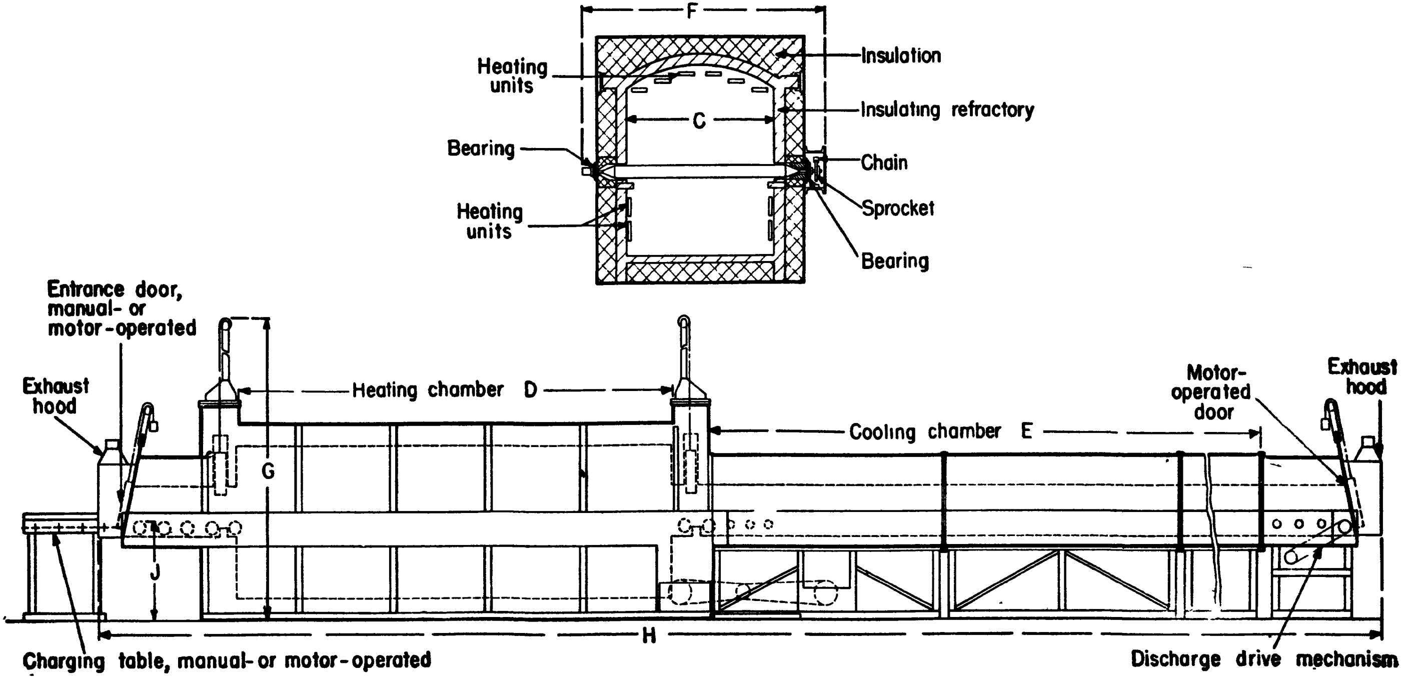


FIG. 16-2. Typical Roller-Hearth Electric Furnace with Cooling Chamber, and for Use with Protective Atmosphere.

needed if the time required to bring the charge to a boil is extended to 1 hr?

A very common type of resistor furnace is used for production brazing, sintering, heat treating, and so on. The product is fed through the furnace either by a continuous chain or by revolving rollers. The resistor units are mounted on the roof of the furnace and low on the sides of the furnace. The construction includes a short entry chamber, a heating chamber,

FIG. 16-3. Mesh-Belt Conveyor-Type Electric Copper-Brazing Furnace (Without End Chamber), Showing Construction of Roof Heating Elements, Hooks, Insulators, and Method of Mounting.

and then a rather long cooling chamber so that no strains are introduced in the material during the cooling period.

Fig. 16-2 shows a drawing of such a brazing furnace. The locations of the resistor units are shown. These are selected so as to produce the most uniform temperature condition in the product. Fig. 16-3 shows the method of suspending the resistor units from the roof of the furnace. A photograph of such a furnace which is installed and operating is shown in Fig. 16-4.

Exercise 16-2. In an oven using nichrome wire one unit has burned out. This unit is operated continuously and provides the main

source of heat. The conductor in the old unit was 60 ft long and drew 150 amp from a 120-v source. Operating experience indicates that more satisfactory operation will be obtained if the power input to this unit is reduced 15 per cent. If nichrome ribbon $\frac{3}{4}$ in. in width may be obtained in thicknesses from 0.03 to 0.10 in steps of 0.01 in., what length and thickness should be ordered to repair this unit?

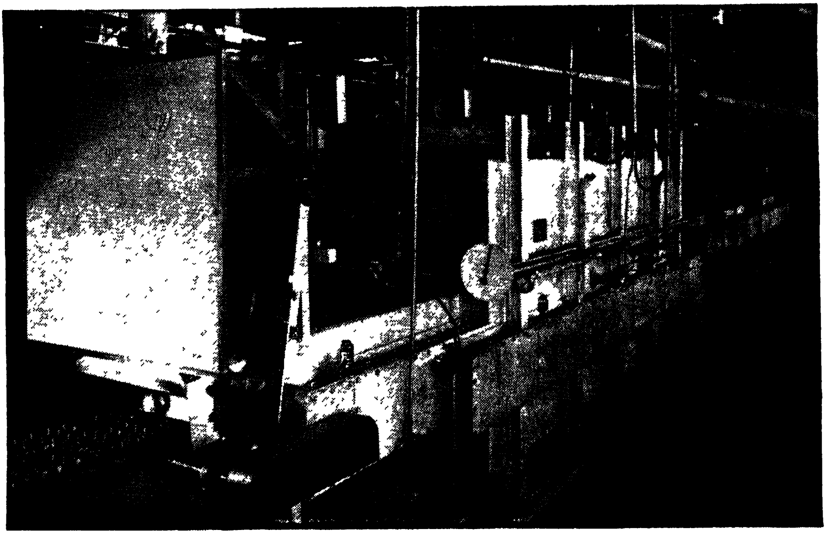


FIG. 16-4. Roller-Hearth Sintering Furnace, Charging End. Door opening, 20 in. wide, 8 in. high; heating chamber, 10 ft long; cooling chamber, 30 ft long; rated 110 kw.

Submerged resistor type of induction furnace. A variation of the resistor type of furnace is found in the induction type, where the metal to be heated acts as the secondary circuit of a transformer. Such a furnace is shown in Fig. 16-5. It is not easy to change alloys in this type of furnace because the old charge must be emptied and the new one started with a batch of molten metal. Furnaces of this type are, therefore, particularly satisfactory for 24-hour service in melting one particular metal or alloy. The secondary is shown in Fig. 16-5 as a *V* slot having a rectangular cross section. A furnace such as this provides a simple and interesting example of a one-turn transformer secondary.

Example. Determine the number of primary turns and size of iron core required for an induction furnace similar to that of Fig. 16-5 under the following operating conditions:

(a) Primary voltage is 220 v.
(b) Power capacity (maximum) is 60 kw used for melting.
(c) Power capacity (minimum) is 8 kw used to hold the temperature overnight.
(d) Cross section of the V slot is 1 in. by 3 in.
(e) Length of the V slot is 40 in.
(f) Resistivity of the molten metal is 40 microhms per inch cube.*

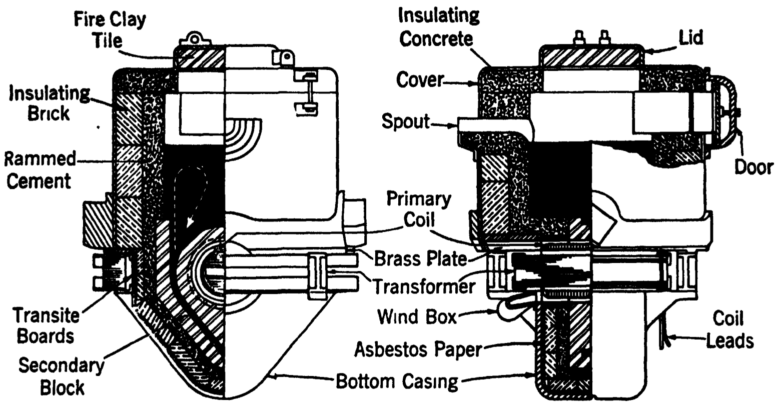


FIG. 16-5. Core-Type Induction Furnace.

(g) Permissible flux density in the core is 75,000 lines per sq in.
Solution: The resistance of the secondary circuit is

$$R = \rho \frac{l}{A} = 40 \times 10^{-6} \times \frac{40}{3}$$

$$= 5.33 \times 10^{-4} \text{ ohms.}$$

The heating in the secondary for maximum power is

$$P = I^2R, \text{ or } 60{,}000 = I^2 \times 5.33 \times 10^{-4}.$$
$$I^2 = 1.13 \times 10^8.$$
$$I = 10{,}600 \text{ amp for maximum power.}$$

In order to produce this current, a voltage will be required equal to

$$IR = 10{,}600 \times 5.33 \times 10^{-4}$$
$$= 5.65 \text{ v.}$$

* This is the resistance between the opposite faces of a 1-in. cube.

The primary turns for maximum power will be

$$\frac{220}{5.65} = 39 \text{ turns,}$$

and the primary current will be

$$\frac{10{,}600}{39} = 270 \text{ amp for maximum power.}$$

The heating in the secondary for minimum power is

$$P = I^2R, \text{ or } 8000 = I^2 \times 5.33 \times 10^{-4}.$$
$$I^2 = 15 \times 10^6.$$
$$I = 3870 \text{ amp.}$$

Voltage per turn will be

$$IR = 3870 \times 5.33 \times 10^{-4} = 2.071 \text{ v.}$$

Primary turns required are

$$\frac{220}{2.071} = 106 \text{ turns.}$$

Primary current is

$$\frac{3870}{106} = 36.6 \text{ amp for minimum power.}$$

To determine the cross-sectional area of the core, it is first necessary to determine the flux required.

$$e \text{ per turn} = \frac{d\phi}{dt} \times 10^{-8} = E_{\max} \sin 2\pi f dt.$$
$$d\phi = 10^8 E_{\max} \sin 2\pi f dt.$$
$$\phi = 10^8 E_{\max} \int \sin 2\pi f dt$$
$$= \frac{10^8 E_{\max}}{2\pi f} \cos 2\pi f.$$
$$\phi_{\max} = \frac{10^8 E_{\max}}{2\pi f} = \frac{5.65 \times \sqrt{2} \times 10^8}{2\pi \times 60}$$
$$= 2{,}120{,}000 \text{ lines.}$$

Cross section required is

$$\frac{2,120,000}{75,000} = 28 \text{ sq in.}$$

A core of 5 by 5.6 in. would be satisfactory.

The above illustrative problem indicates that the power to the induction furnace is reduced by an increase in the number of turns of the primary winding. It also illustrates, in a very elementary way, the method used to design such a furnace. The effects of leakage reactance have been omitted, although in an actual design they would necessarily be considered.

Exercise 16-3. A 100-kw furnace similar to that in Fig. 16-5 has the following data given:

(a) Primary voltage is 440 v.
(b) Maximum power input is 100 kw.
(c) Minimum power input is 10 kw.
(d) V slot is $1\frac{1}{4}$ by 3 in. and is 60 in. long.
(e) Resistivity of the molten metal is 35 microhms per inch cube.
(f) Permissible flux density is 65,000 lines per sq in.

How many turns would be needed for the primary winding for both maximum and minimum power? Would the conductors on the primary winding all be of the same size, or would two different-sized conductors be used? Why? What cross section would be required for the core?

Electric-arc furnaces. Much of the refining of steel for castings is done in electric furnaces which use an arc between three very large carbon rods and the metal surface to supply the heat. In this case the power is supplied through a three-phase transformer of high reactance so that the arc current is easily controlled. The arc itself has a negative resistance—that is, the current will increase indefinitely with only a limited voltage applied. The high reactance acts, therefore, to limit the current to a specific value as adjusted by the furnace control.

The power factor in both the submerged resistor induction furnace and the arc furnace varies from about 50 to 85 per cent. The lower power factors are not usually satisfactory, so it is customary to install static condensers in parallel with the transformer primary to supply the inductive reactive current locally rather than through long electric power lines. Thus a small amount of conducting material made into condensers

will often save the necessity of using many times this amount of material in electric power transmission and distribution lines.

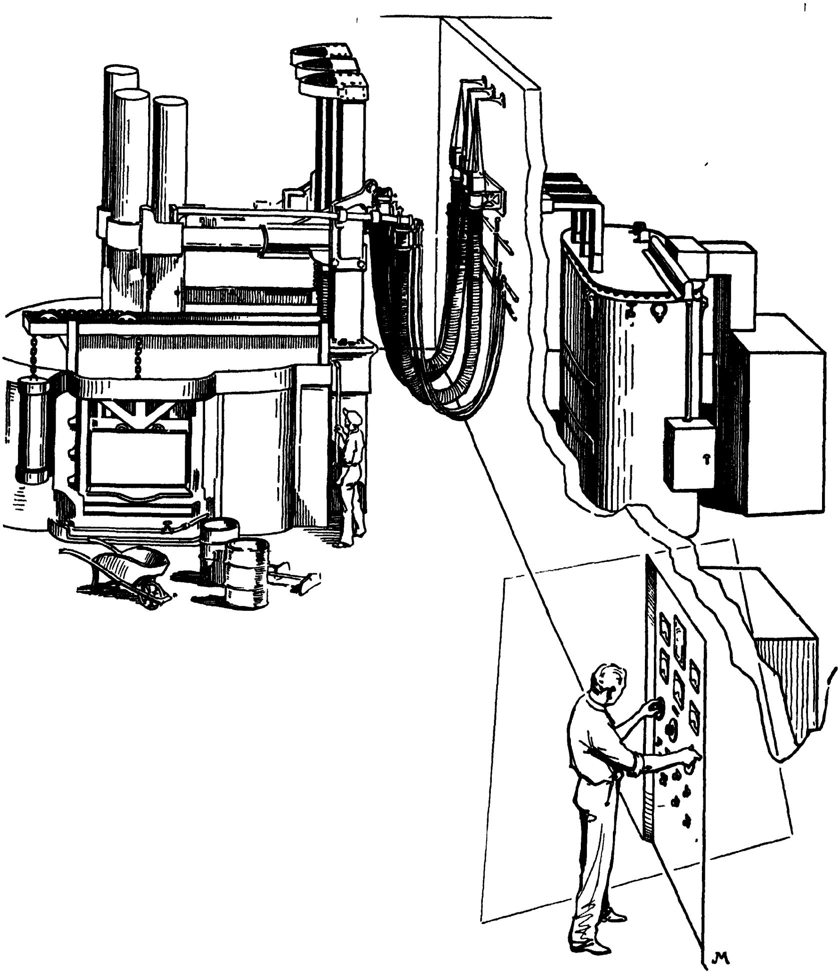
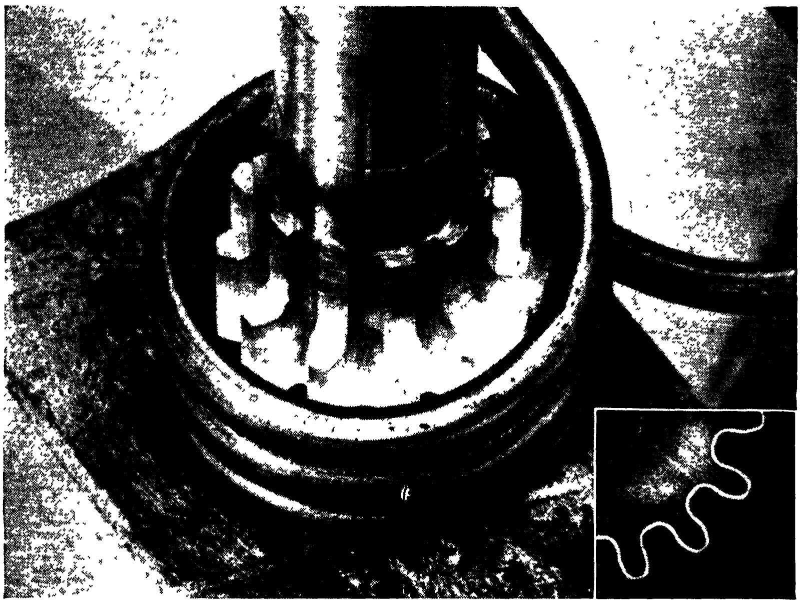


FIG. 16-6. Typical Layout of Electric Equipment for Arc Furnace. Equipment includes furnace transformer, switch gear, and control.

High-frequency induction heaters. High-frequency currents are extensively used to heat the surfaces of gears and rollers for heat treating and also for heating localized areas for brazing and soldering. The high frequency is supplied by

induction from a coil of water-cooled copper tubing to the part to be heated. Thus, if a small gear is placed in the center of a coil as shown in Fig. 16-7 and a high-frequency current flows in the coil, the coil acts as the primary of a transformer and the gear itself acts as the secondary. As is explained in the theory of transformers, a current will tend to flow in the secondary (or gear) which will oppose the magnetomotive force of the primary current. In this type of transformer the current in



(a) (b)

Fig. 16-7. (a) Inductively Heating 1-in. (Outside Diameter) Gear for Hardening, Using a 15-kw Electronic Heater. (b) A Part of a Surface-Hardened Gear, Heated Electronically, and Polished and Etched to Show Depth of Hardened Zone.

the primary is approximately constant, and so the secondary current tends to neutralize most of the magnetomotive force of the primary and thus hold that flux to a minimum. In order to do this the secondary current will flow just as close to the coil as is possible, and in a gear or cylindrical object placed in a solenoid the current will therefore flow on the surface. This is particularly true of iron and steel, which are good magnetic conductors.

With very high frequency and high current flow it is possible to heat the surface of gears to hardening temperature without permitting the heat to penetrate more than a few hundredths of an inch into the part. This is shown in Fig. 16-7(b), where a section of the gear has been cut and polished to show the depth of the hardening. The hardening of the tooth root is prevented and the teeth thus retain their proper strength. This tendency of the secondary current to flow on the outside surface is sometimes called *skin effect.*

Where high frequency is used to provide localized heat for soldering and brazing joints, the high-frequency coils should follow the joint as closely as possible because, even with non-magnetic materials, the current in the secondary or part being heated will flow as close to the coils as possible. In this way, the currents in the secondary may be controlled so as to produce the heat where it is most effective. This is shown in Fig. 16-8, where the primary coils are so placed as to concentrate the heating along the joint to be soldered or brazed.

Although the final solution to the design of high-frequency heating coils usually involves an experimental procedure, the following rules may be helpful as a guide.

(1) The coil should take the approximate shape of the part to be heated.

(2) The part should be centered in the coil if possible.

(3) Sharp corners will tend to heat first, because usually these corners are nearest the coil and, in addition, they have a minimum of mass. Therefore, the shape of the coil should be adjusted to clear the corners as far as possible.

(4) Where dissimilar metals are to be brazed together, the current must be concentrated on the one with the lower resistance and permeability. Thus, copper and silver heat most slowly, while brass heats more rapidly and steel heats most rapidly. Therefore, the coil should be kept close to the silver, copper, and brass, and away from the steel.

(5) The material to be brazed should come to temperature first and draw the brazing alloy into the joint. Therefore, the brazing alloy should be kept as far away from the coil as possible.

(6) To obtain uniform heating around the circumference of a part, it may be necessary to rotate it while being heated.

The majority of induction heating is done at frequencies of about 1000, 3000, or 10,000 cycles. The power for these

frequencies is supplied by motor-generator sets. Where it is desired to limit the heating to a very thin surface layer for case hardening, or to heat a joint for soldering or brazing, it is

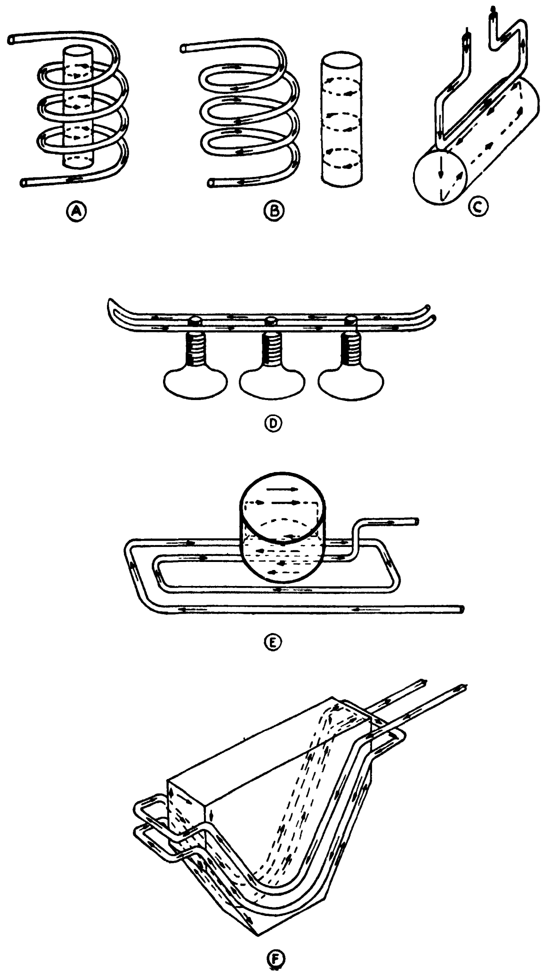
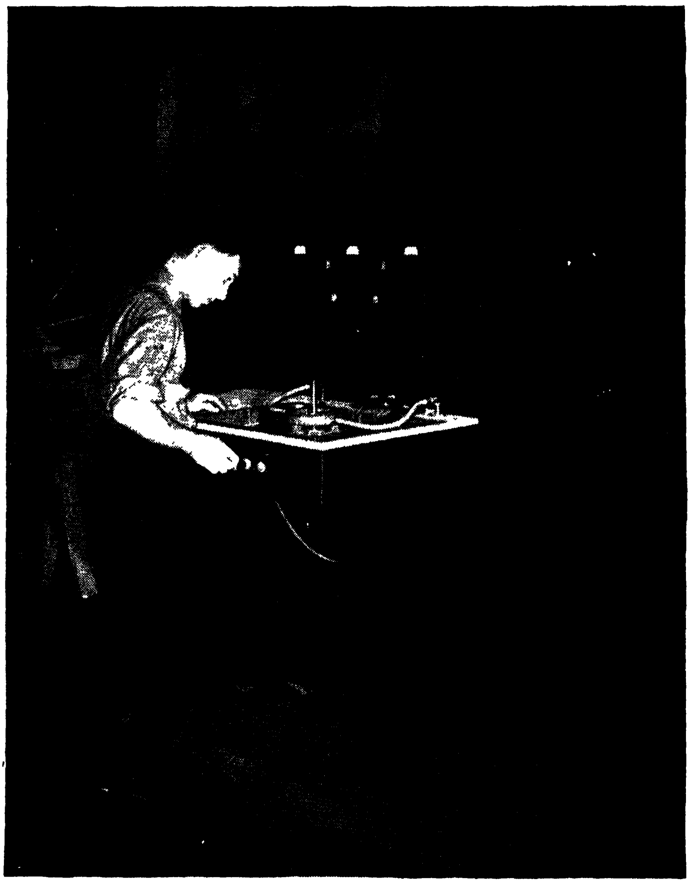


Fig. 16-8. Sketches Showing Direction of Current Flow in Electronic-Heater Coils and in Parts Being Heated.

often necessary to use large inputs with frequencies in the range of 200,000 to 500,000 cycles. In this case, the high-frequency power is supplied by vacuum-tube oscillators. Fig. 16-9 shows both the exterior and interior views of such a power oscillator. The rectifier tubes are shown in the upper right portion of the

interior view, and the high-frequency oscillator tubes are in the central left portion of the interior.

A diagram of the oscillator circuit is shown in Fig. 16-10. This includes the rectifier to supply unidirectional plate voltage to the oscillator tube, the oscillator tube and its circuit, and the tank circuit, which is the oscillatory circuit stimulated by pulses every cycle from the oscillator tube. In this circuit the capacitive portion of the tank circuit is split so that a portion of the voltage may be fed back to the grid to maintain the oscillations. A variation of the grid-leak resistance is used to vary the self-biasing effect of the oscillator tube and thus to control the

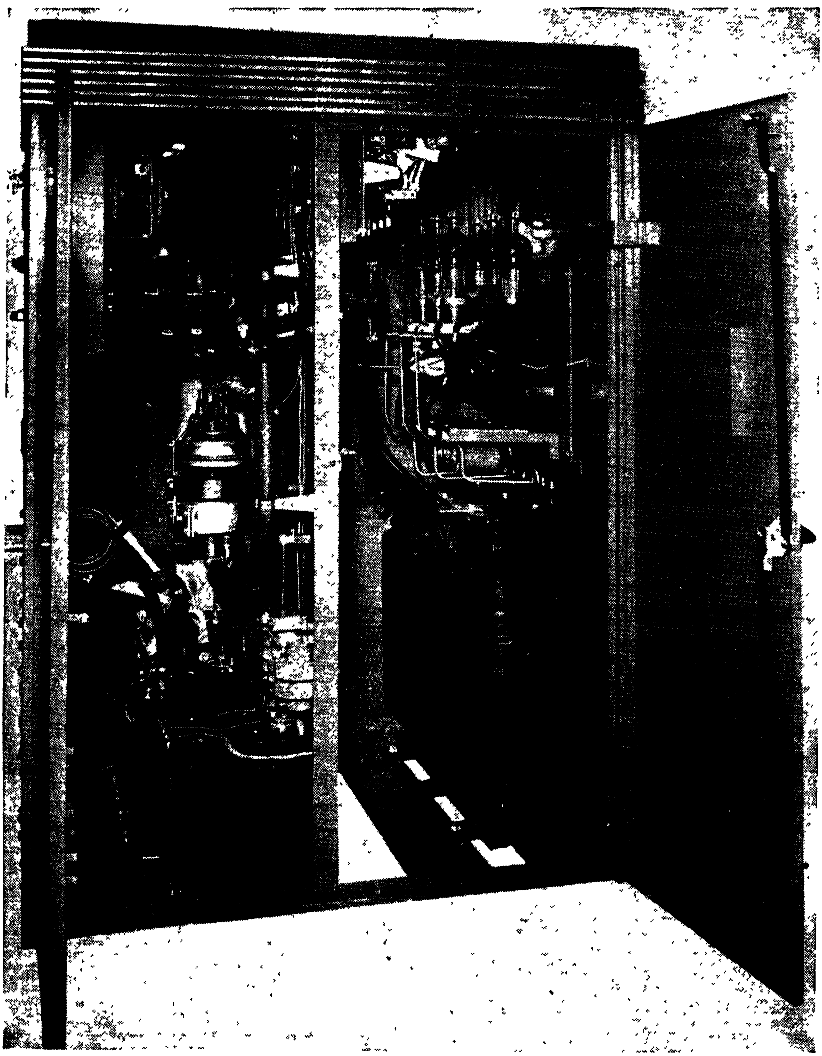


(a)

FIG. 16-9. A 15-kw. Electronic Heater. (a) Exterior View with Work Table and Quenching Fixture Adapted to Surface-Hardening a Wide Variety of Small

output. This variation is indicated as the fine adjustment of the oscillator output. The wave diagram at the top of the figure indicates the type of current flow in the various portions of equipment.

The same or similar equipment is used to preheat plastic blanks before molding. In this case the load is not placed in



(b)

Parts. (b) Interior View Showing Transformer, Rectifier Tubes, Oscillator Tubes, and Wiring.

the inductance portion of the circuit but is placed between plates of the tank-circuit condenser. The losses due to the rapid variation of dielectric stress heat the material rapidly, and since the losses are distributed evenly through the material,

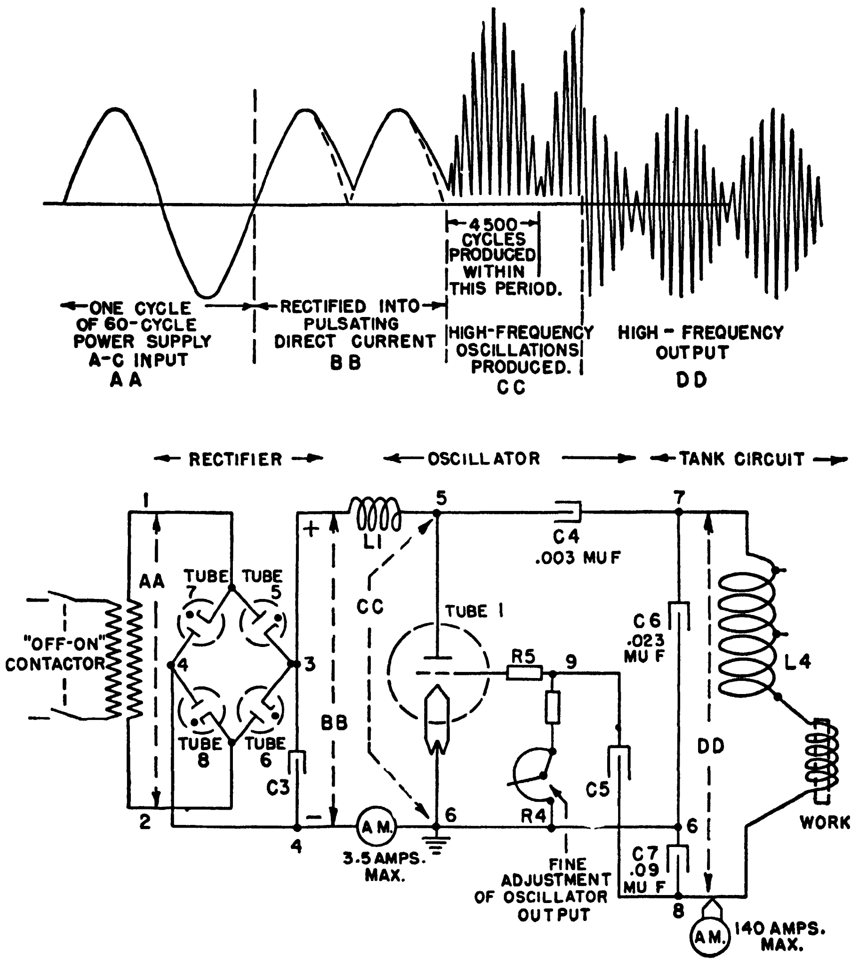


FIG. 16-10. Elementary Diagram of an Electronic Heater.

the plastic blanks are heated uniformly throughout. The frequency used varies from one to one hundred megacycles.

The cost of high-frequency equipment is high, and the power efficiency is, in general, low. However, it often provides heating results which cannot be otherwise obtained, and the

ability to concentrate the heat in a localized area may affect production economies which will pay big dividends on the money invested in this equipment.

Exercise 16-4. Make an analysis of the circuit diagram of Fig. 16-10 to determine the function of each circuit element.

Welding

Arc welding. In electric-arc welding an electric arc is used to produce a concentrated high temperature, which supplies local heat to the material to be welded and at the same time melts the electrode material so that it will flow into the weld. For successful welding, the current should remain reasonably constant in spite of the rather wide variations in voltages across the arc as the electrode is passed back and forth across the weld.

Both direct and alternating current are used for arc welding. Where direct current is used, the power is usually supplied by an individual generator for each welder. This generator is designed to produce an almost constant current over a wide range of voltage. The arc voltage usually is on the order of 20 to 40 v. When alternating current is used for welding, it is supplied through a transformer having a high reactance. Since the arc is resistive in nature, a rather wide variation in voltage across the arc will produce only a small resultant change in impedance and current flow. Secondary or arc current is adjusted by changing the turns ratio of the transformer, as was explained in the discussion of the induction furnace.

Although most arc welding is done manually, the development and application of automatic arc-welding equipment is progressing rapidly in industrial production.

Spot welding. Spot welding is the most common and one of the most simple of many different types of resistance welding. In spot welding a high localized temperature is obtained at the surface between two plates due to the resistance of surface contact. At the same time that the fusion temperature is reached, a high pressure is applied and a weld is obtained. The greatest use of spot welding is in the fabrication of sheet-metal parts.

A low voltage (2 to 12 v) is obtained from the secondary of a transformer, and the current (5000 to 75,000 amp) is conducted by flexible leads to heavy electrodes which are sometimes

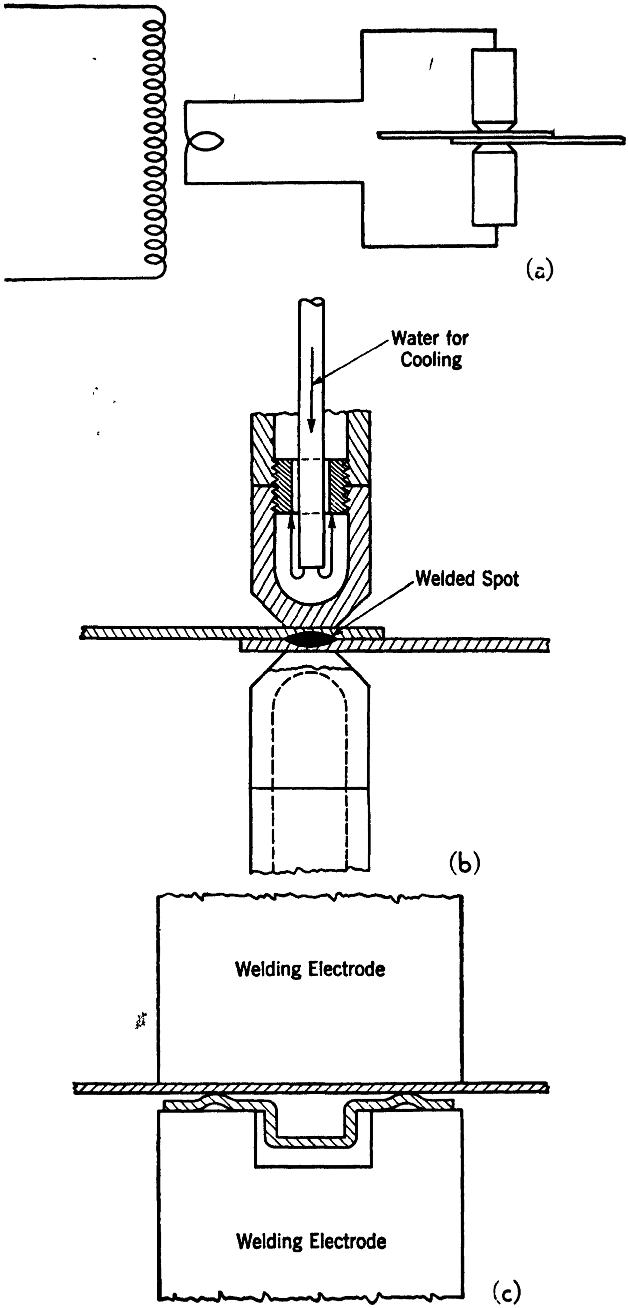


FIG. 16-11. (a) Elementary Diagram for Resistance Welding. (b) Water-Cooled Electrodes and Location of Weld. (c) Projection Welding.

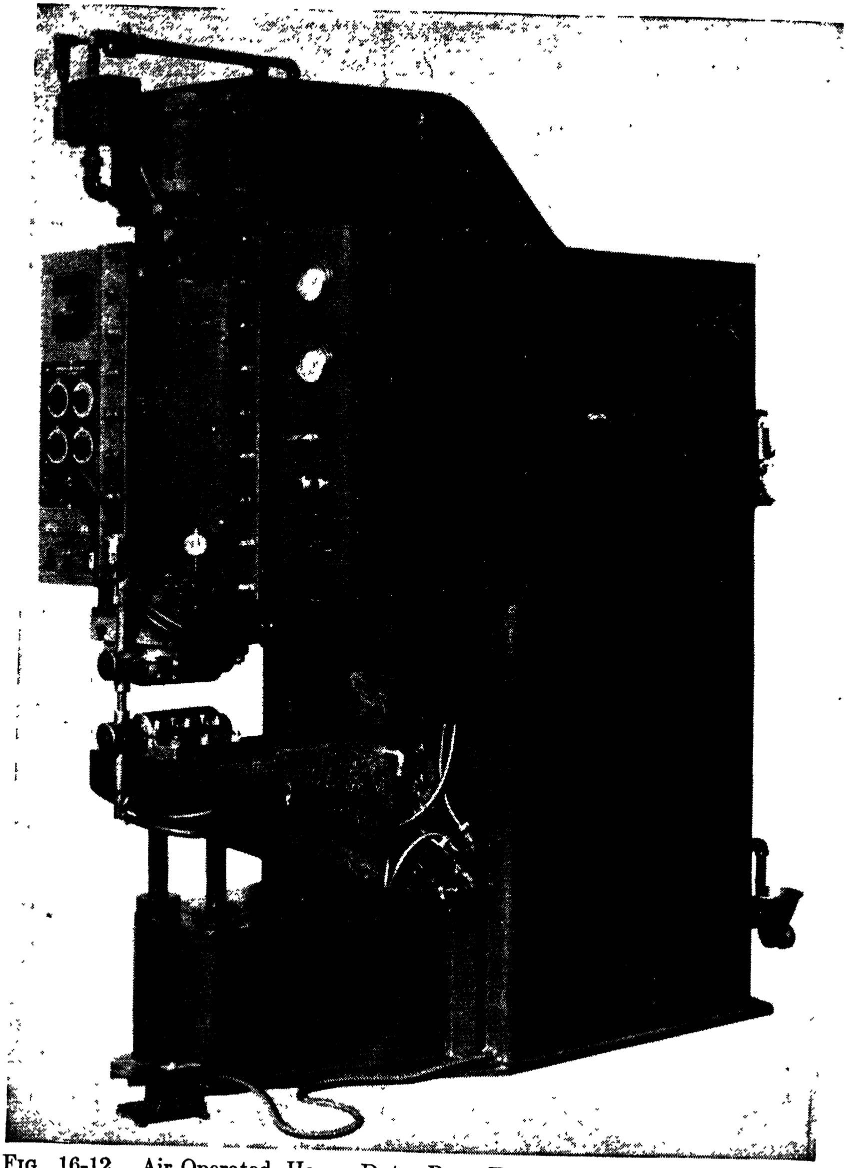


Fig. 16-12. Air-Operated Heavy-Duty Press-Type Spot Welder for General-Purpose Use with Built-In Electronic Control.

water cooled. This is shown diagrammatically in Fig. 16-11(a). An enlarged cross section of the weld is shown in Fig. 16-11(b). The high conductivity and low temperature of the electrodes prevent the outer surfaces from becoming excessively hot.

The relatively high resistance of the contact between the metal sheets causes a hot spot which fuses the metal, and the pressure on the electrodes produced by the welding equipment completes the weld. The timing of the welding current must be controlled quite accurately for best results. The electronic timer is very useful for this purpose. With such a control, enough energy can be supplied to produce effective fusion but not sufficient to burn or melt the metal beyond the proper thickness.

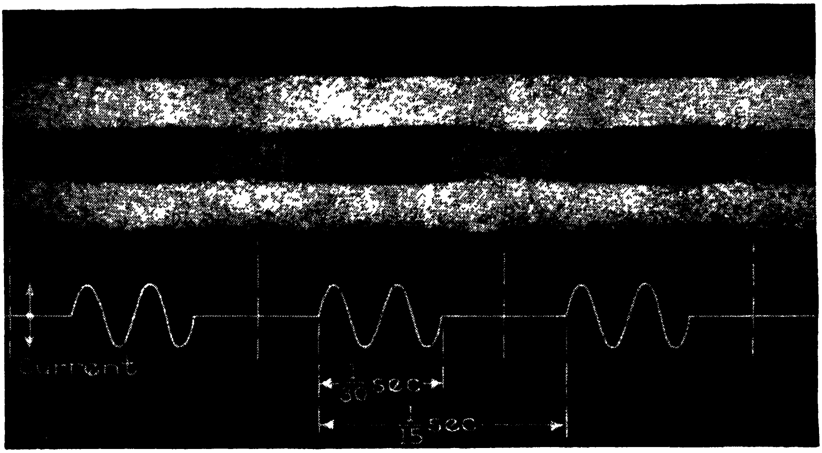


FIG. 16-13. Two .037-in. Stainless-Steel Sheets, Gas-Tight Seam Welded, at 72 in. per Minute.

Fig. 16-11(c) shows a variation of spot-welding procedure in which a small projection is placed on one of the sheets. This procedure assures better concentration of current and more accurate welds. When fusion is reached, the pressure of the electrodes forces the plates together so that the final result is essentially the same as in *b* of the figure. Commercial spot welders are usually designed with automatic timing control for the current, and pressure is supplied by a hydraulic piston. A photograph of a large welder with automatic controls is shown in Fig. 16-12.

Seam welding. Seam welding is a variation of spot welding. The electrodes are rolls and the material is fed through the rolls as in a sewing machine. Pressure is maintained continuously on the rolls, and periodic surges of current produce overlapping spot welds to form a continuous seam weld. A cross section of a weld between two sheets with the associated pulses of current is shown in Fig. 16-13. The current is con-

trolled by thyratron circuits to produce the on-and-off periods necessary to meet the needs of the particular materials being welded. Fig. 16-14 shows equipment for seam welding the two

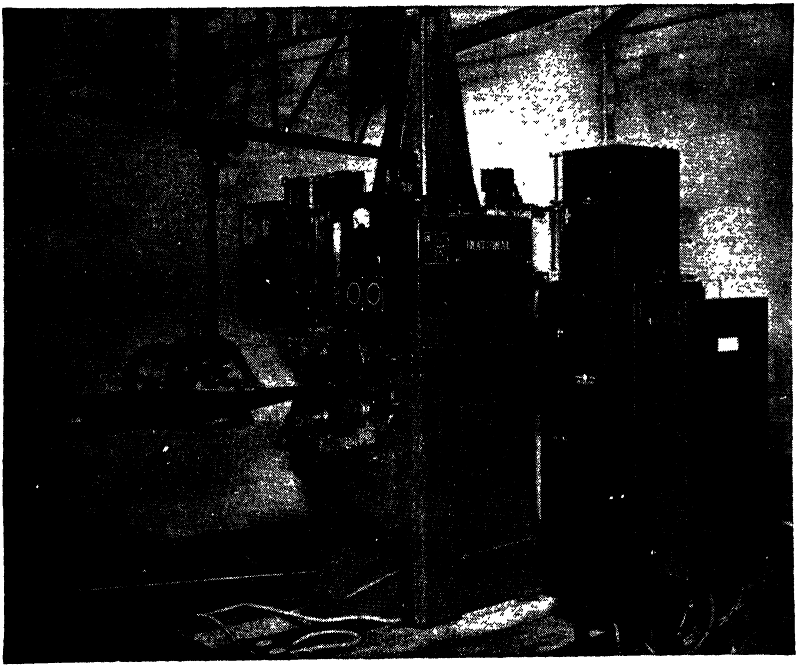


Fig. 16-14. Synchronous-Spot-Welding Control and Current-Regulating Compensator Used in Resistance Welding of Aircraft Propeller.

halves of a hollow steel airplane propeller. The electronic control equipment is shown in the case to the right of the welder.

Electrochemical processes

From 15 to 20 per cent of all the electrical power produced in the United States is consumed in electrochemical processes. Most of it is used in the production of aluminum, magnesium, copper, zinc, sodium, and chlorine. Other chemical products using electrochemical methods include fused sodium chloride, hydrogen, manganese, potassium perchlorate, and sodium chlorate. There are many minor variations in the processes used in manufacturing these different materials, but they all employ direct current flowing through a series of electrochemical cells, and so, from the point of view of the electrical supply, they are all similar.

Mercury-arc rectifiers operate most satisfactorily at about 600 v, so it is usual to use enough cells in series to operate at this voltage. Currents range from one thousand to sixty thousand amperes. Most of the installations since 1937 have been rectifiers, and, as most of the growth in the field of electrochemical production has been since that time, rectifiers supply nearly all of the power for these processes. The present discussion will, therefore, be limited to the basic theory and some of the more important operating problems of rectifiers.

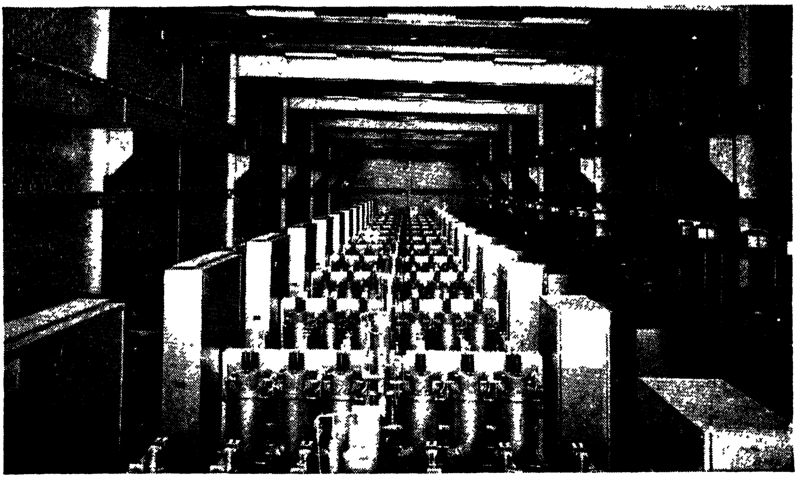
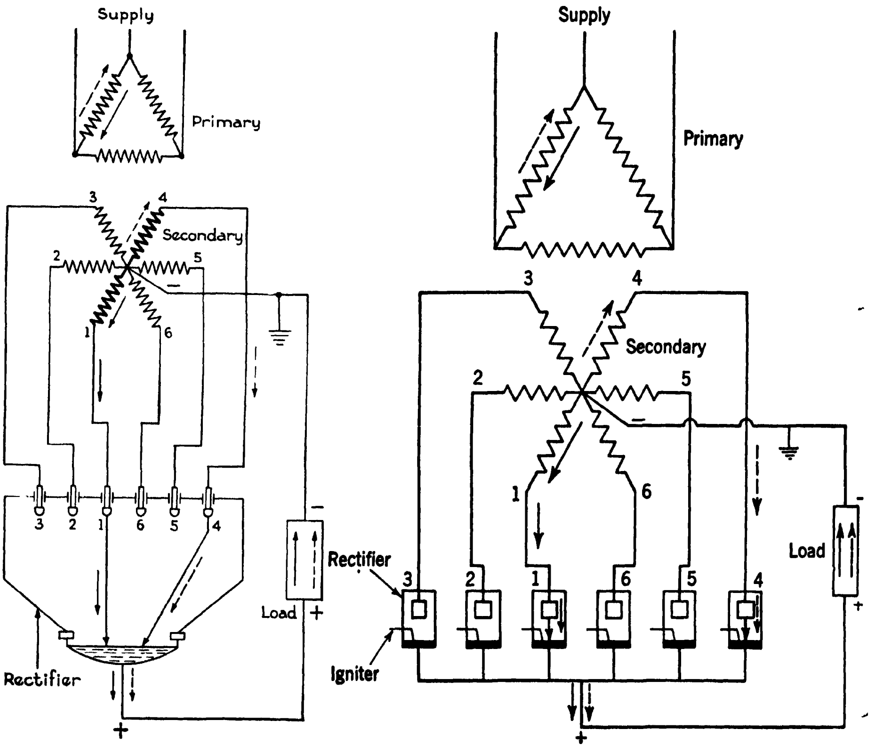


FIG. 16-15. Twelve-Unit Ignitron Mercury-Arc Rectifier Installation. Rectifier units are rated type RSW, 12-anode, 5000 amp, 645 v (3225 kw). In reduction plant (electrometallurgical) of Aluminum Company of America, Vancouver, Wash.

Polyphase rectifier theory. A very brief discussion of polyphase mercury-arc rectifiers is included in Chapter 14. The mercury pool located in the bottom of the rectifier supplies a continuous source of electrons, and the arc follows around the tank as first one and then another of the anodes becomes most positive. The voltage of these anodes is determined by the polyphase transformers supplying power to the rectifier. There are many variations of these transformer connections, but in all of them the anodes reach a maximum positive value in time sequence. In a three-phase 60-cycle rectifier the sequence is 120° or every $\frac{1}{180}$ of a second. In the six-phase rectifier, the sequence is at a 60° interval, or every $\frac{1}{360}$ of a second, while in the twelve-phase rectifier the sequence is 30°, or $\frac{1}{720}$ of a second.

Since the twelve-phase rectifier can usually be considered as two six-phase rectifiers in parallel on the d-c side, but with a 30-degree phase shift on the a-c side, this study will be made on the basis of a six-phase rectifier. Furthermore, since most of the recent installations have used the more efficient single-



(a) (b)

FIG. 16-16. (a) Six-Anode Mercury-Arc Rectifier. (b) Six Single-Anode Ignitrons Connected to Perform the Same Function.

anode *ignitron* tube, it will be assumed that this type of rectifier is under discussion—although most of the theory will apply equally well to the multianode type of mercury-arc rectifier. The equivalence of the two types can be seen from the diagrams of Fig. 16-16, where part (a) shows the six-anode mercury-arc rectifier and part (b) shows six single-anode units connected to perform the same function.

Since the arc drop in the mercury-arc rectifier is only 20 to 30 v, the mercury pool will remain within 20 to 30 v of the potential of the most positive anode, as indicated in Fig. 16-17.

Since the mercury pools of the single anode tubes are all at the same potential, and since the arc drop is usually limited to about 20 v, the voltage relations are similar for the ignitron type of rectifier. In the mercury-arc rectifier there is a continuous source of electrons, and the arc passes from one electrode to the next as the electrodes become successively most positive. In the ignitron type, however, an arc must be ignited at the proper time in each cycle by an auxiliary circuit.

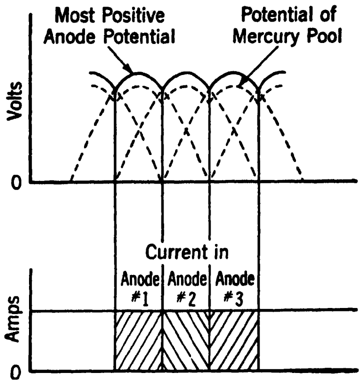


FIG. 16-17. Current and Voltage in a Six-Phase Rectifier—Assuming Constant Current and an Instantaneous Shift of Current Between Anodes.

The voltage condition shown in Fig. 16-17 assumes that the arc suddenly shifts completely from one anode to the next, as shown in the lower portion of the diagram. This is not possible, because the inductance of the anode circuit tends to continue the current in the anode that has been carrying current and to oppose the increase of the current in the anode that is picking up the current.

This effect results in a set of current and voltage curves similar to that shown in Fig. 16-18, in which each anode carries current for approximately one sixth of the cycle. The short period during which the d-c current is divided between the two anodes is called the *angle of overlap* or the *time of commutation.* The cathode voltage during this time is the average of the voltages of the two anodes carrying current.

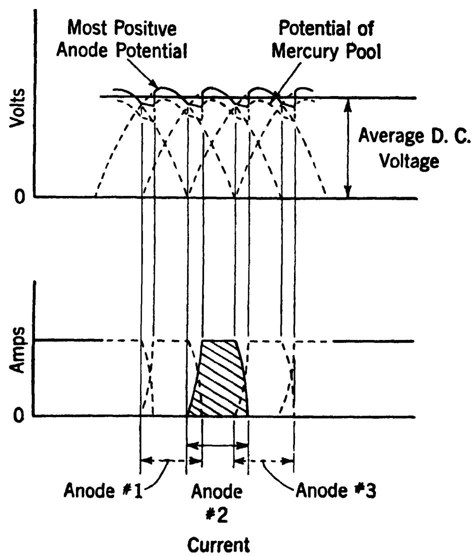


FIG. 16-18. Current and Voltage in a Six-Phase Rectifier.

If the time of ignition is delayed in the ignitron tubes, or if a high negative potential is placed on the grids of the mercury-arc rectifiers, the time at which the anode will pick up the current can be delayed, and thus the average d-c voltage will be reduced, as shown in Fig. 16-19. Since the arc drop is such a small percentage of the total voltage, it is common practice to neglect it in diagrams, and this practice will be followed hereafter.

The nonsinusoidal character of the current in the anodes sets up harmonic currents in the power supply which interfere with telephone communication. In the smaller rectifiers, such as those supplying 5000 to 10,000 amp d-c current, this interference has not proved too objectionable. In the larger sizes, such as those supplying 50,000 to 80,000 amp, it is necessary to place a number of rectifier units in parallel, as the current per tube is usually limited to about 5000 amp. Where there are several units operating in parallel, it is possible to shift the phase of the various units so that they will be evenly spaced in time. Thus, if five units are used in parallel, they are spaced 12° apart; and if six units are used in parallel, they are spaced 10° apart.

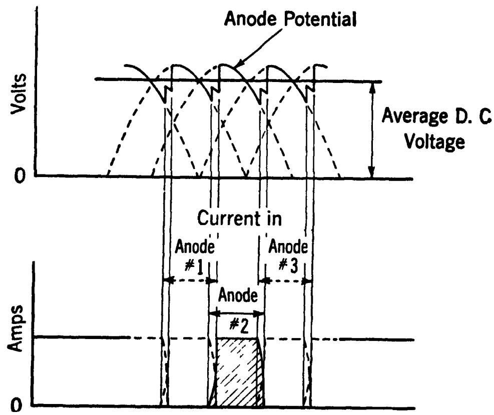


FIG. 16-19. Current and Voltage in a Six-Phase Rectifier– Delayed Ignition.

This need for even spacing necessitates the use of phase-shifting transformers of a type indicated diagrammatically in Fig. 16-20. In this case, the primaries of three transformers are connected in Δ, and the secondary of transformer (1) is connected in series with lead A' going to the primary of the rectifier transformer. Similarly, the secondary of transformer (2) is connected in series with lead B', and the secondary of transformer (3) is in series with lead C′. A voltage of about 10 per cent of the line voltage in these secondaries will be sufficient to rotate the phase angle by 10°. By reversing the connections of the secondary, this angle may be made to

lead or lag the original voltage supply. Thus, if six units are operating in parallel, a Δ-connected primary can be used with three of them. One would be carried straight through, and phase-shifting transformers would supply one unit on either side by 10°. A Y-connected primary would be used on the other three, which would give an inherent shift of 30°. One of these primaries would be carried straight through, and phase-shifting transformers would be used to shift the other two to either side by 10°. This would then provide six units separated by 10° in time phase and would thus neutralize most of the current harmonics.

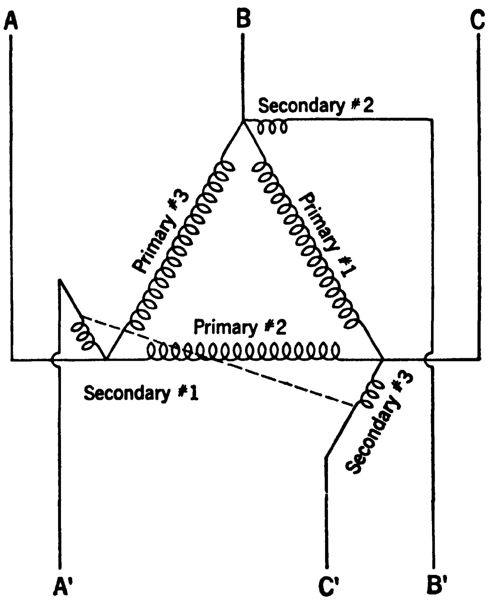


FIG. 16-20.

The above method of phase shifting is illustrative of many possible arrangements of transformer connections to obtain shifts in phase and magnitude of voltages. The transformer connections here shown are the most simple of the rectifier transformer connections. Other arrangements will undoubtedly be met in many commercial installations.

A single-line circuit diagram of the installation shown in Fig. 16-15 is given in Fig. 16-21. This shows the sequence of transformers, the location of the phase-shifting transformers, and the protective circuit breakers on both the a-c and d-c sides of the equipment.

Electrostatic precipitation equipment. Some chemical processes require the precipitation of fine particles either in the chemical process or as an adjunct to the process. This is often done by blowing the material through an ionized zone in which the particles collect a positive charge. They then pass between plates which are charged, and the particles are drawn to the

negative plates as illustrated in Fig. 16-22. The precipitated material is then collected from the plates.

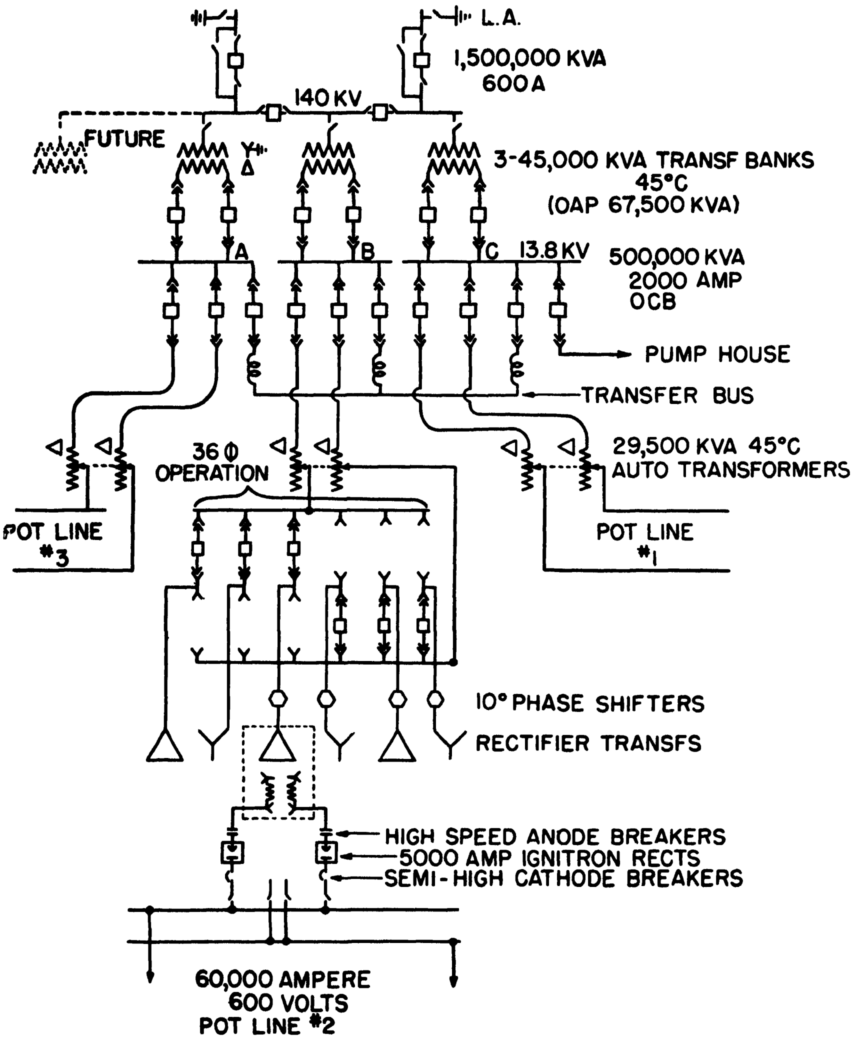


FIG. 16-21. Diagram of Rectifier Installation to Supply 60,000 amp at 600 v.

Ionization is produced by a high d-c voltage on a fine wire which is maintained at the ionizing potential by a high-voltage rectifier. Sometimes this rectifier is mechanical, but very often

it is a vacuum-tube rectifier which supplies 10,000 or more volts to produce ionization.

Summary

The illustrations of heating, welding, and electrochemical uses of electricity which have been covered in this chapter have been chosen both for their importance in industrial operations and because of the effectiveness of demonstrating the application of fundamental theory to these problems. The chemical

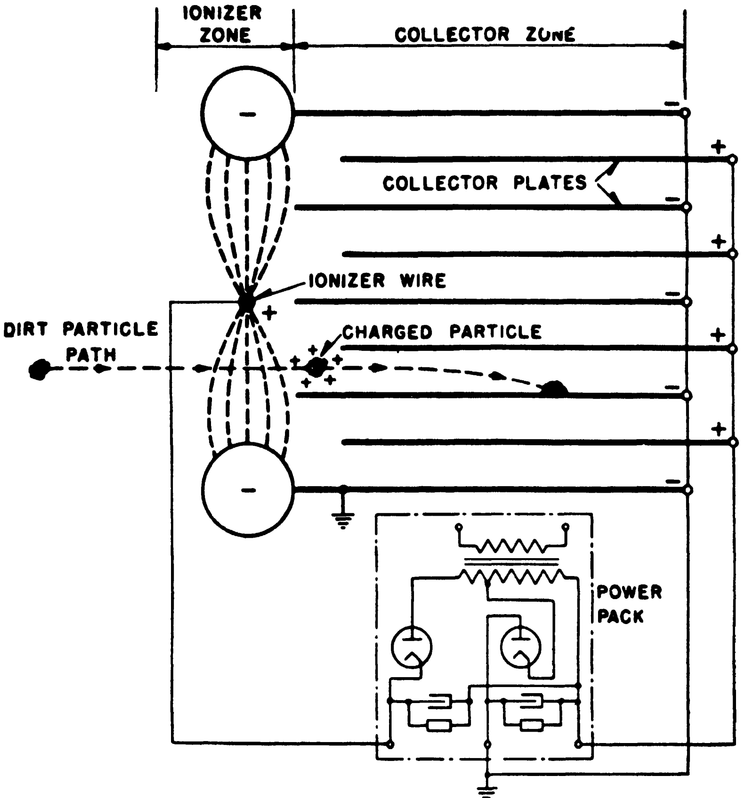


Fig. 16-22. Simplified Sketch Illustrating the Principle of the Precipitron.

engineer will find many additional illustrations in the current literature of his field. Similarly, mechanical and metallurgical engineers will find a wealth of illustrative material in their current literature.

Suggested Supplementary Reading

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CHAPTER 17

Electrical Illumination

Character of light and illumination

The science of physics defines light as a form of radiant energy. It is similar to the electromagnetic waves used for radio and radar except that the frequency is much higher. Light waves are propagated in straight lines in a homogeneous medium, and therefore the intensity of the light from a point source is inversely proportional to the square of the distance from the source unless altered by materials in its path. Light may be reflected by surfaces and refracted by lenses and prisms. A knowledge of these and many other properties of light is important in planning the illumination of any area.

It is not enough, however, to consider only the magnitude of light intensity. This light is to be used for seeing, and the seeing mechanism of the body has characteristics which also must be considered. The fundamentals of efficient and comfortable seeing, as determined by the eyes and the whole human seeing process, must enter into a successful lighting design. The four main factors of seeing are (1) the size of the object, (2) its brightness, (3) its brightness contrast with surroundings, and (4) the time that is available for seeing. The light must be so arranged that it will accentuate the differences in brightness and will not cause fatigue to the eye. The intensity of illumination must therefore be such that sufficient brightness is obtained to register on the eye without effort.

A single spot of unusual brightness caused by a direct reflection of a high-intensity light source will cause severe eye fatigue. Thus, undue contrasts are known as glare and must be prevented if the illumination is to be satisfactory. The science of physics, as it applies to light, and the physiological and psychological reactions of the individual to light, provide the foundations of illumination design.

Definitions of light units

The basis of all the quantitative light measurements is a *candle*, the specifications for which were agreed upon by the

standardizing laboratories of several of the leading countries.* The *candle* represents an amount of light that is given by this standard candle. The intensity of illumination of an object is measured in *foot-candles.* This unit is defined as the illumination received by an object which is at a distance of one foot from a standard candle. If the candle were located at a distance of two feet, the intensity of illumination would be but one quarter of a foot-candle, since, without reflectors, the intensity of

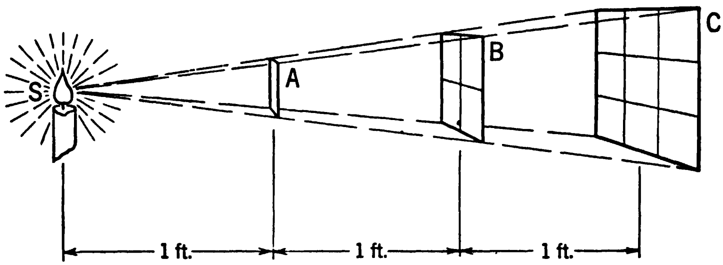

FIG. 17-1. The Illumination on a Surface Varies Inversely as the Square of the Distance from the Source to the Surface.

illumination varies inversely as the square of the distance from such a point source.

The amount of light required to illuminate an object will depend upon its size. The unit of light most commonly used is the *lumen,* which is defined as the light required to supply an illumination of one foot-candle to one square foot of area. If the candle were a source of light that was equally bright in all directions, and if it were located in the center of a sphere of one foot radius, then each point on the interior of the sphere would have an illumination of one foot-candle. Since each square foot of the surface would receive one lumen of light, then the whole surface would receive 4π lumens. The total light output of a standard candle can therefore be said to equal 4π lumens.

The number of lumens required to illuminate an area is equal to the product of the brightness in foot-candles and the area in square feet. Thus, if an area of 200 sq ft were to be illuminated to an intensity of 10 foot-candles, it would require 2000 lumens evenly distributed over the area.

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* This unit is now standardized and maintained by the much more constant incandescent lamp.

Control of light

The control of light is necessary to increase efficiency and furnish comfortable seeing conditions. In some factories, for instance, it would be undesirable to use a large portion of the light to illuminate the dirty and unsightly rafters of the building. It is common practice, therefore, to provide reflectors which redirect the light downward, where it can be used in illuminating the surface of the work. Reflection is therefore one of the very important methods of controlling light.

Where possible, it is desirable to clean the roof structure and paint it white. In this case a much more pleasing, comfortable, and effective lighting situation is obtained. The light that falls on the ceiling structure reduces the brightness ratio between the lighting units and the ceiling, which forms the background.

It is also necessary to reduce the brilliance of the light source in order to reduce glare. This is done by placing the light in a unit designed to keep the brightness within the comfort range, or by reflecting the light to a ceiling painted a light color, where it is again reflected to the working surface.

Where the direct rays of the light plus such light as is redirected by the lighting fixture are used to illuminate the working surface, the illumination is said to be *direct*. Where all of the light is first reflected to the ceiling, the illumination is said to be *indirect*. Between these two extremes is a wide variety of units ranging from the semidirect, where most of the light is directed downward but a little goes to the ceiling, through the general diffuse, where enclosing globes are commonly used, to the semi-indirect, where most of the light is directed to the ceiling and only a small portion comes through the translucent bowl of the unit.

Characteristics of reflecting surfaces

When a beam of light strikes a polished metal surface, it is reflected as shown in Fig. 17-2. The reflected beam remains concentrated, and the angle of incidence X is equal to the angle of reflection Y. Very little light is reflected in directions other than that of the main reflected beam. Reflectors made of polished metal or mirrored glass are, therefore, desirable for accurate focusing of light as in automobile headlights and in floodlights. These reflectors are sometimes enclosed and protected from dirt and dust, especially in headlights and floodlights.

When a beam of light strikes a rough-mat surface such as white blotting paper, the reflected light (as shown in Fig. 17-3) is reflected uniformly in all directions. This is said to be a *diffused reflection.* Enameled reflectors and the paint of walls and ceilings should provide this type of reflection. Ceilings

FIG. 17-2. Regular or Spectoral Reflection of Light.

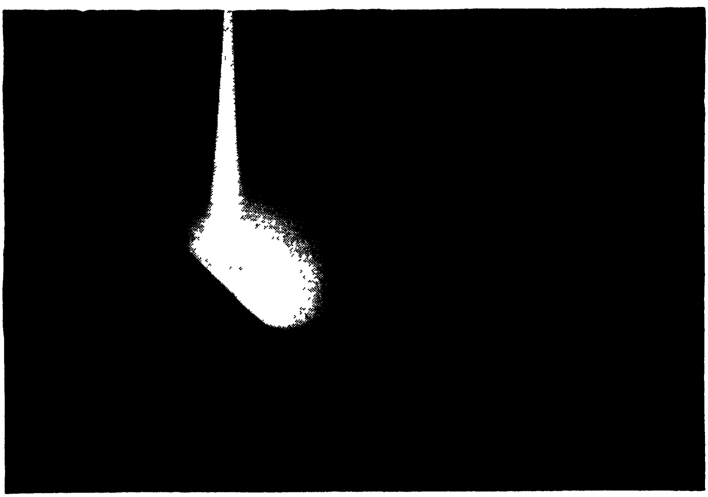


FIG. 17-3. Diffused Reflection of Light.

should be of a flat white or off-white color having a reflection factor of 80 per cent or more. Walls may have a reflection factor as low as 50 or 60 per cent. The colors most used for walls are green, blue, tan, coral, or gray—the first two being psychologically cool, the second two warm, and the last a neutral color.

Etched aluminum is used extensively as a reflector because of its continued high efficiency. It produces a semispecular reflection known as *spread reflection,* which is intermediate between the specular and diffused reflection.

Translucent materials

If a beam of light strikes a sheet of white or milk glass, a small part of it will be reflected as a beam from the surface. Most of it, however, will enter the glass; and when it strikes the white particles of the glass, it will be reflected in all directions

FIG. 17-4. Combined Reflection and Diffused Transmission of Light by a Translucent Material.

as shown in Fig. 17-4. About half of the light will get through the glass, and the remainder will be absorbed or reflected. A material that has light transmission similar to white glass is said to be translucent. Many plastics have this characteristic and so have come to be used in controlling light.

Brightness and glare

The eye responds logarithmically to variations in intensity of illumination. This response covers a wide range of intensity variation. If the general level of illumination is high, as it is on a bright sunny day, then the pupil of the eye contracts so that only a small amount of the light is permitted to enter the sensitive portions of the eye; and if the general level of illumination is low, the pupil enlarges to allow a greater portion of the light to enter. The pupil responds to average illumination, and, if a restricted area has a high intensity of illumination, it will overstimulate the sensitive parts of the eye and dis-

comfort will result. Such discomfort is referred to in illumination terminology as *glare.* Glare may be considered as any brightness in the visual field of such a character as to cause discomfort, annoyance, interference with vision, or eye fatigue. Experiments have determined that light intensities or brightness of greater than two or three candles per square inch produce immediate discomfort when in the central portion of the vision, while a brightness of more than 0.5 candle per square inch may produce discomfort under repeated exposure, especially if the luminous source is large.

Since all present sources of electric light, including the fluorescent lights, are above this lower level, it is necessary to (a) raise the lights above the ordinary range of vision, (b) reduce the unit brightness by enclosing the light in a translucent fixture, or by shielding with louvers or baffles, or (c) reduce the unit brightness by reflecting a large part or all of the light to the ceiling, where it is reflected at a low surface brightness.

A type of glare that is particularly annoying and difficult to eliminate is that which is reflected from the working surface. This is known as reflected glare. It may be caused by the reflection of the light source in the glossiness of white paper or other highly reflecting surface on a desk or workbench. For this reason it is never desirable to have a light in such a position that it reflects directly into the eyes. Desk or bench lamps for local illumination are likely to be particularly objectionable in this regard.

Light sources

Three types of light sources are available for industrial and office illumination. The oldest is the incandescent lamp, and it is still the correct solution in many instances. The mercury-vapor lamps are very efficient, but the color of their light is not pleasing. Just as efficient, and with much more pleasing color characteristics, are the popular fluorescent lamps. The important characteristics of each type are reviewed below.

Incandescent lamps. Incandescent lamps may usually be considered as a point source. They are available in a wide range of sizes varying from 10 w to 1500 w. The efficiency ranges from ten to twenty-two lumens per watt of electric power. The color of incandescent lamps, although slightly yellow in the lower wattages, has one advantage over the mercury and fluorescent lamps in that radiation from the incandescent lamps is

continuous throughout the visible spectrum rather than concentrated in certain lines as in the other two types of lamps. The life of incandescent lamps used for general service ranges from 750 to 1500 hr, according to lamp design.

Mercury-vapor lamps. The medium- and high-pressure mercury-vapor lamps (with the exception of the 3-kw lamp) are essentially point sources of light. The commercial sizes range from 100 w to 3000 w. Most of these lamps, however,

FIG. 17-5. General Lighting (Direct). Alternate staggering mercury and filament system. Concentrated high-bay Alzac reflectors with 1000-w bi-post lamps, and two-lamp high-bay Alzac mercury reflectors with two 400-w mercury lamps. Mounting height, 44 ft; spacing, 20 ft by 16 ft. Illumination, 42 foot-candles.

are in the 400- and 1000-w sizes, in which the efficiency is about 40 lumens per watt. The color is a blue-green and should ordinarily be combined with incandescent lamps to equalize the color. The life of these lamps is usually over 2000 hr and may be as high as 3000 hr. The replacement cost is high per unit, but the cost per lumen-hour is about the same as for incandescent lamps.

Fluorescent lamps. Fluorescent lamps are normally diffuse line sources. The color depends upon the fluorescent coating,

and so this type of lamp may be used in a variety of colors for decorative effects. The white and the daylight are the two most popular colors, the white being slightly more efficient than the daylight. The lamp life ranges from 2500 to 6000 hr, and so, although the cost of the lamp is considerably greater than the cost of an equal output incandescent lamp, the replace-

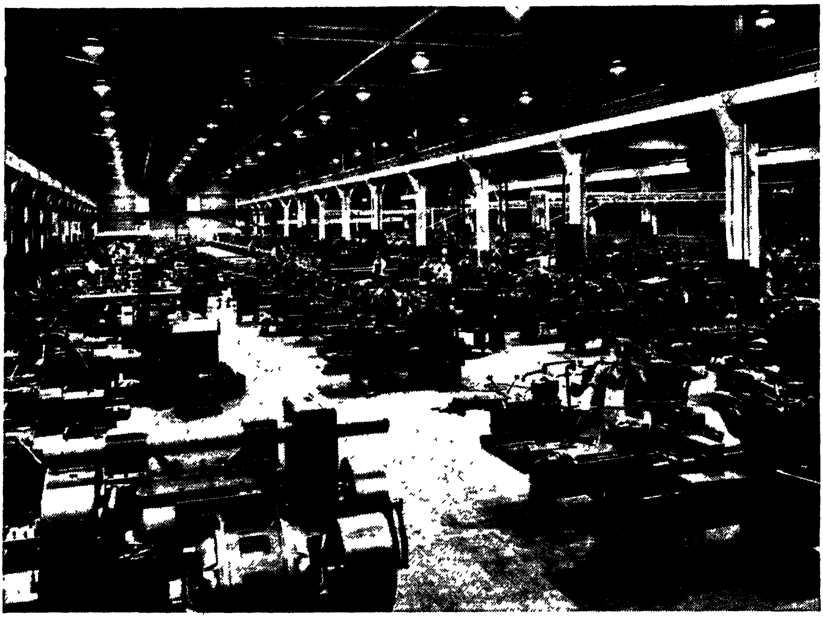


FIG. 17-6. Good Seeing Conditions in This Machine-Tool Plant. 750-w incandescent lamps in prismatic reflectors give uniform illumination of 30 foot-candles at bench height.

ment cost per lumen-hour is about the same as incandescent and mercury-vapor lamps. The efficiency ranges from 30 to 60 lumens per watt, which is about twice the efficiency of the incandescent lamps. Where illumination is used many hours per year, or where the energy rate is high, the fluorescent installation will usually prove most economical.

The lighting plan

In setting up the lighting plan one should first determine the seeing problem. That is, find out what must be seen and whether it is seen by brightness or by color contrast. It may also be seen by a contrast of detail against some background or by silhouette or even by a reflected image. After the seeing

problem has been carefully analyzed and its many ramifications studied, then the lighting results required to meet this seeing need are determined. The lighting levels required for the more usual factory and office tasks have been determined by joint committees of the Illuminating Engineering Society and the American Standards Association. These recommendations have

FIG. 17-7. Precision Tool Room. Industrial luminaires installed in continuous rows spaced 10 ft apart. Two 49-in. fluorescent lamps per luminaire. Mounting height, 10 ft. Illumination, 40 foot-candles.

been published and are available through these societies. The titles are given in the bibliography at the end of the chapter.

When the desirable type and level of illumination has been determined, several arrangements of the lighting installation should be planned. These should be so designed as to give as nearly similar illumination results as possible.

When the alternate designs are satisfactory from the point of view of illumination, they can be compared on a cost basis. This should include the first cost, power cost, and maintenance cost and may be calculated on an annual or capitalized cost basis as desired.

The details of illumination design, which include the knowledge of *American Recommended Practice of Industrial Lighting,*

the method of determining the room index, and utilization factors of various types of luminaires is beyond the scope of this text. Those who are faced with illumination design problems should make an extensive study of the bibliography at the end of the chapter or retain a competent illumination engineer to complete the design.

Typical illumination designs

It is dangerous to generalize regarding illumination design procedures. However, two rather typical illumination designs

FIG. 17-8. Good Office Illumination. Single-lamp 40-w aluminum troffers on 4-ft centers.

have been developed to meet different types of industrial situations.

The first is the large industrial building with ceiling height of twenty to fifty feet where cranes require that the lights be mounted twenty feet or more above the floor. Here it is common to use incandescent lamps alone or high-intensity mercury-vapor lamps, alternating with large incandescent lamps. These lamps are mounted in open or enclosed luminaires, which direct the light to the work below. The maintenance is low, the color is satisfactory, and the over-all efficiency is good.

Two examples of such installations are shown in Figs. 17-5 and 17-6.

Where the ceilings are lower, it is more common to use incandescent or fluorescent lamps in suitably shielded luminaires, as shown in Figs. 17-7 and 17-8. Here with the proper shielding the low surface brightness of the light source permits high levels of illumination without glare. Efficiency is high, and with adequate maintenance the benefits of good illumination are sustained.

There is often a temptation to cut the cost by lowering the quality of illumination. Usually the illumination cost in an industrial enterprise is such a small part of the over-all production cost that lowering the quality of illumination is false economy.

Suggested Supplementary Reading

Books:

Kraehenbuhl, J. O., *Electrical Illumination.* New York: John Wiley & Sons, Inc., 1942.

Luckiesh, M., *Light, Vision, and Seeing.* New York: D. Van Nostrand Company, Inc., 1944.

Sharp, H., *An Introduction to Lighting.* New York: Prentice-Hall, Inc. 1951.

Bulletins of the Illuminating Engineering Society:

Recommended Practice of Office Lighting.

Bulletins of the General Electric Company:

Mazda Lamps (LD-1).

Illumination Design Data (LD-6H).

Essential Data for General Lighting Design (Folder D).

CHAPTER 18

Electrical Methods of Industrial Measurement

Instrumentation in industry

An important application of electricity is its use in measuring and controlling industrial quantities and processes. Many large manufacturing companies train recent engineering graduates by placing them in charge of the operation and maintenance of the instruments in the plant. The intimate contact with processing problems, gained in maintaining the instruments and control, soon permits them to assume responsibilities for process control. That responsibility is usually the first step in the climb to positions as plant superintendents.

Many of these instruments are not electrical instruments. However, it has been found that electrical methods of measurement are so versatile and so reliable that instrument companies that restricted themselves entirely to mechanical types only a few years ago are now selling a high proportion of electrical instruments. The electrical method seems to be most advantageous where the problems of measurement are difficult and complex or where measurement and control are associated in the same instrument. It is often comparatively easy and inexpensive to arrange a complex electrical circuit to accomplish results which could not otherwise be obtained or which could be achieved only with considerable difficulty and expense by mechanical and hydraulic methods.

Conversion from industrial to electrical quantity

The first problem in industrial instrumentation is to convert the variation of the industrial quantity into a variation of some electrical quantity. Temperature is one of the most important of the quantities in industrial production, and there are several ways in which a temperature variation is converted into a variation which can be measured electrically. For comparatively low temperatures, the variation of resistance is usually most convenient. For intermediate temperatures the thermo-

couple is most often used, while for very high temperatures the preference goes to one of several methods of converting radiation into a variation of voltage or current.

In the measurement of speed of rotation it is usual to use a small d-c or a-c generator with a permanent magnet so that the flux is constant. The voltage generated is then proportional to speed.

Strains and the corresponding stresses can be measured by converting the small change of length into a variation of resistance, inductance, or capacitance. These variations of circuit parameters can then be measured and the results converted back into the original stresses and strains. Pressure can usually be measured in terms of a strain, and so deflections of a diaphragm are sometimes converted by means of carbon buttons or piezoelectric crystals to voltage variations.

These are only a few of the many types of conversion to electrical quantities, but they serve to illustrate the process. This summary also emphasizes that the specific illustrations given later in the chapter are only a restricted sampling of the very broad field of industrial instrumentation using electrical methods.

Measurement of the electrical quantity

After the industrial quantity has been converted to an electrical variation, it is necessary to measure the electrical quantity. Two general types of measurement are available. One provides an indication of the magnitude and is called an *indicating meter;* the other not only indicates but makes a record of the variation of magnitude and is called a *recording meter.* Except in aircraft operation, where it is necessary to concentrate all measurable quantities on one instrument panel, it may be expected that, when it is worth while to convert to electrical methods of measuring, it is probably worth while to make a permanent record of the measurement. Subsequent illustrations, however, will include both indicating and recording types of electrical measuring instruments.

Electrical measuring instruments may also be classified under the electrical principle used in the measurement procedure. The Wheatstone bridge, which was briefly described in Chapter 3, is one of the most generally used methods of measurement. The potentiometer, the principles of which are explained in connection with an illustrative application in a later paragraph,

is so similar in the measurement procedure that the same equipment may be used equally well with the Wheatstone bridge and the potentiometer. The bridge circuit is ordinarily used to measure a variation in a circuit element such as resistance, inductance, or capacitance, while the potentiometer measures small changes in voltage with great accuracy.

Both the bridge and potentiometer involve making changes in a calibrated variable circuit element until it has compensated for a change in voltage produced by the industrially activated circuit element. When this compensation has been made, there is no voltage difference, and the measurement is said to be a *null measurement* because the final reading is the reading of the calibrated circuit element when the voltage difference is zero.

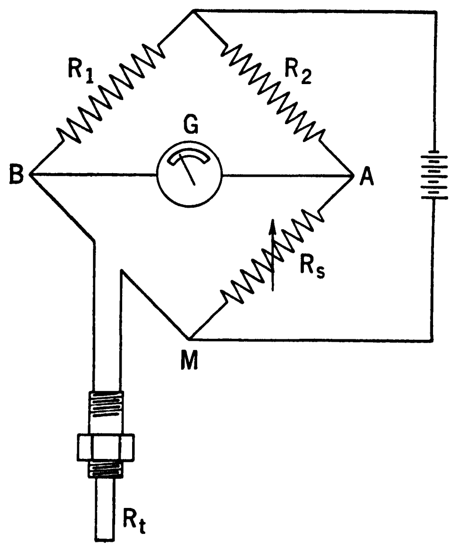


FIG. 18-1. Simple Wheatstone Bridge for Measuring Temperature with Resistance Coil.

Permanent-magnet moving-coil types of meters are used most extensively for indicating meters for measuring the current or voltage variation. They are also used to measure the magnitude of the unbalance in the Wheatstone bridge and potentiometer circuits. Occasionally these meters have recording pens to make a permanent record.

Where the quantity to be measured varies rapidly, as in the case of vibrations in machinery, it is often necessary to detect the variations, amplify them by means of a vacuum-tube amplifier, and then observe or record them by means of an oscillograph. This instrument is one in which the current- and voltage-measuring elements have very low inertia so that they can respond to rapid variations.

Temperature measurement using resistance coils

As mentioned above, the variation of the resistance of a coil with temperature is often used as the basis of temperature

measurement. In this method the resistance coil is placed in a thermometer well or other location whose temperature is to be determined. Usually the coil is used as one element of a Wheatstone bridge, as indicated in Fig. 18-1. When the resistance of the coil has been measured, the temperatures may be obtained from a calibration chart, or the variable resistance used to balance the bridge may be calibrated directly in temperature.

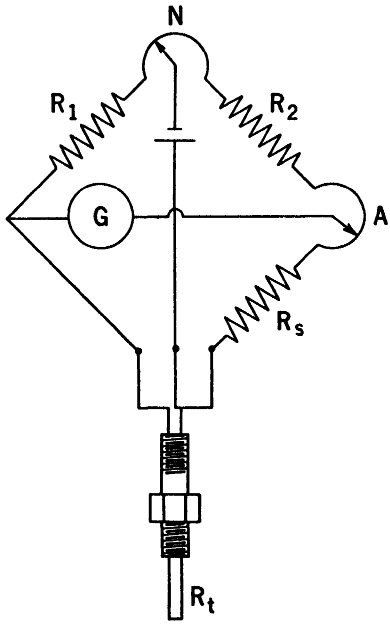


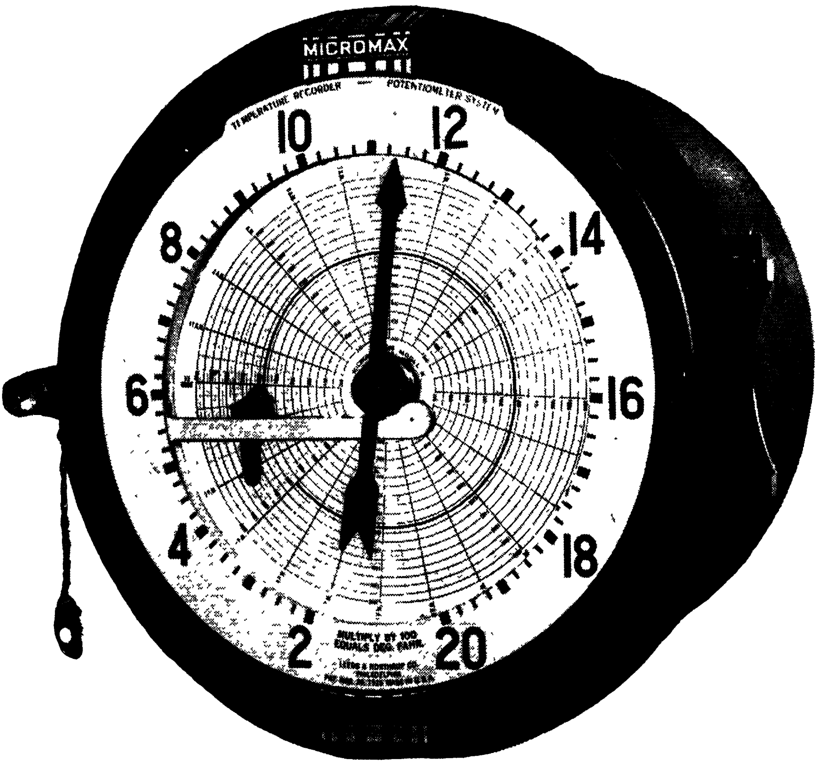
FIG. 18-2 Circuit Used to Eliminate Contact Errors in the Wheatstone Bridge.

A record may be obtained by any of a large number of recording meters available. Most of them are similar, in that they have a sensitive galvanometer to indicate the unbalance. They likewise have a motor mechanism (actuated by this galvanometer) which varies R_s in order to maintain the bridge in balance. A recording pen is geared to the slidewire adjustment of R_s, and this recording pen both indicates and records the variation in R_s directly in terms of temperature variation. Refinements to obtain greater accuracy are often introduced into the circuit. One manufacturer uses the circuit indicated in Fig. 18-2. In order to obtain a high degree of accuracy, several precautions have been taken which are typical of the methods used in accurate recording meters.

In the first place, a three-conductor cable with all conductors the same size is used to connect the temperature-sensitive coil to the bridge. Since the cable introduces the same amount of resistance in the R_s circuit as in the R_t circuit, no correction is necessary for the distance between the recorder and the thermometer well.

Second, the recorder uses a double slide wire, one wire located at A and the other at N, in order to eliminate errors due to contact resistance. The contact at A is in the galvanometer circuit, and, since it is a null detector, a slight

additional resistance has no effect on the measurement. Similarly, the contact at N is in series with the battery, and so does not enter into the accuracy of the measurement. When the slide contact at A increases the resistance of R_s, it simultaneously decreases the resistance of R_2. Since it is desirable that R_1 be equal to R_2 at all times, the slide contact at N is so arranged that the decrease will be divided equally between the two

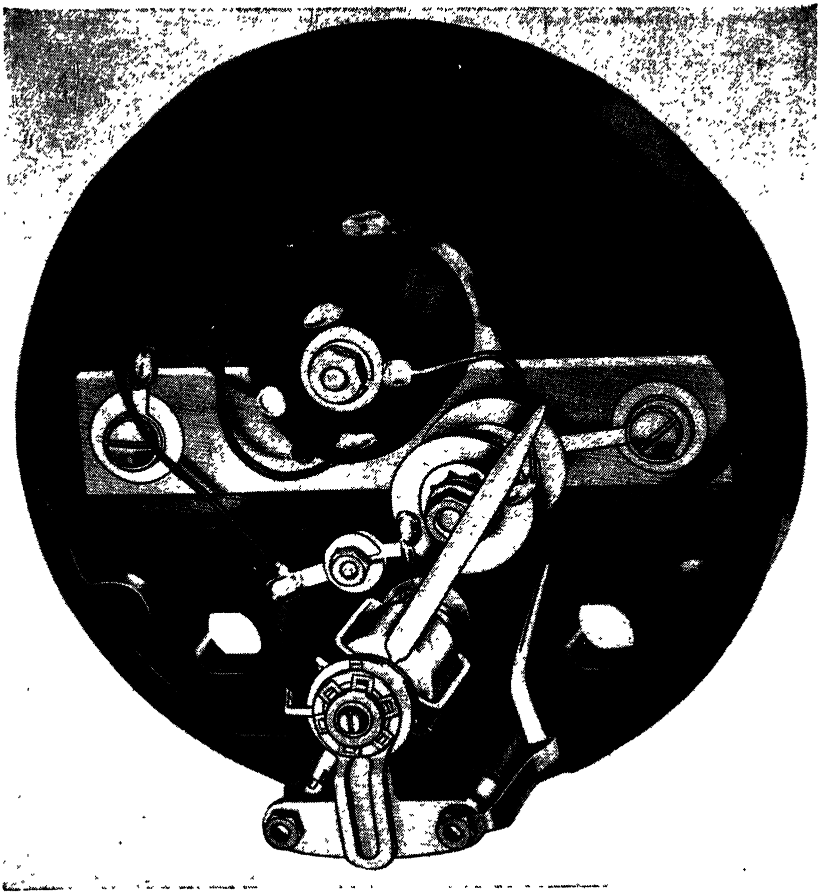


Courtesy of Leeds and Northrup Company

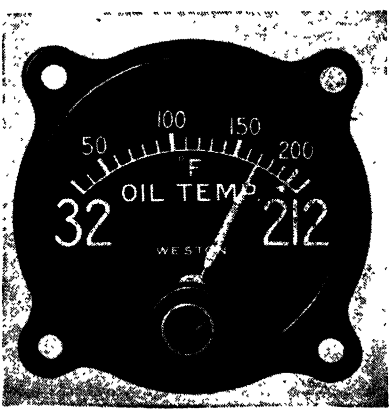
FIG. 18-3. Temperature Indicator and Recorder with Circular Chart.

resistances R_1 and R_2. In order to accomplish this, the slide wire at N must have one half the resistance of the one at A. The meter using this circuit and a section of the chart drawn by it is shown in Fig. 18-3.

Switching arrangements may be made so that a recorder of this type can be used to measure successively the temperature of eight or ten of these resistance coils and make a continuous record of them on one chart. Since a recording instrument of this type is quite expensive, this is an important feature in



(a)



(b)

FIG. 18-4. (a) Wheatstone-Bridge-Type Aircraft Resistance Thermometer. The circuit is not easy to trace, because three arms of the bridge are wound on a single spool to save space. The second spool shown adjusts the instrument for a certain supply voltage. (b) A Wheatstone-Bridge-Type Oil Temperature Indicator.

preventing excessive instumentation costs. It is possible to place control contacts on such a meter in order to maintain constant temperature automatically.

When an indication of the temperature is adequate, and when a high degree of accuracy over the entire dial range is not required, the deflection of the galvanometer may be used as a measure of the variation of resistance. Instruments of this type are particularly useful on airplane instrument panels. In this application, the resistances R_1, R_2, and R_3 are all fixed in magnitude (usually being wound on a single spool and mounted inside the galvanometer or millivoltmeter case) and are adjusted so that the bridge is balanced at or near a temperature where maximum accuracy is desired. The millivoltmeter is set to indicate this temperature on the scale when no current flows, or when the bridge is balanced. If the resistance coil in the temperature well is of a lower temperature, then the voltage (as indicated in Fig. 18-1) across MB will be reduced and a current will flow from A to B. Since the voltage variation and therefore the meter deflection will be essentially proportional to the deviation from balance, the meter scale may be made to read temperature rather than voltage difference.

Fig. 18-4(a) shows an enlarged phantom view of an instrument with the bridge resistances wound on a spool and located within the meter case. Fig. 18-4 (b) shows an assembled instrument designed for aircraft service.

Temperature measurement using a thermocouple

Another common method of measuring temperature is that which uses a thermocouple. A thermocouple is made by welding two different metals together. When this junction is heated, a small voltage is developed between the two metals. Since the magnitude of this voltage is approximately proportional to the temperature, it may be used to measure the temperature. Stated more exactly, the magnitude of the voltage is approximately proportional to the difference in temperature between the "hot junction" and the "cold junction" of the thermocouple wires. The voltages generated by six commonly used thermocouple materials are given in Table 18-1 for various temperatures of the hot junction when the cold junction is kept at the temperature of melting ice.

When an indication of the temperature is all that is required, an indicating meter of the permanent-magnet moving-coil type

can be used with a circuit similar to that of Fig. 18-5(a). In this circuit the cold ends of the leads are at the meter, and, since that temperature may vary considerably, some arrangement must be made to compensate for this variation in temperature. One manufacturer does this by inserting a bimetallic spring between the zero adjustment and the hairspring of the meter. The bimetallic spring, as shown in Fig. 18-5(b), will rotate the zero setting of the instrument so that it will read the temperature of the meter. In this way the meter is made to read true temperature in spite of wide variations in the temperature of the cold junction. It is necessary also to compensate for variation in the resistance of the leads with temperature, and this is done by inserting a resistance which has a negative temperature coefficient.

TABLE 18-1
VOLTAGES GENERATED BY COMMON THERMOCOUPLES
(Temperature of Cold Junction—0° C)

	Degrees C				Degrees C		
EMF, mv	Platinum to platinum-(10% rhodium)	Platinum to platinum-(13% rhodium)	Copper to constantan	EMF, mv	Chromel to alumel	Iron to constantan	Chromel to constantan
0	0	0	0	0	0	0	0
2	265	259	49	5	122	93	80
4	478	457	94	10	246	182	153
6	678	638	136	15	367	272	221
8	861	806	176	20	485	362	286
10	1037	964	213	25	602	453	350
12	1206	1114	250	30	720	543	413
14	1374	1259	285	40	966	711	537
16	1543	1404	319	50	1232	865	661
18	....	1550	353	60	....	...	786
				70	....	...	915

Sometimes difficulty is experienced due to resistance developing in the hot junction of the thermocouple. In order to overcome this resistance, to obtain greater accuracy, and to make possible a satisfactory recording meter, a potentiometer is often used to measure the thermocouple voltage. The basic principle of the potentiometer involves balancing the voltage to be measured against the potential drop in a standardized

resistance carrying a known and standard amount of current. When used as a measuring device, it is customary for the current to be a decimal fraction of an ampere, as, for instance, 1 ma. To standardize the current to this correct amount, the variable contact C shown in Fig. 18-6 is set so that the resistance between C and A is equal to 1018.3 ohms. When 1 ma flows, the voltage drop will be 1.0183 v, which is the voltage of the standard cell. The double-throw switch is thrown to the "up" position, and the resistance R_2 is adjusted until no deflection is observed in the galvanometer. The double-throw switch is then thrown down, and the variable contact C is adjusted until the galvanometer does not deflect. The magnitude of the unknown voltage V is then equal to the resistance between C', the new position of the variable contact, and A multiplied by 0.001.

Since the voltage of the standard cell is approximately one volt and the voltage of the thermocouple is only a few millivolts, this circuit is changed slightly

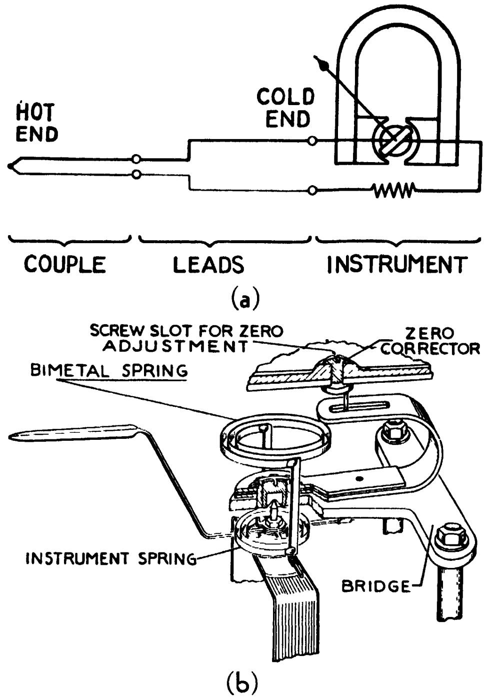


FIG. 18-5 (a) Thermocouple Circuit for Measuring Temperature. (b) Temperature Compensator. Instrument (cold-end) compensation is accomplished by the bimetallic spring, which winds or unwinds with changes in temperature and acts like an automatic zero compensator.

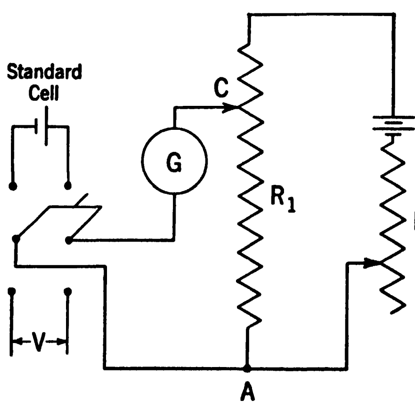


FIG. 18-6. Elementary Potentiometer Circuit.

in some instruments. Thus the circuit that is employed by one manufacturer is illustrated in Fig. 18-7. Here two parallel circuits are used, and the drop across these is standardized against the standard cell as explained previously. (The standardizing connections are omitted in order to simplify the diagram.) Point 6 of the circuit may be assumed to be of fixed potential, and this potential corresponds to the potential of point 2 on the parallel circuit. Between points 2 and 3 there is a resistance wire along which the variable contact can slide. The thermocouple T is connected from point 6 to the variable contact through the galvanometer G. When the variable contact is adjusted so that the galvanometer deflection is zero, the voltage of the thermocouple is just equal to the voltage drop of the slide wire between points 2 and 4. The slide wire may be calibrated in terms of thermocouple voltage or directly in terms of temperature.

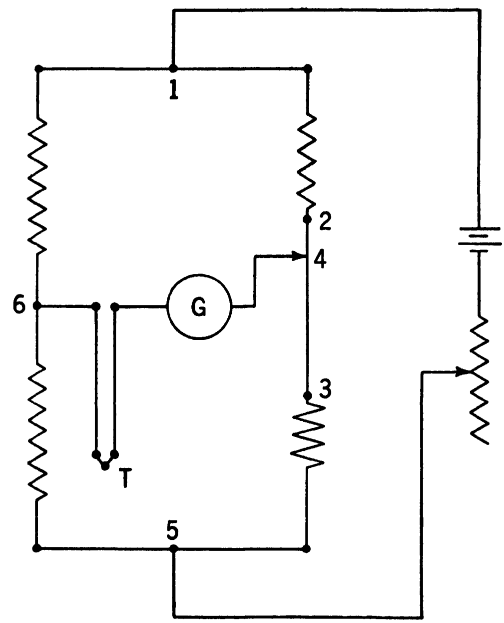
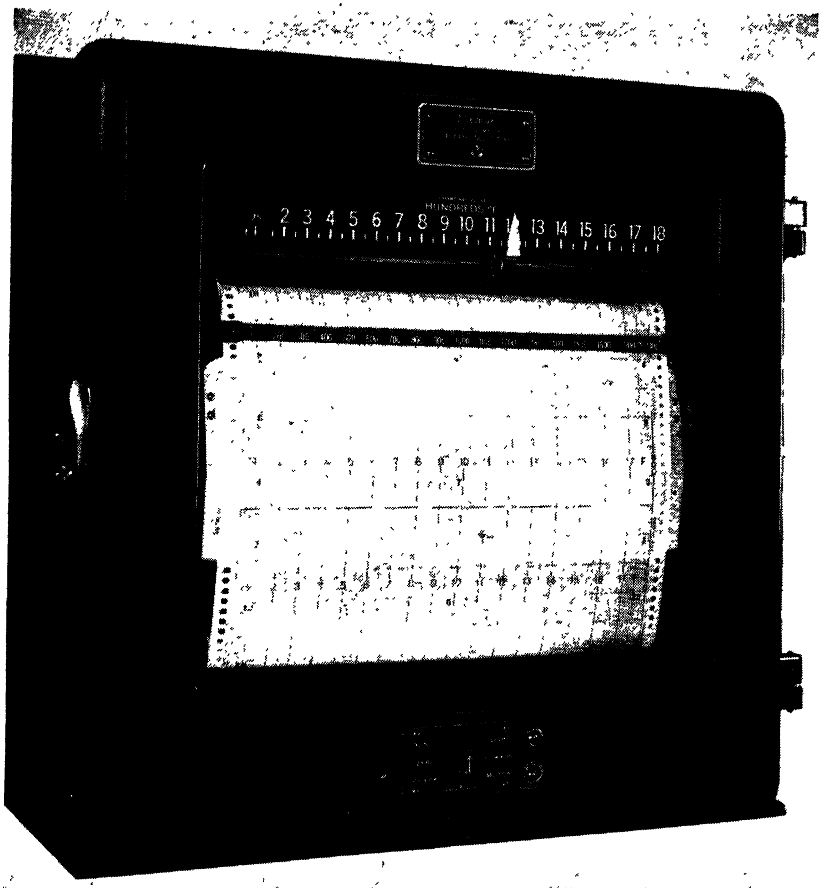


FIG. 18-7. Commercial Potentiometer Circuit Used with Thermocouple to Measure Temperature.

Cold-junction compensation is needed in the potentiometer method, just as it is in the indicating meter. This is usually accomplished by inserting a resistance with a temperature coefficient in one of the balancing circuits. For instance, the resistance between points 1 and 6 can be made so that the increase in voltage drop is just equal to the reduction in thermocouple voltage when the temperature of the cold junction increases. This will then continue to give the correct reading on the slide wire at point 4.

Fig. 18-8 shows a self-balancing and recording meter which uses this type of circuit. There is little if any difference in

the balancing operation of this type of instrument and of the Wheatstone bridge studied previously.



Courtesy of the Foxboro Company.

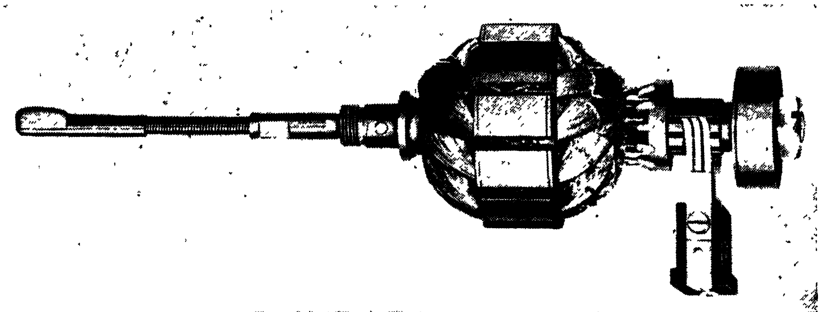
FIG. 18-8. Temperature Indicator and Recorder with Strip Chart.

Electrical measurement of speed

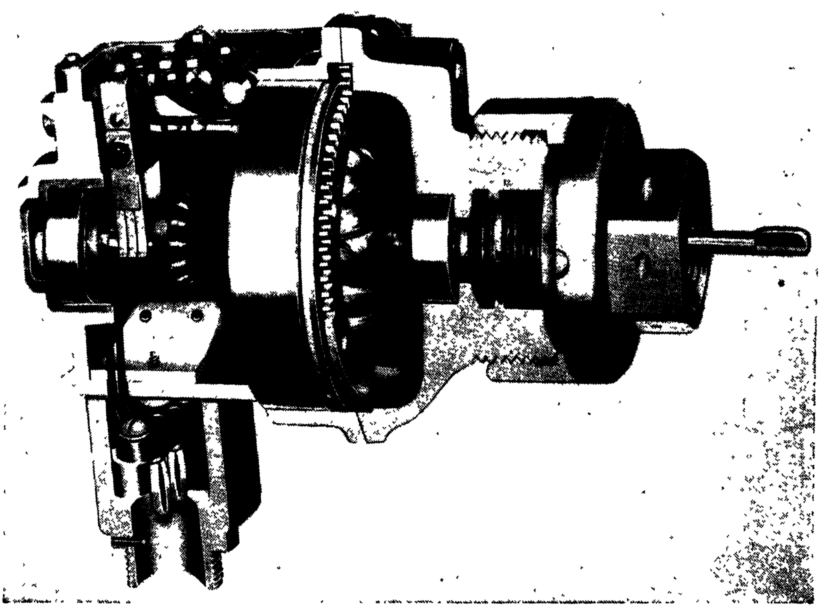
Speed measurements usually refer to measurements of rotational velocity. Where it is desired to measure a linear velocity, it is usually possible to convert this linear velocity to a rotational velocity by means of gears, belts, or other mechanical devices. Rotational velocity is usually measured by connecting the armature of a permanent-magnet generator, often called a *magneto*, to the rotating element. Since the field flux is constant, the voltage is directly proportional to the

speed. A voltmeter that has a scale calibrated in revolutions per minute is connected to the magneto and provides a satisfactory indicating meter.

The construction of a magneto-generator for speed measurements is shown in Fig. 18-9. In part *a* of the figure, the d-c



(a)



(b)

FIG. 18-9. Internal View of a Tachometer Magneto Generator. (a) The Coils of the Armature Are Connected to the Commutator's Segments, on Which the Brushes Rest to Pick Up a Direct Current. (b) The Circular Magnet Completely Surrounds the Armature.

armature, commutator, and the brushes are shown. Part *b* of the figure shows a cutaway view of the assembled magneto. The poles are on the opposite sides of the circular magnet, so that the machine is a two-pole generator. This construction is made possible because of the high retentivity of modern magnet steels. Since the only current delivered by the generator is that needed to operate the indicating voltmeter, commutation difficulties are minimized.

One example of the use of such a tachometer indicator is in large bakeries where ovens are in continuous operation, with the pans of bread and cookies placed on a conveyor. The length of time the baked goods are in the oven depends upon the speed of the conveyor. The meter shown in Fig. 18-10 has a range in baking time of from 15 to 60 min. It is noted that the maximum indication gives the minimum time. Thus, if the maximum motor speed is 1800 rpm, and if this gives 15 min baking time, the maximum scale reading which corresponds to 1800 rpm would be 15 min. Similarly, half speed, or 900 rpm, would give 30 min, and one-fourth speed, or 450 rpm, would give 60 min.

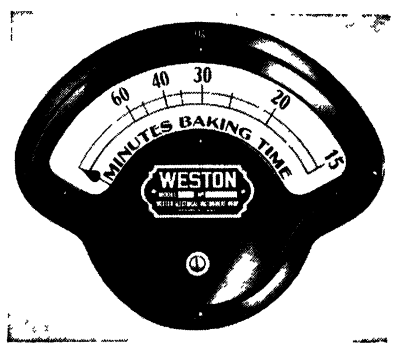


FIG. 18-10. Tachometer Indicator Used to Measure Baking Time.

Measurement of stress—strain gages

The extensive use of strain gages in solving the problems of vibration in machines and determining stresses of structures in service, makes the understanding of their use an important item in the training of civil, mechanical, and chemical engineers. As mentioned in Chapter 15, fine wires may be cemented to the surface of machine or structural elements. These wires are elongated or compressed to the same degree that the surface of the machine element is elongated. If the variation of resistance with variation in length is known, this can be used to measure the surface deformation and thus the surface strain. Such strain gages, mounted on thin paper, ready to be cemented to the machine surface and already standardized and calibrated, are now commercially available. (See Fig. 18-11.) The wire

from which the gage is made is usually of a diameter of 0.001 in. and made of an alloy. The favorite material for static gages is a copper-nickel alloy, while that preferred for dynamic strain investigations is known as isoelastic. The copper-nickel alloy has a negligible temperature coefficient of resistance and has a 2 per cent variation in resistance for every 1 per cent variation in length. The isoelastic alloy gives a 3.5 per cent variation in resistance for every 1 per cent variation in length but has a high temperature coefficient.

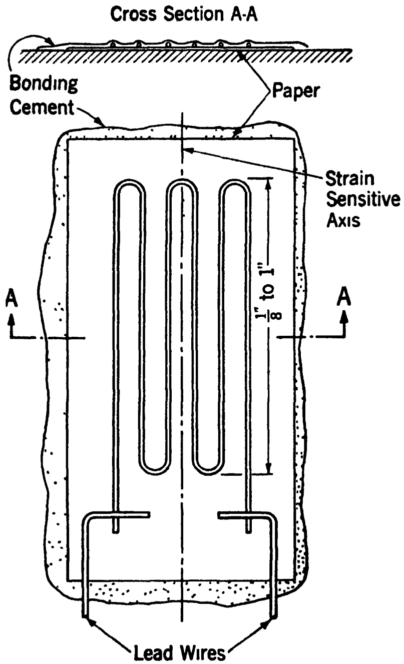


FIG. 18-11. Plan and Cross-Section Views Showing Construction of a Typical Wire Resistance Strain Gage.

When the strain gage is to be used for the study of vibration or dynamic strain measurements, the circuit shown in Fig. 18-12 may be used. The resistance R should be adjusted so that about 30 ma flow in the strain gage. This gives a voltage across the gage of 15 v for a 500-ohm gage. If a 0.2 per cent variation in length of the machine part is to be measured, the voltage variation across the gage (when made of isoelastic alloy) will be

$$0.002 \times 3.5 \times 15 = 0.105 \text{ v.}$$

This voltage is transmitted through the condenser C to the grid of the first tube in the amplifier, as described in Chapter 8.

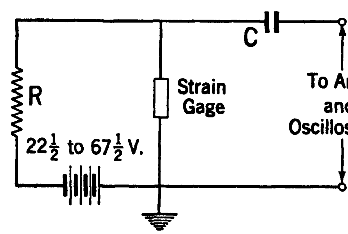


FIG. 18-12. Voltage-Divider Circuit for Dynamic Strain Measurement.

In Fig. 18-13 the circuit of a type of commercial instrument for measuring static strains is shown. The Wheatstone bridge is shown at the left inclosed in the dotted lines marked A. The zero-set adjustment can be composed of two very high

resistances as shown to compensate for small variation in the resistance of the gages and give the correct zero setting on the slide wire. The actual measurement is made by manually adjusting the slide wire to obtain a balance.

The oscillator, shown inclosed in the dotted lines marked *B*, supplies the audio-frequency voltage to the bridge. The balance is determined by taking the unbalance or detector current through a transformer and into a normal three-stage

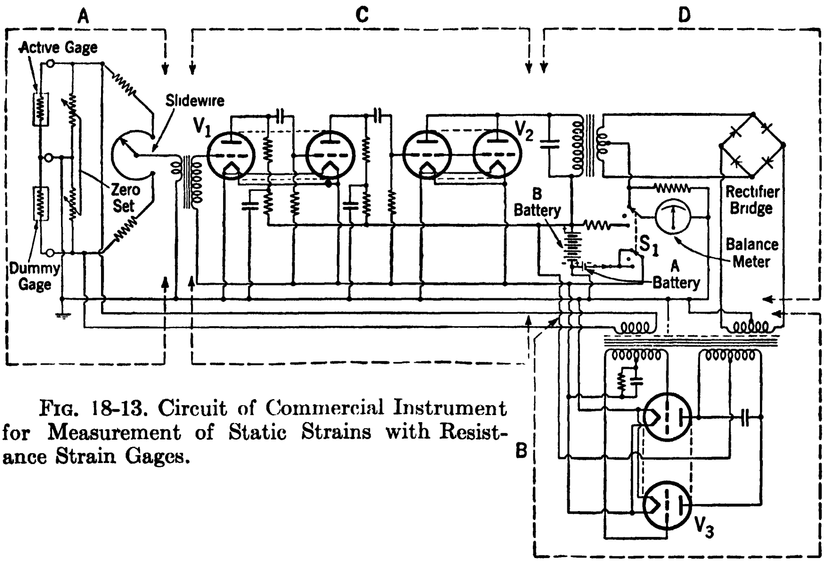


FIG. 18-13. Circuit of Commercial Instrument for Measurement of Static Strains with Resistance Strain Gages.

amplifier inclosed in the dotted lines *C*. It is noted that the first two stages are in one tube envelope, and in the other tube envelope the tube elements are connected in parallel. The output of the amplifier feeds into the detecting section *D*, so that even a very minute unbalance at *A* becomes sufficiently large for the balance meter to detect.

The operation of the detecting element is somewhat complicated by the ring-rectifier element,* which is used with

* The operation of this ring-rectifier bridge is based on the fact that the voltage from the oscillator is always considerably larger than that from the amplifier detector. It therefore determines which of the bridge elements are of low resistance and high resistance each half cycle. The output of the amplifier is impressed upon a circuit which from its corners is unsymmetrical, and so a current flows through the balance meter, the direction depending upon the polarity of the amplified voltage with respect to the oscillator voltage.

a zero-center balance meter to indicate the direction in which the bridge is unbalanced. This circuit is illustrative of many ingenious but somewhat complicated circuits which are continuously associated with electronic equipment. Such circuits are very useful in making electronic equipment accomplish particularly difficult objectives, but they are discouraging to the beginner. When analyzing electronic circuits, the beginner is urged to group the circuit on a functional basis, much as has been done in the above circuit, and then the analysis of each element is much simplified.

Measurement of pressure

The measurement of pressure is usually accomplished by translating the pressure to a movement or deflection which is proportional to the pressure. This movement is then used to cause a variation in some electrical element, such as resistance, inductance, capacity, or voltage across a piezoelectric crystal.

One method uses a wire resistance strain gage to measure the expansion of a very small cylinder (about one-fourth inch in diameter) which is made subject to the pressure. This appears to have real merit for pressures in the range of 1000 lb per sq in. The inertia is low, and therefore the frequency response is satisfactory up to several thousand cycles per second. This application shows the wide adaptability of a single technique of instrumentation, such as the strain gage.

Other pressure pickups use the variation of inductance or capacitance. These all require the use of amplifiers, and in some cases circuits, which act as integrators or differentiators. As a result, the use of these electrical pressure instruments is largely restricted to research problems involving high pressure, as in the study of the cylinder pressures of internal combustion engines.

Electrical indication of position

A method of transmitting position electrically from one location to another, which is finding extensive use, is that system known by such trade names as *Selsyn*, *Autosyn*, and so on. In this system two identical stators, similar to induction motor stators with three-phase windings, are used with bipolar fields which may be rotated and which are excited from a common 110-volt a-c line. The stators are connected in symmetrical

fashion as shown in Fig. 18-14. If the unit at the left of the diagram is the sender or generator and the one on the right is the receiver or motor, then the rotor position of the sender will be determined by the position of the machine element which it is desired to indicate at the remote position. When this rotor has assumed a position, it will generate voltages in the stator windings by transformer action. The voltages of the stator windings will all have the same time phase but will differ in

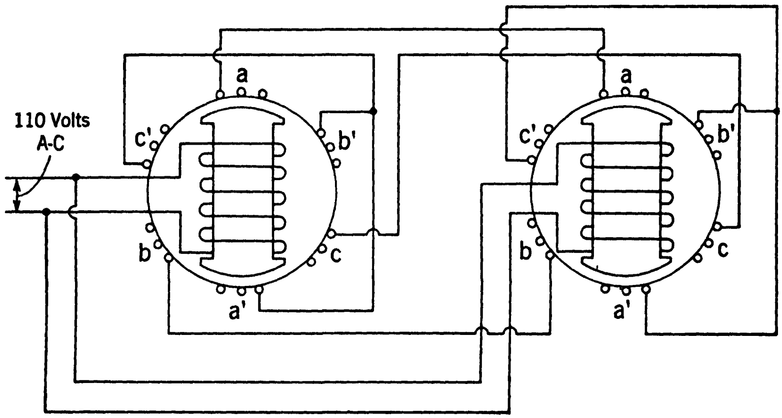


FIG. 18-14. Diagram of Selsyn Connections.

magnitude. These voltages will cause currents to flow in the stator of the receiver, and these currents will produce a torque that will cause the rotor of the receiver to turn. This movement will continue until the rotor of the receiver assumes the same position as the rotor of the sender. When this occurs, the voltages in both stators, produced by the transformer action of the identical rotors, will be equal, and no stator current will flow. The rotor of the receiver, being free to move to any position, will therefore always assume the same position as the sender.

Selection of electrical instruments

Electrical instrumentation should never be chosen because it is *electrical.* Unless it is more *accurate*, more *reliable*, or can provide indication and recording at a *central location*, or in some other way will reduce operating costs or improve the product, it should not be installed. Good instrumentation will usually pay big dividends, but it should not be overdone.

Local indication and recording save in the original cost of the

installation, but savings in cost of changing charts, the advantage of being able to supervise the plant operation from a central location, and the reduction in maintenance resulting from removal of instruments from intimate contact with corrosive fumes, and so on, may well justify a central instrument room.

Automatic control is usually associated with the instrumentation. This subject is discussed in Chapter 19. Automatic control features will often determine the selection of equipment which combines control and instrumentation.

Instrument maintenance

Any system of instrumentation is only as good as the maintenance of the equipment. Modern equipment involving delicate and highly accurate instrument mechanisms must be serviced by intelligent and well-trained personnel. It is not necessary that they be capable of designing the instruments, but they should understand their operation in order that they may know what to do in case trouble develops.

Exercise 18-1. Design a meter that will give the cargo weight of an airplane. (*Hint:* Strain gages on the supports of the landing wheels is one possibility.)

Exercise 18-2. Design an accelerometer using a spring-controlled iron armature to vary the inductance of a solenoid.

Exercise 18-3. Show how a strain gage could be used to measure draw-bar pull of a locomotive. What precautions would have to be observed to get accurate results? How could it be calibrated?

Exercise 18-4. Explain why two strain gages are often used in series, one on each side of a structural member under stress.

CHAPTER 19

Industrial Wiring Systems

Types of industrial installations

Industrial installations vary in size from the small shoe-repair shop using a single fractional-hp motor to the huge factories, refineries, mills, and so on, which use more power than many large cities. The problem of an electric power installation will likewise vary from a simple single-phase wiring plan to a very complex and carefully engineered distribution system. Little difficulty is usually experienced with either extreme of this industrial wiring problem. The simple system can be adequately handled by the local electrical contractor guided by the National Electrical Code and the rules of the public utility serving the territory. The large and complex installation will be handled by competent electrical engineers in the employ of the industrial organization, aided very often by expert consulting engineers. In the intermediate group of industrial concerns, however, there is opportunity for much improvement. In most of these industries they cannot afford, or at least do not hire, competent electrical engineers, and the supervision of the wiring plan is the responsibility of the chemical, metallurgical, or civil engineer who is the technical advisor of the management.

Power sources

Most of the intermediate-size industrial installations will use power from the local public utility, and *any plans for new installations or radical changes in present equipment should be discussed with the power company engineers.* In general, the power company will have available a supply of three-phase power at 2400, 4160, 6900, or 13,200 v. For small installations or for intermediate installations in regions of heavy load density there will also be available three phase at 240 v or three phase four wire at 120–208 v.

Where a power plant has already been installed as a part

of the industrial plant, it is likely to be one of the above voltages, although 460 v three phase is not uncommon. The problem of distribution to the buildings of the industry and to individual

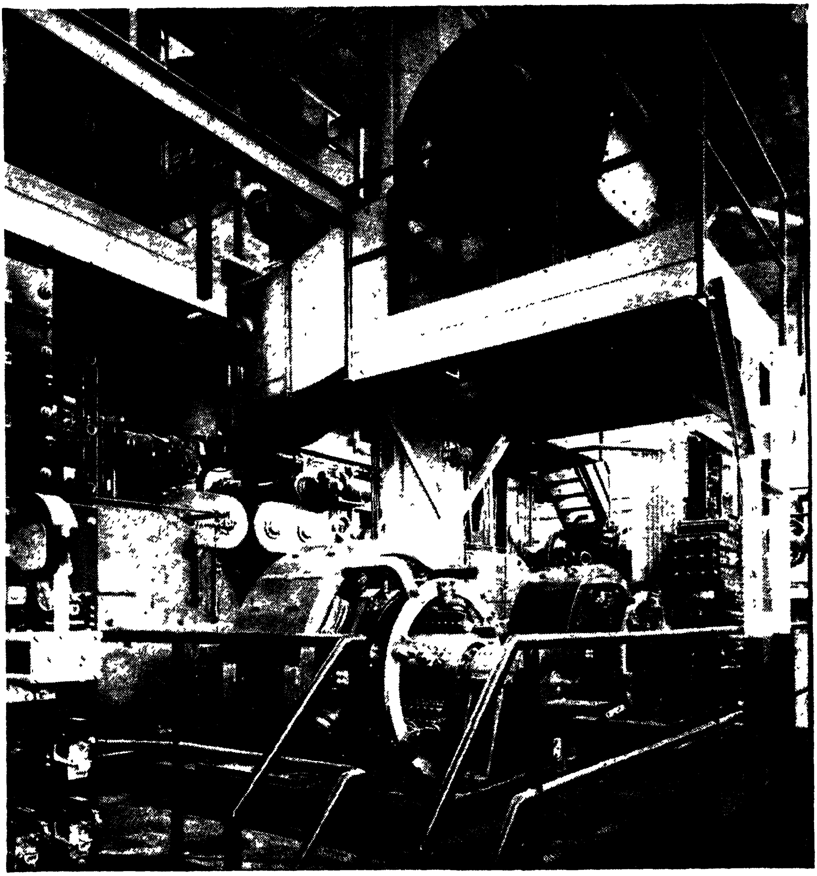


Fig. 19-1. Typical Illustration of the Application of Electrical Equipment to a Processing Industry (Paper Mill).

operating processes within the buildings is pretty much the same, regardless of whether the power is purchased from a public utility or taken from a local power plant.

Design considerations in industrial wiring systems

Industrial wiring systems have many of the design objectives common to all engineering design. This includes economy of first cost, low operating cost, satisfactory service continuity,

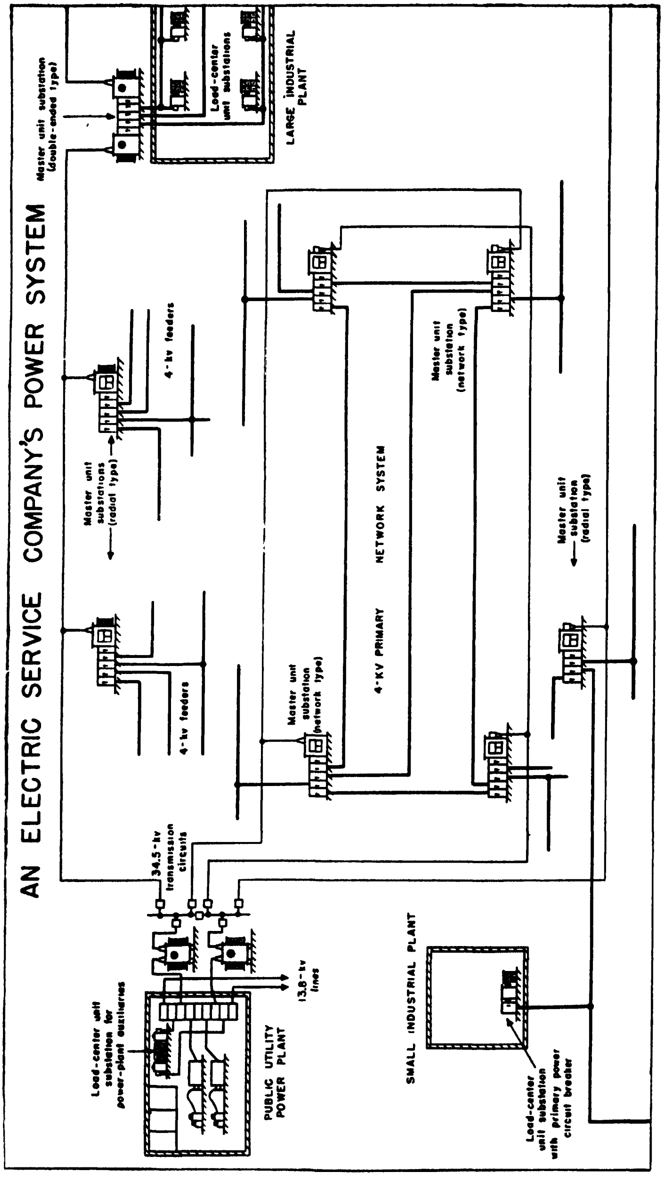


FIG. 19-2. A One-Line Diagram of a Typical Power System Supplying an Industrial Load.

reliability, expansion, and safety considerations. In addition, the design must provide for satisfactory voltage regulation, must conform to the National Electrical Code, and should follow the recommendations of recognized national committees which have indicated standards based on the cumulative experience of the members.*

The voltage regulation on lighting circuits should not exceed 5 per cent, as at the lower voltage a reduction of approximately 15 per cent in illumination is obtained with incandescent lamps. Fluorescent lamps are not as sensitive to voltage variation, but these should not have a voltage variation of more than 10 per cent above or below their rated voltage. Electric motors will also operate satisfactorily on voltages 10 per cent above or below rated voltage. In general, however, the voltage variation on a new installation should not exceed 3 per cent above or below the normal or rated voltage.

Where the wiring problem represents an increase of load on the present system or changes in the location of equipment, consideration must be given to the fundamental adequacy of the present wiring system. The decision to discard a wiring system which has become inadequate is always difficult, since it involves an item of major expense which must be justified on the basis of reduced operating cost, improved electrical performance (which can be evaluated in terms of lowered production costs), the elimination of fire hazards, greater safety to employees, reduction of future expenditures (if the inadequacy of the present system may be definitely predicted), or other similar considerations.

It is very difficult if not impossible to give a dollar value to some of these considerations, and usually all economic predictions are subject to widely varying interpretation by different individuals in the organization. It is necessary, therefore, that the engineer work closely with the management in evaluating intangible factors and that he have a clear idea of most probable future development as estimated by the management.

If the contemplated addition or change to the wiring system does not justify a major change, but if such a major change is anticipated in the future, the present change must be made in a manner that will fit into the future plan, or made as a temporary

* The A.I.E.E. report *Electric Power Distribution for Industrial Plants* is one such report which might be used to advantage in studying design standards for any particular installation.

expedient. *Temporary expedients are responsible for much of the bad industrial wiring at present, as the temporary expedient has a way of continuing temporary over a long period of years.* If the present wiring system is adequate, then there is no problem, since the voltage, phase, general wiring method, and other features are determined.

Selection of voltage for distribution

Where a new installation, or a major revision of wiring plan, is under consideration, one of the first decisions to be made is

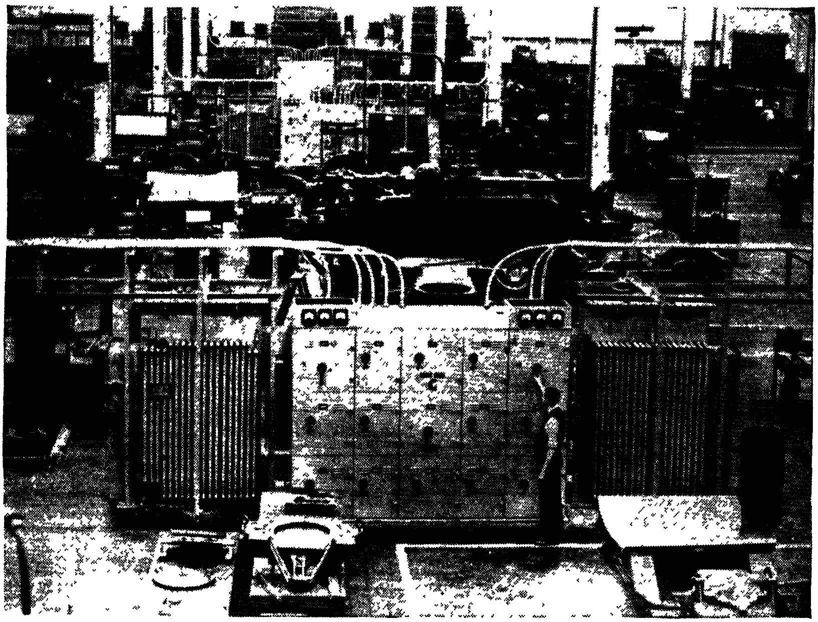


FIG. 19-3. 2000-kva Load-Center Unit Substations for Secondary Service, Showing 3 of 18 Units Installed in an Industrial Plant.

whether the system shall be entirely low voltage or whether a combination of high voltage and low voltage is to be preferred. The design criteria mentioned above will determine the final decision on this question.

The use of a combination of voltages cannot be justified in small installations. Recent developments in transformer design and in packaged or standard industrial substations have, however, made the combination of voltages much more attractive for intermediate-sized installations. Transformers are now

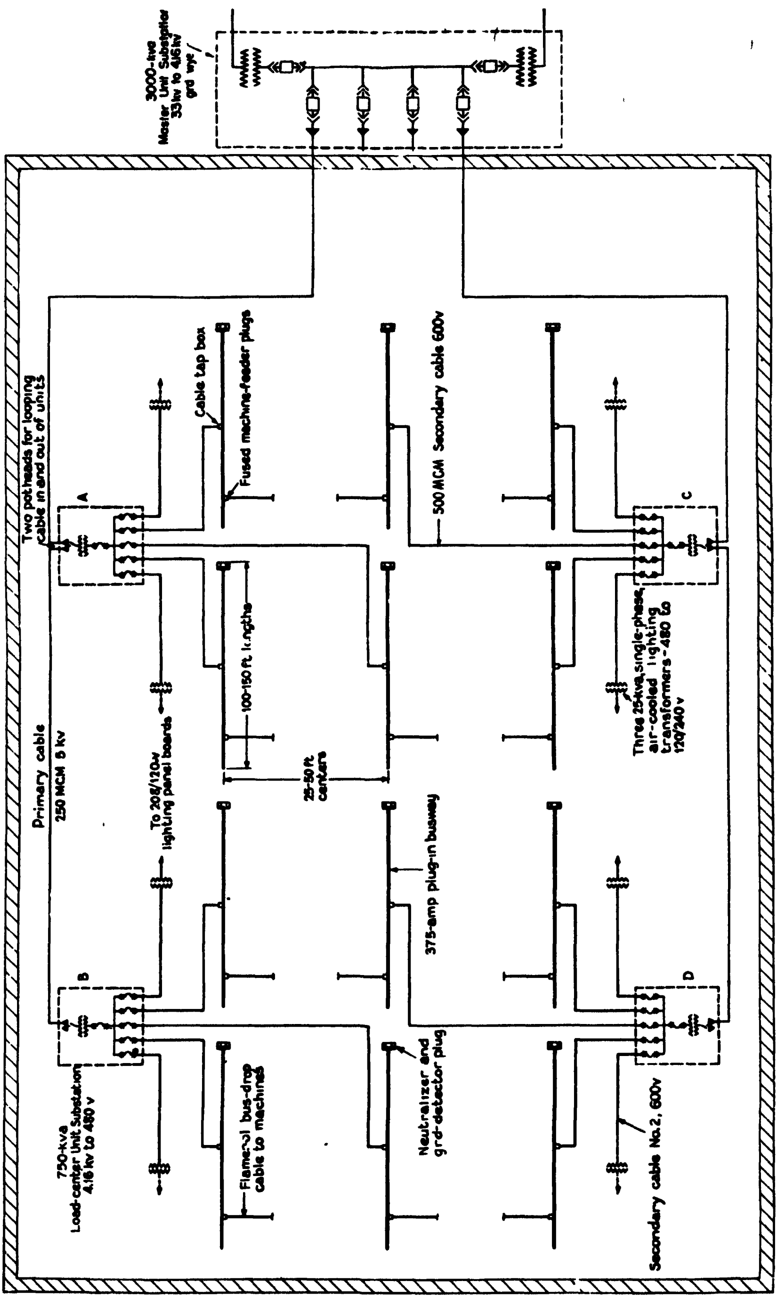


FIG. 19-4. A One-Line Diagram of a Typical Power Distribution System for Industrial Use.

available in air-cooled styles and with noninflammable cooling liquid which may be located in the center of a production floor without a transformer vault. The transformer and associated low-voltage switches or panel board may be located in a neat and compact metal enclosure which occupies little floor space and which gives a minimum distance from the transformer to the points of power utilization.

The advantages of supplying power at high voltages to small transformer substations located at the load center include increased efficiency and savings in the power bill, improved voltage regulation, which will usually be reflected in better illumination and increased production, and greater flexibility for load expansion or changes in production processes. In many cases it will be found that in addition to the above advantages the first cost is lower than the secondary or low-voltage installation with its associated long circuits of heavy copper wire.

For most industrial installations of intermediate size using high and low voltages, the high voltage will be 2400 v or 4160 v, whichever is available. The use of combined high and low voltages will not ordinarily be justified for installations of less than 100-kva capacity unless this load is located at some distance from the power source. Where several hundred kilovolt-amperes of load are located over a fairly large floor area, the possibilities of establishing transformer substations in different load centers should be carefully considered. The chief disadvantage of an intermediate voltage such as 460 v has been the danger to employees who may come in contact with this voltage through defective insulation. With improvements in equipment design and wiring techniques, however, the popularity of 460 v is increasing.

Selection of the type of secondary system

The single-phase system supplying 115–230 v may be found to be economical for lighting load only or for power loads below five kilowatts. In general, however, all industrial wiring systems requiring the attention of the engineer will be three phase.

The 120-v single-phase, 208-v three-phase four-wire system was described in some detail in Chapter 8. This system has the great advantage that both light and power can be taken from the same wires. Where the lighting load is predominant,

it is usually the most satisfactory secondary system. It does have the disadvantage that the nominal three-phase voltage is below the normal 220-v rating of the motors. Since induction motors will not operate satisfactorily on a voltage more than 10 per cent below rated voltage, only an 8-v total line drop is permissible from the substation to the motor.

Where the majority of the load consists of motors, it is usually preferable to use a nominal 220-v three-phase three-wire system with an actual no-load voltage of about 240 v. This gives much better performance on the motors but normally requires that the lights be supplied through insulating transformers. This limitation is not serious where the lighting load is small.

Where both lighting load and power load are heavy, it may be found advantageous to use 460 v for both the motor load and the main lighting load. This is permissible if the lights are not less than 8 feet from the floor and do not have switches in the lighting fixture.

Automatic switches for circuit protection

All electric circuits are subject to short circuits and overloads which may become a dangerous fire hazard. Each circuit is provided with a fuse or an automatic switch which will disconnect the circuit from the power source in case excess current flows. The oldest, the cheapest, and still a very important method of providing this circuit protection is by fuses that will melt and disconnect the circuit when the current exceeds the rated value. The disadvantage of the fuse is that it is expensive to replace and that it usually requires the services of an electrical maintenance man to put the circuit back in operation. This is costly in waste time and lowered production, while the workmen and machines being served by the circuit wait for power to be restored.

Usually when a circuit fails due to a short circuit or overload, the workmen know what caused the difficulty. If the particular motor involved is disconnected, the rest can continue to work, provided power is returned to the circuit. It is quite usual, therefore, for industrial concerns to use automatic switches instead of fuses. When this is done, the foreman can supervise the disconnection of the faulty equipment from the line and then close the switch so that the remainder of the circuit can immediately continue operation.

The National Electrical Code

The National Board of Fire Underwriters has prepared and maintains continual revisions of the National Electrical Code. This code establishes minimum safe practices for wiring under

FIG. 19-5. An Under-Floor Conduit Installation Before Concrete Has Been Completely Poured.

various conditions in order that hazards of fire and personal injury due to electrical causes may be reduced to a minimum. This document should be carefully studied by each engineer who is responsible for decisions on wiring policy.

The National Electrical Code does not specify general

FIG. 19-6. An Under-Floor Duct System in Process of Installation.

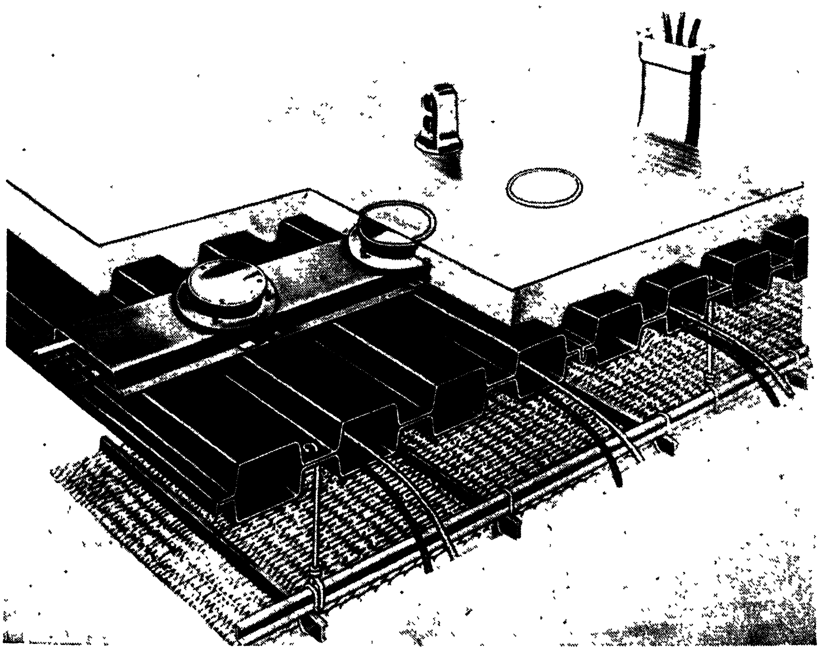


FIG. 19-7. A Sectional Sketch of Cellular Steel Floors with Raceways.

design considerations, but it does indicate safe practices in wiring. It is necessary to abide by the code in order to conform to most municipal laws and in order that fire insurance contracts may not be voided.

Methods of wiring

Since a number of methods of installing electrical conductors in buildings are approved by the National Electrical Code, it is necessary to choose among them.

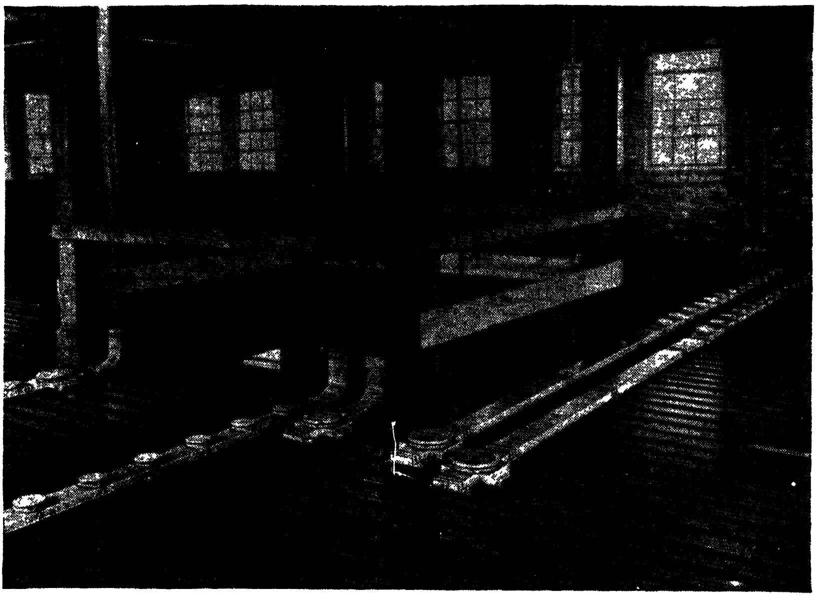


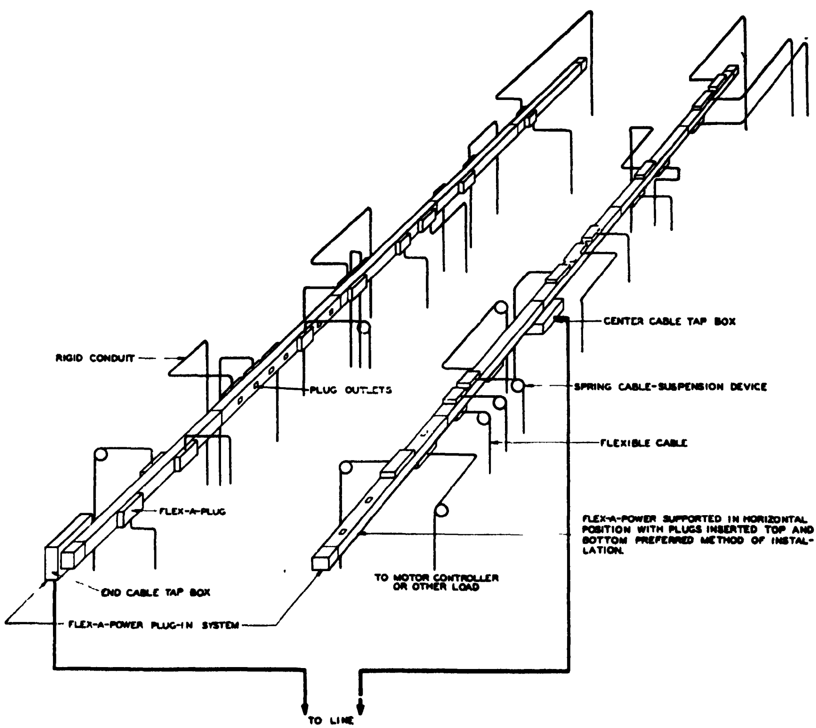
FIG. 19-8. An Installation of Cellular Steel Floors Before Concrete Surface Has Been Poured.

The most popular method of wiring for industrial service is known as conduit wiring. In this system the wires are enclosed in steel pipe or conduit, which protects them from mechanical injury and which also protects the building from fire in case of short circuits within the conduit. This system has the advantage of being the least expensive of the really satisfactory methods of installing electrical wiring. It is not flexible, since, if a load is moved, it is necessary to tear out the conduit and reinstall it in a new location. It has the further disadvantages that it interferes with passageways if laid on the floor, and, unless

carefully planned, is unsightly and interferes with illumination when mounted on the ceiling. Flexible conduit is usually limited in its use to making connections from the rigid conduit to the individual machines.

To meet these objections, in many modern buildings underfloor conduit is laid out in grids so that no portion of the floor is more than five or six feet from an available conduit opening. The flexibility of this system is high. The cost is higher than conduit but is justified when new high-quality industrial buildings are being constructed.

Cellular steel floors are now available, which not only provide the structural support for the floor but also act as raceways for electric wiring, air, gas, and water services. This type of flooring is a part of the building structure and is ideal for many types of laboratory and industrial buildings. The construction details are shown in the drawing of Fig. 19-7. An actual

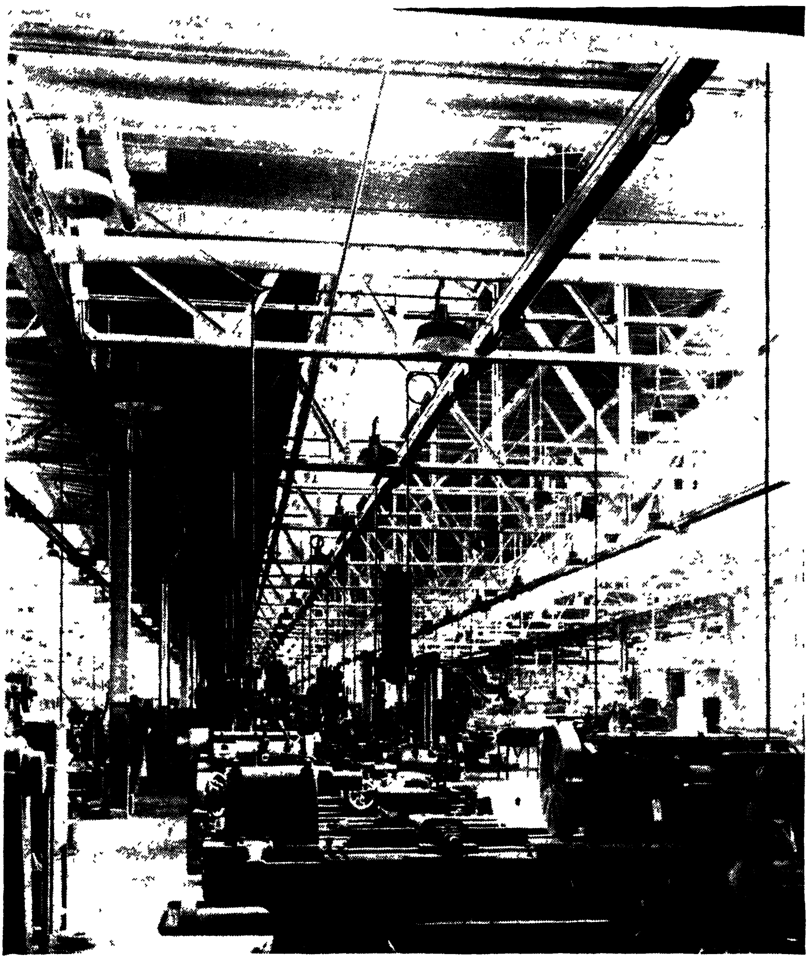


(a)

Fig. 19-9. (a) Typical Industrial Distribution System Utilizing Enclosed Busbar

installation is shown in Fig. 19-8, which is ready for the concrete surface to be poured. In such a floor, the wiring plan can be arranged so that no point on the entire floor area is more than a foot from a point of electrical service.

Several manufacturers have available a system of construction known as *busways.* In these, which are illustrated in Fig. 19-9, heavy bus bars are enclosed in steel or aluminum duct and supplied in prefabricated form. Regularly spaced openings are provided in the ducts so that connections can be made



(b)

(Flex-a-Power) Method of Wiring. (b) Typical Commercial Installation.

to supply local feeders through approved protective devices. Since the busway comes in prefabricated sections, which are connected in installation, it is not difficult to disconnect the sections and reinstall them in a new location or in a different pattern if the assembly line or production machines are moved or reorganized. For many installations this system is advantageous in spite of its greater cost as compared to conduit.

Armored cable is used for temporary construction and for installations of small capacity. It is not, however, commonly used for medium- and large-sized industrial installations.

Although open wiring is not entirely forbidden in industrial buildings, it is not favorably considered except in the most temporary construction. Even in temporary work, the engineer who approves this type of wiring is assuming considerable responsibility, since in temporary construction the fire hazards and the possibility of mechanical damage to the wiring are often unusually high.

Summary

Since the cost of electrical distribution is only about 2 to 10 per cent of the total cost of the industrial installation, and has so great an effect on the reliability and satisfactory operation of the entire plant, the best system possible is likely to be justified. Consult the public-utility engineers regarding the power sources available. Take advantage of the knowledge and experience of the sales engineers who supply electrical equipment and if the job is large, retain the services of a consulting electrical engineer. Plan for possible expansion and change in the industrial program. Get the best electrical system possible, because it will pay big dividends!

Short problems. (1) It is necessary to supply ten 300-w lamps from a 120 v single-phase source. What size wire would be required by the National Electrical Code?

(2) If the lights in problem 1 were located in a room 150 ft from the source or panelboard, what would be the voltage drop? Is this satisfactory? What size wire should be used?

(3) What size wire would be required by the code to supply ten 300 w lamps on a 120-240 v three-wire single-phase system? What would the voltage drop be if the lights are 170 ft from the supply and they are equally divided between the two sides of the line? Is this voltage drop satisfactory? What wire size would you actually recommend?

(4) A small assembly plant requiring 2 kw in incandescent lamps is to be located 500 ft from the main building of the industry. The power supply available in the main building is 220 v three phase three wire. How would you supply this load? Would you use the same system if the load were 10 kw? Give reasons for your answer.

(5) Two floors of an industrial building are fed by a single-phase three-wire 120–240-v line which is 400 ft long. The wire size is #8. The load on the upper floor, which is supplied by one side of the line, is 3 kva. The load on the lower floor, which is supplied by the other side of the line, is 2 kva. Determine the current in each of the three feed wires and the voltage supplied to the upper and the lower floor, if the voltage at the source is 120–240 on a three wire single phase line.

(6) It is necessary to supply a load of 10 kva in lights and fractional-horsepower motors to a new addition to the plant located 600 ft from the power source. The power available at this source is 230 v three phase three wire. Would you recommend that the building be supplied by single phase or three phase? Transformers to step from 230 v to 115–230 v may be assumed to cost $15 per kva in small sizes and wire to cost approximately 75¢ per pound of bare copper in the smaller sizes.

(7) A pump requiring a 50 hp motor is to be located on a canal 500 yd from the plant. The power supply to the plant is 2300 v three phase, which is stepped down to 120–208 v three phase four wire. (a) If the motor is supplied from the 208-v secondary, what sized wire is required to maintain the minimum voltage of 200, which is needed for satisfactory motor operation? (b) A step-up transformer to boost the voltage from 208 to 230 v may be obtained for $100. How much could be saved in the cost of the installation by stepping the voltage up so that the wire size required by the code would be adequate? Assume that the price of wire for these sizes is about $1 per thousand feet for each 1000 cir. mils of cross-sectional area. (c) What considerations would favor and what oppose the plan of supplying the motor by a 2300-v line with a step-down transformer whose cost is $600? (d) Would you consider the use of a 50-hp 2300-v motor?

Project problems. (1) Redesign the wiring system for a small industrial concern with which you are familiar.

(2) A large one-story building is available for use. It is divided into four equal sections, each 100 ft by 150 ft. Power at 2300 v three phase is available. The process has been laid out, and section D is for receiving and storage of raw materials. It requires 15 kw of lights and 5 kw of fractional-horsepower motors. Section C is the chief processing space and requires 30 kw in lights and 200 kw in medium-size induction motors for processing machines. Section B is

final processing and subassembly. It requires 30 kw for lights and 100 kw for power in medium and small motors. Section A is final assembly, packaging, and shipping. Lighting requirements are 20 kw and power is only 50 kw, half of which is in a baking oven for drying the final finish.

Make recommendations on a power-supply system which will give most satisfactory service with minimum cost. (The completeness of this can be gaged to meet the time available for study.)

Suggested Supplementary Reading

Electric Power Distribution for Industrial Plants, Bulletin of the American Institute of Electrical Engineers.

National Electrical Code of the National Board of Fire Underwriters. 1947. 85 John St., New York or 222 West Adams St., Chicago.

Tarboux, J. G., *Introduction to Electric Power Systems.* Scranton: International Textbook Company, 1944.

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CHAPTER 20

Electric Power—Economics and Maintenance

Elements in the cost of electric power

The basic elements of cost are the same for power generated by public-utility companies and by industrial plants. These are fuel, labor, investment costs, and maintenance. Their relative magnitude varies over a wide range depending upon the conditions of generation. Fuel costs are usually properly allocated, but sometimes lubrication cost, which is an associated item of operating expense, is not. In hydroelectric plants the item of fuel cost is omitted, but the investment costs are correspondingly increased. Investment costs are usually most difficult to evaluate correctly, particularly in industrial plants. Labor cost is likewise difficult to determine, where the operation and maintenance personnel work on both the power plant and the general plant operation. Maintenance is often deferred, and so the operation of the first few years may not be a correct criterion of average maintenance costs. Each of these items will be considered separately, and then some operating methods will be discussed which may assist in keeping these costs at a minimum.

Fuel costs. Except for hydroelectric-power generating stations, fuel is one of the most important costs. From the operating point of view it is probably the most important because it is a cost that can be reduced by maintaining maximum efficiency. It might be said that measurement is the first step to economy. Therefore, it is important to know the unit cost of fuel from week to week and month to month. When this is known, it is possible to determine whether operational efficiency is improving or dropping and thus to interpret the effects of different operating procedures.

Where electric power is a by-product, as where large amounts of process steam are needed, it is necessary in some way to allocate costs to electric power and to steam. Metering is usually a difficult problem, and important engineering decisions

often depend on costs, which in turn have been based on a division of charges made years previously without adequate information and without a realization of their importance. *Accurate information is the first prerequisite to intelligent engineering decisions.*

Labor costs. Labor costs are another important part of power costs. Particularly is this true in a small industrial power plant. Here the ratio of man hours to power output is much higher because of the smaller machines, and thus the labor charges are usually higher. When operating personnel are responsible for duties outside of the power plant, an allocation of costs is necessary, and considerable judgment is required to obtain an equitable distribution. In general it may be said that the little additional expense necessary to obtain high-quality operating personnel will pay big dividends in improved efficiency and reduced maintenance costs.

Investment costs. Investment costs involve depreciation rates, rates of return on investment, and so on, all of which are more dependent upon the stability of the enterprise than on the life of the generating equipment. Where the power plant is housed in the manufacturing buildings, it is always a question as to how much of the building charges should be assigned to the power plant.

It is quite reasonable to assume that the normal life of electrical equipment is twenty years, although much equipment is still in use after thirty or forty years. Usually the functional life of equipment is not as long as the life on the basis of "wearing out," and so the "functional life" is more important in determining investment costs. For instance, a power plant installed in an industrial plant might cost a million dollars. If the industrial plant became inoperative after five years, the salvage value of the power plant might be less than one hundred thousand dollars, and therefore a depreciation charge of 20 per cent of the investment would have been justified. Except in the more stabilized industries, an estimated life of industrial equipment beyond ten years is seldom justified. In the case of public utilities, it may be assumed that electric power will continue to be required in one factory or another. Thus the longer life and smaller depreciation charges are justified.

Rates of return on investment, sometimes referred to as interest and profit, are reflected also in terms of the functional life of the equipment. The profit part of the return is to com-

pensate for the risk of loss, and in the less stabilized industry the shorter probable life and the greater risks are closely associated. Therefore, rates of return on the investment of 6 to 10 per cent may be justified. This, added to a 10 per cent annual depreciation charge, will bring the annual investment charges to 15 or 20 per cent of the initial investment. In an industry that has been stabilized over a period of twenty years or more, these charges may be reduced on the basis of their experience.

In a steam-generating plant the costs will include the costs of boilers, turbines, generators, switchboards, and associated auxiliaries with the necessary wiring. Cost of a building to house the power plant and of land on which to place the building must also be included.

Maintenance. When estimating probable costs of a new installation, maintenance is one of the most difficult elements to determine. If equipment of a similar type has been in operation for ten years or more, and if accurate costs have been kept on such an installation, and, furthermore, if such costs are available to the engineer making the estimate, a reasonably accurate determination may be expected. Usually, however, these ideal conditions do not exist.

Operating maintenance has a habit of being greater than originally estimated, and so it is wise to be pessimistic rather than optimistic in making these estimates. In industrial power plants this is particularly true, since the job of the industry is to manufacture a product rather than to generate power; therefore the maintenance schedule of the power plant is usually subservient to the production schedule of the industrial plant. That this is proper does not make it any the less true, nor does it reduce the actual maintenance costs.

Consideration of such points emphasizes the importance of a carefully organized maintenance program. Although a comparatively small part of the total cost of power, maintenance can have a marked effect on the economy of the industrial plant by assuring continuous and uninterrupted electric service.

Power-generation equipment

The most important part of electrical generating equipment from the point of view of the production or operating engineer is the prime mover, which is beyond the scope of this text. For industrial plants these prime movers are usually steam turbines or Diesel engines. The turbine is a very high-

speed machine, while the Diesel engine is inherently a low-speed machine. This difference in speed affects considerably the proportions of the electrical generator to which they are connected, but either generator may be expected to give many years of trouble-free service with a minimum of maintenance if treated intelligently.

The electrical generator will in practically all cases be a 60-cycle alternator, since that frequency is standard for the nation. An exception might be a small plant which needed d-c motors and therefore chose to generate direct current for use throughout the plant. Since a-c generators are assumed, they will supply a distribution system as described in Chapter 19, and so, except for the generator circuit breakers to supply the power bus and distribution breakers to supply the power-plant auxiliaries, the distribution system will be the same.

Purchased power vs. manufactured power

Producing electric power is a specialized business just as is any other manufacturing process. It is not surprising, therefore, to find that plant superintendents chosen for their intimate knowledge of the industrial process should not normally be particularly familiar with the tricks of the trade of power generation. Furthermore, the responsibility of having to supervise a power-generation program with which they are not too familiar may detract from their efficiency in supervising the industrial program.[2] From the point of view of the operating personnel of an industrial plant, a reliable supply of purchased power is preferable to generating their own power, since they can then concentrate their energies on the main job.

From the point of view of investment costs, the purchased power is much to be preferred. The power plant will usually cost from one hundred to two hundred dollars per kilowatt of capacity, whereas a substation to take care of purchased power will be limited to ten or twenty dollars per kilowatt.[3] With most industrial projects, this reduction in capital required will be of considerable importance. They can usually invest their capital at a return that is greater than that invested in the power-generating equipment, and so the purchased power is attractive from the investment angle.

Another advantage of purchased power is that a power plant may block future desirable expansion of the manufacturing plant.[4] This is particularly true where space available for expansion is definitely limited.

To offset these advantages of purchased power, there are occasionally situations in connection with the manufacturing processes which permit the power to be generated as a by-product. In this case, the industrial company cannot afford to purchase power at a cost that is much higher than power that they themselves can produce. Cases of this kind include situations in which much process steam is used.[1] Here power can be produced by high-pressure or bleeder turbines supplied by high-pressure boilers and exhausting into the process steam lines. In other cases waste products can be used as fuel.[2] The decision to purchase or generate should be based on a careful and detailed engineering study if there is any doubt as to the correct decision.

Preventive maintenance

The maintenance of electrical equipment differs little in basic principles from any equipment-maintenance problem. A regular schedule of inspection is necessary to determine whether any deterioration has occurred and whether any repair or adjustment is required.

In the discussion of electric motors, mention was made of the generally low maintenance of these items of electrical equipment. Because they require attention so seldom, it is sometimes assumed that they never need to be inspected. This assumption is not correct. A proper maintenance schedule is necessary.

Where the loads are heavy and the motors become quite hot, the volatile material in the insulation is baked out and the insulation then becomes brittle. It is desirable that this volatile material be replaced periodically by new coatings of insulating varnish. The frequency of this treatment will depend upon the operating temperature and may vary from twice a year to once every five years.

In d-c motors, most of the maintenance centers around the commutator and brushes. It is important to keep the commutator smooth and clean, to keep the mica properly undercut, to maintain correct brush pressure, and to see that the brush surface is smooth and clean.

The important part of a maintenance program is to establish and follow vigorously a regular schedule of inspection and routine maintenance operations. If this is done, it is not likely that failure will occur at critical times during production.

Preventive maintenance is better than rebuilding machines after they have failed.

The maximum-demand charge

The power company must make an investment in power plant, substations, and power lines adequate to handle satisfactorily the maximum power required by each of its customers. Since this cost to the power company is not proportional to the energy used, most companies make a special charge to cover it. The charge is dependent upon the maximum load or maximum demand for power made by the customer. This maximum demand is measured by a meter and placed on the bill along with other elements of the cost.

The *maximum demand* is defined as the highest average load for any short specified period (usually fifteen minutes) during the billing period. Thus, a large load that lasted only two minutes might not cause an increase in the maximum demand if the average load at that same period were less than the maximum. The maximum demand may be determined on the basis of either kilowatts or kilovolt-amperes, depending upon the terms of the contract.

The young engineer can often save his company a considerable amount of money and make a good reputation for himself by intelligently scheduling heavy loads so that, at the time they are on, the nonessential loads are off. This procedure holds down the maximum demand and reduces the monthly power bill.

Example. An industrial plant is buying power on a contract that specifies a charge of \$1.50 per month per kva of maximum 15-min average demand within the month. The present maximum demand is averaging 875 kva and is found to occur at the beginning of the day when all motors are started at the same time. It is found that, if these motors are started in two groups spaced at one-half hour intervals, the maximum demand (for fifteen minutes average) is reduced to 700 kva. What is the annual savings?

Solution: (1) Monthly savings amount to

$$175 \text{ kva @ } \$1.50 = \$262.00.$$

$$(2) \text{ Annual savings} = 12 \times 262 = \$3,160.00.$$

Exercise 20-1. Determine the cost of starting and running a large milling machine for one hour per month if it draws 10 kva from the power line. Assume that this will increase the demand by 10 kva and neglect the energy charges.

Exercise 20-2. How much will a 10-kva load lasting 3 min increase a 15-min maximum demand?

Power-factor correction

The amount of equipment needed to supply a given load will depend upon the current, since the limitation on most electrical equipment is that of heating, an important element of which is proportional to the square of the current. Thus it will require approximately 20 per cent more equipment to supply a given load at 80 per cent power factor than it would at 100 per cent power factor. The maximum demand is often based, therefore, on maximum kilovolt-amperes rather than on kilowatts. When it is based on kilowatts, there is usually a clause in the power contract which penalizes for low power factor, or gives a bonus for high power factor.

Thus it becomes of considerable importance to the industrial plant to maintain as high a power factor as possible. Since most of the power load is usually composed of induction motors which have an inherently lagging power factor, this is not always an easy problem. As was mentioned in the chapter on induction motors, the exciting current which lags by almost 90 degrees is approximately constant, while the power input or in-phase component of the current varies with the characteristics of the load. Thus, if an induction motor is lightly loaded, it will have a low power factor. The first thing to do in correcting low power factor is to see that the machines are not run by motors that are larger than necessary.[1] Not only will the power factor be improved if the induction motors are properly loaded, but the efficiency will be improved as well. Thus, in selecting a motor to do a specific job, it should be expected that it will operate normally at nearly full load. If there are short periods (10 minutes or less) in which 150 per cent of full load or even more is required, the average induction motor will take care of it satisfactorily. The torque-speed curves of induction motors in Chapter 12 showed that these motors would provide 200 per cent or more of full-load torque for short periods of time.

Although it is desirable to maintain proper loading on induction motors, this will not always solve the problem. It is often economical to install capacitors to supply the exciting current locally and thus improve the power factor.[2] Sometimes it is advisable to do this as a unit at the substation, but often it is advisable to install the capacitors at the ends of the line or

near the load center on a particular feeder. The closer the capacitors are placed to the induction motors, the induction furnaces or arc furnaces, or other low-power-factor loads, the more effective they will be.

If there is a pump, or some similar load of considerable size that operates continuously at constant speed, it is often desirable to use a synchronous motor drive. If the motor has been designed for this purpose, it is possible to overexcite it, and thus it will take a leading current which will neutralize a portion of the lagging load.

Example. An industrial plant draws 150 kw from a 208-v three-phase power line. The power factor is 0.7.

(a) How many kva of capacitors are needed to bring the power factor to 0.9?

(b) If, instead of using static condensers, an additional load of 50 kw is to be supplied by a synchronous motor at 0.7 power factor leading, determine the final power factor of the plant load.

Solution: If θ is the power-factor angle,

$$\cos \theta = \frac{\text{kw}}{\text{kva}},$$

and

$$\sin \theta = \frac{\text{rkva}}{\text{kva}},$$

where rkva is the reactive kilovolt-amperes.

(1)
$$\text{kva} = \frac{\text{kw}}{\cos \theta} = \frac{150}{0.7} = 214.$$

(2) The reactive kilowatt-amperes at 0.7 power factor is

$$\begin{aligned}\text{rkva} &= 214 \sin \theta = 214 \sin 45.5° \\ &= 153.\end{aligned}$$

(3) The reactive kva at 0.9 power factor is

$$\begin{aligned}\text{rkva} &= \frac{150}{0.9} \sin (\cos^{-1} 0.9) = \frac{150}{0.9} \sin 26° \\ &= 73.5.\end{aligned}$$

(4) Reactive kva which must be neutralized to obtain 0.9 power factor is

$$153 - 73.5 = 79.5 \text{ rkva}.$$

Since capacitors have approximately 90° leading current, 80 kva of capacitors would neutralize the same amount of inductive rkva. (*Ans.*)

(5) In part (b), the additional load would have 50 kw and a leading rkva of

$$\text{rkva} = \frac{50}{0.7}\sin(\cos^{-1} 0.7) = \frac{50}{0.7}\sin 45.5$$
$$= 51.$$

(6) The total rkva is the difference between the lagging and leading rkva which is

$$153 - 51 = 102 \text{ rkva lagging.}$$

(7) Total kw = 150 + 50 = 200.

$$\text{power factor} = \cos\left(\tan^{-1} \tfrac{102}{200}\right) = \cos 27^\circ$$
$$= 0.89. \qquad (\textit{Ans.})$$

Exercise 20-3. A group of induction motors is drawing 22 kw from a 440-v three-phase power line. The power factor is measured and found to be 0.65. What kva of capacitors is needed to raise the power factor to 0.8?

Exercise 20-4. An industrial plant is being penalized (in the power bill) for a low power factor. The present total load is 250 kw at 0.6 power factor. A 75-hp induction motor is driving a compressor at an average input of 50 kw and a power factor of 0.75 lagging. This motor can be replaced by a synchronous motor operating at 0.7 power factor leading. Will this correct to total power factor of 0.80 or better, which is required for a normal rate by the power company?

Exercise 20-5. Determine the annual savings due to the capacitors in exercise 20-3 if the demand charge is $1.40 per kva per month.

Exercise 20-6. If the penalty in exercise 20-4 is in the form of a kva charge for demand, and if the demand charge is $1.50 per kva per month, determine the maximum cost that would be justified in shifting to a synchronous motor. Assume that a return on the investment of 18 per cent is required. This includes depreciation.

Suggested Supplementary Reading

Justin, J. D., and Mervine, W. G., *Power Supply Economics.* New York: John Wiley & Sons, Inc., 1934.

Morrow, L. W. W., *Electric Power Stations.* New York: McGraw-Hill Book Company, Inc., 1927.

Rogers, P. L., *Power-Factor Economics.* New York: John Wiley & Sons, Inc., 1939.

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